How many financial advisers do you need?

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Abstract

We study a rational expectations equilibrium economy where investors rely on suggestions of financial advisers to construct their investment strategies. Advisers observe signals about fundamentals and communicate the strategy that optimizes the expected utility of the investors given that information. Investors are of bounded rationality as they use the sum-of-weights-equals-one heuristic and are subject to price information neglect. Under these constraints, they optimally aggregate all strategies suggested by their advisers. We study how many advisers an investor should consult with and how much to expend on each of the advisers. When information is exogenous, investors should consult at least two advisers even if there is a large difference in the precision of their signals. However, it is not optimal to consult with all possible advisers unless their signal precision is relatively homogenous. When information is endogenous, it is typically the case that investors consult with a small number of advisers and spend an equal amount on each of their advisers under sufficient convexity of the information acquisition cost function. For example, under quadratic information acquisition costs, it is optimal to consult with exactly two advisers.

Keywords: Rational Expectations Equilibrium, Investment Advice, Optimal Aggregation, Information Acquisition

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1 Introduction

Many investors rely on financial advisers to form their investment strategies. How many financial advisers should one consult with? And how much to spend on each adviser? To goal of this paper is to formally study these questions within the framework of a rational expectations equilibrium economy.

We extend the classical model of Hellwig (1980) by considering two types of agents: Investors and financial advisers. Investors trade over a single investment period between a risk-free asset that is in perfect elastic supply and a risky asset in stochastic, normal supply. The key feature of our model is that investors cannot construct their own investment strategies. Instead, they rely on their financial advisers to do so. Advisers observe private signals about the fundamental of the risky asset and, based on this information, suggest an investment strategy back to investors. We suppose that advisers act in the best interest of their clients by suggesting an investment strategy that optimizes the expected utility of the investor they are consulting.

Investors in our model are not fully financially literate and of bounded rationality. They cannot process information about fundamentals themselves and consequently do not attempt to infer signals from the strategies suggested by their advisers. Investors are assumed to only understand and operate on investment strategies. Upon receiving suggested investment strategies from all of their advisers, investors then optimally aggregate suggested strategies into the strategy that is eventually implemented under two constraints of bounded rationality: the sum-of-weights-equals-one heuristic and price information neglect. The sum-of-weights-equals-one heuristic states that the weights given to suggested strategies must add up to one, while price information neglect states that investors do not take into account information contained in the price beyond what is already incorporated in the strategies suggested by their advisers.

We herein consider two basic setups: One where the precision of signals obtained by advisers is exogenously given and another where it is endogenous and controlled by the investors. The first setup applies for investors that, for example, read about a suggested investment strategy in the financial press. In this setting, it is plausible that the investor has knowledge about the precision of the advisers inferred from their reputation and past performance. But the investor cannot influence this precision. In the second setup, investors can affect the precision of their advisers at a cost. The larger the expense on a given adviser, the more precise the signal based on which the adviser suggests an investment strategy. This setting applies when investors can direct and reimburse advisers to do further market research before suggesting investment strategies.

In the first setup with exogenous information, investors can observe suggested strategies of up to n financial advisers, and doing so is free of charge. We suppose that each investor knows the precision of signals based upon which advisers suggest investment strategies. With how many advisers should the investor consult in this setting? And how to optimally aggregate suggested strategies? We are able to answer both questions analytically and gain the following insights. First, the investor should always consult with at least two advisers, no matter how large the difference in the precision of their signals. This result is remarkable since all advisers in our model suggest investment strategies that are in the best interest of the investor and there are no safety or fraud concerns. Still, it is never optimal to only consult with a single adviser. Second, investors should not always consult with all available advisers. When the precision of an additional adviser is low relative to the typical precision of existing advisers it is best to ignore the suggestions of the additional adviser. Third, when the number of potential advisers is large, a given investor should consult with all of them if and only if they are relatively homogenous in terms of signal precision. In the case that the signal precision of advisers is completely identical it is optimal to consult with all of them and equal-weight their suggestions. Finally, we compare implied market quality measures with a benchmark economy that is identical to that of our model except that advisers directly invest themselves based on the signals they observe and gain the following insights: consultation improves price informativeness, and, when all the advisers in our economy have the same signal precision, increases investor welfare, but no impact on market liquidity and return volatility.

In the second setup with endogenous information, investors can again consult with up to n financial advisers and face a cost depending on the signal precisions of their advisers. This creates a tradeoff between accuracy of suggested strategies and resulting cost. Our goal is to investigate with how many advisers investors should consult with, how much to spend on each adviser, and how to aggregate their suggested investment strategies. We first address the latter two questions: Given that the cost function mapping expenses on advisers to precision of signals is sufficiently convex, it is optimal to spend an equal amount on each adviser and to give an equal weight to their suggested strategies. We then provide a characterization of the optimal number of advisers investors should consult with which only on the cost function mapping expenses on advisers to precision of signals, but not on other model parameters. For common choices of this cost function, it is optimal to consult with a small number of advisers. For example, under quadratic information acquisition cost, a common choice in the literature, it is optimal to consult with exactly two advisers, spend an equal amount on both of them, and equal weight their suggested strategies. Finally, we again compare implied market quality measures with a benchmark economy where advisers directly acquire information and invest themselves based on the signals they observe: Consultation reduces price informativeness and increases return volatility. Furthermore, consultation improves investor welfare and market liquidity in informationally inefficient markets.

Our paper is related to the vast literature on information sales (Admati and Pfleiderer 1986, 1988, 1990; Allen 1990; Cespa 2008; García and Sangiorgi 2011; Naik 1997). Admati and Pfleiderer (1986) analyze an information sale model where there is a monopolistic seller and regular traders. The seller can get some information about the fundamental but cannot trade in a speculative market. In contrast, regular traders cannot obtain private information unless they purchase information from the monopolistic seller. The authors find that adding personalized noise to the seller's information is optimal for the seller to maximize his/her profits within a broad set of selling policies. Following the pioneering work of Admati and Pfleiderer (1986), some related problems that have been considered are: Indirect sale where the seller creates a portfolio based on his private information and then sells shares to traders (Admati and Pfleiderer 1988, 1990); reliability problem where the buyers are uncertain about whether the seller has superior knowledge (Allen 1990); continuous-time/dynamic settings (Cespa 2008; Naik 1997); non-competitive economies (Admati and Pfleiderer 1988; García and Sangiorgi 2011); among others. The information seller in the above literature plays a similar role as advisers in our model. For example, the seller does not invest in the market and only sells information or shares of a portfolio to investors. However, there are two main differences. First, instead of buying information from the seller, investors in our model are assumed to be financially illiterate and cannot process information and create investment strategies, so that they have to consult with financial advisers for their suggestions on investment strategies. Second, the focus of the discussed stream of the literature is on how the seller designs selling strategies in order to maximize his/her profits, while our focus is on how many advisers an investor should consult with and how much to spend on each adviser.

Our paper also contributes to the recent strand of theoretical (Colla and Antonio 2010; Han and Yang 2013; Manela 2014; Ozsoylev and Walden 2011; Walden 2019) and experimental research (Halim et al. 2019) on the implications of voluntary[1] information sharing on market outcomes.[2] Ozsoylev and Walden (2011) analyze how the network connectedness of a large economy influences price volatility, trading volume, welfare, and other measures of interest. They find that the ex ante certainty equivalent of investors is either globally decreasing, or initially increasing and eventually decreasing in network connectedness. Manela (2014) analyzes the effect of the speed of information diffusion on the welfare of investors and shows that the value of information is hump-shaped in the diffusion speed. Walden (2019) considers a dynamic model for a rational expectations economy with decentralized information diffusion through a general network. He shows that more central investors make higher profits, and, consistent with the findings in Colla and Antonio (2010) and Ozsoylev and Walden (2011), that investors that are close to each other have more positively correlated trades. While these papers assume that information is exogenously given, Han and Yang (2013) and Halim et al. (2019) investigate the effect of social communication on market outcomes when information acquisition is endogenous. Han and Yang (2013) show that social communication reduces the endogenous fraction of informed investors and thereby harms market efficiency, reduces trading volume, and improves welfare. Halim et al. (2019) show that social communication provides an incentive for investors to free ride on other investors' information and consequently reduces the overall information in the market. Although our work is related to this literature, there are three main differences. The first main difference is that while investors in the above mentioned

[1]Differently, Bushman and Indjejikian (1995), Indjejikian et al. (2014), and Goldstein et al. (2021) consider strategic settings where some investors have endogenous incentive to voluntarily leak their information to other investors to increase their welfare by impacting prices.

[2]There also has been some empirical work on the effects of social communication on trading behavior of investors, see for instance, Hong et al. (2004), Hong et al. (2005), Heimer (2016), Pool et al. (2015), Ozsoylev et al. (2014), etc.

literature can process the information received from other investors, investors in our model are not fully financially literate and cannot process information themselves. The second difference is that while there are direct interactions among investors in the above mentioned literature, there is no direct interaction among investors in our model. Instead investors only interact with their advisers and then aggregate the suggested strategies. The third difference is that while the focus of the existing literature is on the impact of information sharing on market equilibrium, our focus is on the question of how many financial advisers investors need.

The remainder of this paper is organized as follows. In Section 2, we introduce the model of a rational expectations equilibrium economy populated by investors and their financial advisers. Our main results are in Sections 3 and 4, which treat the cases of exogenous and endogenous information acquisition respectively. We conclude the paper in Section 5. Further discussions on modelling assumptions are delegated to Appendix A. Appendix B contains all proofs.

2 The Economy

Our model builds on the finite-agent noisy rational expectations equilibrium economy of Hellwig (1980). This is a single-period model where a risk-free asset and a risky asset are traded by $h \in \mathbb{N}$ investors. The risky asset has fundamental value $\theta \sim N(0, 1/\tau_\theta)$, $\tau_\theta > 0$. The preferences of the investors in our model are represented by CARA utility functions, i.e., the utility investor i derives from the (stochastic) terminal wealth $W(x_i) = x_i(\theta - p)$ is given by

$$U_i\left(W(x_i)\right) = -\exp\left(-\rho_i x_i(\theta - p)\right),$$

where ρ_i is investor i's risk aversion coefficient and p is the publicly observable price of the risky asset. Due to the assumption of CARA preferences, we can assume without further loss of generality that the wealth of all investors is zero.

The key feature of our model is that investors do not create investment strategies themselves. This can be because they are either not fully financially literate, or because they face time and resource constraints.[3] Instead, investors in our model consult with financial advis-

[3]For example, financially illiterate investors cannot construct strategies themselves because they cannot correctly understand and interpret their observed information and are unable to translate them into meaningful signals about fundamentals.

ers, for example wealth managers, market experts, or robo-advisors, on how to invest in the market. Specifically, we assume that each investor i can consult up to n advisers indexed by $(i,1),(i,2),...,(i,n)$. We assume that the maximal number of advisers investors can consult with is identical for each investor for notational simplicity. Our results also hold when $n = n(i)$ differs across investors. In particular, our model allows for a subset of investors that are financially literate and construct their own investment strategies by setting $n(i) = 1$ for some $i \in \mathcal{I} \subseteq \mathbb{N}$ with the interpretation that adviser and investor coincide in this case. In other words, our results also hold in a more general setting where only a fraction of investors are financially illiterate, and financially literate investors observe private signals and construct their own strategies without consulting financial advisers.

The advisers of our model do not invest in the market themselves, but only provide advice to their clients in terms of suggested investment strategies.[4] After observing strategies suggested by their advisers, investors aggregate these strategies as a weighted average to determine their final, individual investment strategies. We assume that investors can communicate their risk preferences to their advisers[5] and that the advisers provide suggestions that are in the best interest of the investors given their preferences. That is, adviser (i, j) provides an investment suggestion to the investor i maximizing the expected utility of investor i given her own information further specified below.

Each adviser (i, j), $i = 1, \ldots, h$, $j = 1, ..., n$, observes a private signal $y_{ij} = \theta + \epsilon_{ij}$ about the fundamental θ, where the noise $\epsilon_{ij} \sim N(0, 1/\tau_{ij})$ is assumed to be independent across advisers and $\tau_{ij} > 0$ denotes the information precision of adviser (i, j).[6] The strategy constructed by

[4]In practice, investors typically obtain a combination of investment strategies and information from their advisers. But the investors of our model are assumed to be not fully financially literate and thus cannot interpret information correctly. Hence, investors directly follow suggested strategies when making investment decisions.

[5]Typically this is done by asking the investor a series of questions designed to infer the investor's risk preferences.

[6]Our results also hold under a more general setting where advisers can suggest strategies to multiple investors (i.e., $y_{i_1 j_1} = y_{i_2 j_2}$ for some $i_1 \neq i_2$ and $j_1 \neq j_2$) provided that the total number of advisers in the economy is sufficiently large, each adviser suggests strategies to a limited number of investors, and information is either exogenous or investors are homogenous in terms of risk aversion. For example, our results also apply to the setting where multiple investors share the same adviser pool. In this case, the number of advisers can be smaller than the number of investors.

adviser (i, j) and communicated to investor i is allowed to depend on both the private signal and the public price of the risky asset, i.e., $x_{ij} = x_{ij}(y_{ij}, p)$. We will subsequently consider economies with *exogenous information* (Section 3), where τ_{ij} is exogenously given for each (i, j), and economies with *endogenous information* (Section 4), where each investor pays an amount $c(\tau)$ to an adviser such that the adviser gathers information resulting in a signal with precision τ. To prevent prices from fully revealing, there is per-capita random supply u in the market satisfying $u \sim N(0, 1/\tau_u)$, $\tau_u > 0$. We suppose that the random supply is independent of other random variables θ and ϵ_{ij}, $i = 1, ..., h, j = 1, ..., n$.

We assume that a single adviser does not impact prices by suggesting an investment strategy and therefore does not consider price impact. To justify this assumption, we adopt the large economy setting by considering $h \to \infty$ as in Hellwig (1980) and Ozsoylev and Walden (2011).

Assumption 1. *We assume that the following holds as the number of investors goes to infinity, $h \to \infty$.*

(i) There exists $m \in \mathbb{N}$ and a finite number of coefficients of risk aversion $\rho_k^\diamond > 0$, $k = 1, \dots, m$ such that $\rho_i \in \{\rho_1^\diamond, ..., \rho_m^\diamond\}$ for all $i \in \mathbb{N}$.

(ii) There exists $0 < \lambda_k < 1$, $k = 1, \dots, m$, with $\sum_{k=1}^m \lambda_k = 1$ such that the fraction of investors with $\rho_k^\diamond$ converges to λ_k, $k = 1, \dots, m$, i.e., $\lim_{h\to\infty} \frac{1}{h} \sum_{i=1}^h \mathbf{1}_{\{\rho_i = \rho_k^\diamond\}} = \lambda_k$.

When information is exogenous, we make the following additional assumptions.

(iii) There exists $v \in \mathbb{N}$ and a finite number of signal precision profiles $\boldsymbol{\tau}_z^\diamond = (\tau_{z1}^\diamond, ..., \tau_{zn}^\diamond) \in \mathbb{R}^n_{>0}$, $z = 1, \dots, v$, such that $(\tau_{i1}, ..., \tau_{in}) \in \{\boldsymbol{\tau}_1^\diamond, ..., \boldsymbol{\tau}_v^\diamond\}$ for all $i \in \mathbb{N}$.

(iv) There exists $0 \le \omega_{kz} \le 1$, $k = 1, \dots, m$, $z = 1, \dots, v$, such that $\sum_{z=1}^v \omega_{kz} = 1$ for every $k = 1, \dots, m$ and the fraction of investors with risk aversion $\rho_k^\diamond$ that are consulted by advisers with signal precision profile $\boldsymbol{\tau}_z^\diamond$ converges to ω_{kz}, i.e.,

$$\lim_{h\to\infty} \frac{1}{h} \sum_{i=1}^h \mathbf{1}_{\{\rho_i = \rho_k^\diamond\}} \mathbf{1}_{\{(\tau_{i1}, \dots, \tau_{in}) = \boldsymbol{\tau}_z^\diamond\}} = \lambda_k \omega_{kz}, \quad k = 1, \dots, m, \;\; z = 1, \dots, v.$$

Requiring that the number of profiles of risk aversion and signal precisions remains finite is only necessary when information acquisition is exogenous. In the case of endogenous information acquisition, it is sufficient to assume a finite number of coefficients of risk aversion across the economy. We will later observe that this then automatically leads to the finite types of signal precision when information is endogenous, cf. Section 4.

Each investor i observes the investment strategies suggested by his/her advisers and then aggregates them in order to maximize expected utility. A fully rational investor knowing how his/her advisers derived their suggested strategies would first infer the signals they observed and then compute rationally optimal investment strategy based on all available information. Because of linearity, the resulting rationally optimal investment strategy would be of the following form

$$\sum_{j=1}^{n} a_{ij} x_{ij} - \varphi p, \tag{1}$$

for some $a_{ij} \geq 0$ and $\varphi \in \mathbb{R}$. For fully rational investors, the setting where advisers communicate strategies is essentially equivalent to models of information sharing studied in the literature (Han and Yang (2013); Ozsoylev and Walden (2011)).

However, investors in our model are not fully rational due to a lack of financial literacy or time and resource restrictions. We consider two sources of bounded rationality in our model: The *sum-of-weights-equals-one* heuristic and *price information neglect*.

The sum-of-weights-equals-one heuristic states that the weights given to suggested strategies must add up to one, formally that $\sum_{j=1}^{n} a_{ij} = 1$. This heuristic is based on the principles of normalization and relative weighting that are well-established in cognitive psychology. The constraint $\sum_{j=1}^{n} a_{ij} = 1$ also follows when, in the event that every adviser suggests the same strategy, the investor will adopt it as his/her investment strategy.

Naturally, the strategies advisers suggest depend on their private signals and the price of the risky asset. Investors know this, i.e., they are aware that suggested strategies already depend on the price of the risky asset. Price information neglect then states that they do not take into account further information contained in the price *beyond* what is already incorporated in the strategies suggested by their advisers, formally that $\varphi = 0$.

As we will show in Proposition 10 in Appendix A.1, for an investor adopting the sum-of-weights-equals-one heuristic, price information neglect is optimal and thus no further loss in rationality. Motivated by this observation, we herein focus on investors that are following the sum-of-weights-equals-one heuristic and exhibit the price information neglect. That is, after observing the strategies suggested by his/her advisers x_{ij}, $j = 1, \ldots, n$, investor i decides on the weights $a_{ij} \geq 0$[7] satisfying the sum-of-weight-equals-one constraint $\sum_{j=1}^{n} a_{ij} = 1$ and then aggregates suggested strategies to

$$x_i^* := \sum_{j=1}^{n} a_{ij} x_{ij}.$$

In Appendix A.2, we demonstrate that our main results qualitatively still hold when investors exhibit price information neglect alone, i.e., without the constraint $\sum_{j=1}^{n} a_{ij} = 1$ that weights must sum up to one.

The investors determines the weights $a_{ij} \geq 0$ in order to maximize his/her expected utility. Note that, in order to calculate the expected utility resulting from an aggregation of suggested strategies, it suffices that the investor knows the joint distribution of suggested strategies x_{ij} of his/her advisers, the fundamental θ and the price p. In particular, knowledge about the distribution of signals is not required. It seems plausible that the investor learns this joint distribution from past observations and experience.

Some financial advisers do not only provide investment suggestions but also help investors to implement investment strategies. This is in particular the case for the emerging industry of robo-advisors, see, e.g., Capponi et al. (2022); D'Acunto et al. (2019); D'Acunto and Rossi (2021); Dai et al. (2021); Liang et al. (2023) for a recent literature discussing the interaction between robo-advisors and their human clients. Note that this setting can also be covered by our model by interpreting $a_{ij} x_{ij}$ as the amount investor i transfers to adviser (i, j) which is then invested in the risky asset by the adviser on behalf of the investor.

3 Exogenous Information

In this section, we consider the setting where the signal precision of advisers are exogenously given. We for now assume that investors know the signal precisions of their advisers. A discussion of the case where investors do not have full knowledge of the signal precisions of their advisers is postponed to the end of this section. We first introduce the notion of an

[7] The constraint $a_{ij} \geq 0$ must hold when the investor buys (or sells) the stock, provided that all advisers suggest to buy (or sell) the stock (no matter how much).

advice-based equilibrium with exogenous information and then discuss results on equilibrium existence, suggested equilibrium strategies, and equilibrium prices.

Definition 1. *An advice-based equilibrium with exogenous information is a tuple* $\big((x_{ij}, a^*_{ij})_{i=1,...,\infty,j=1,...,n}, p\big)$ *such that*

(i) for each i *and* j*,* x_{ij} *maximizes the expected utility conditional on the private signal of adviser* (i, j) *and price* p*, i.e.,*

$$x_{ij}(y_{ij}, p) \in \arg\max_x \mathbb{E}[-\exp\left(-\rho_i x(\theta - p)\right) |y_{ij}, p],$$

(ii) for each i*, the weights* $(a^*_{ij})_{j=1,...,n}$ *maximize the expected utility of the weighted average strategy taken by investor* i*, i.e.,*

$$(a^*_{ij})_{j=1,...,n} \in \underset{a_{ij}\geq 0, \sum_{j=1}^n a_{ij}=1}{\arg\max} \ \mathbb{E}\left[U_i\left(W\left(\sum_{j=1}^n a_{ij}x_{ij}(y_{ij}, p)\right)\right)\right],$$

(iii) the market clears, i.e.,

$$\lim_{h\to\infty} \frac{1}{h}\sum_{i=1}^h \left(\sum_{j=1}^n a^*_{ij}x_{ij}(y_{ij}, p)\right) = u.$$

Recall that we assumed a finite number of combinations of signal precisions of advisers servicing to the same investor in Assumption 1. We conjecture that in an advice-based equilibrium with exogenous information, all investors who consult with advisers with the same combination of signal precisions impose the same weight to the suggested strategies independent of their respective risk aversion. That is, $(a^*_{i_1 1}, ..., a^*_{i_1 n}) = (a^*_{i_2 1}, ..., a^*_{i_2 n})$, denoted as $(a^*_{z1}, ..., a^*_{zn})$, for any investor i_1 and i_2 whose advisers have the same combination $\boldsymbol{\tau}^\diamond_z = (\tau^\diamond_{z1}, ..., \tau^\diamond_{zn})$ of signal precisions, $1 \leq z \leq v$. This conjecture will later be verified.

We will show the existence of advice-based equilibria with exogenous information through the following two steps. The first step is to show that an equilibrium exists for any given weights $(a_{ij})_{i,j}$ that satisfy the above conjectured homogenous condition, i.e., there exists a tuple $((x_{ij})_{i=1,...,\infty,j=1,...,n}, p)$ such that the conditions (i) and (iii) (with a replacement of (a^*_{ij}) with the given (a_{ij})) in Definition 1 hold. The second step is to show that the optimal weights in (ii) depend only on the signal precisions of advisers, but not other model parameters, so that the conjecture above indeed holds. Substituting the optimal weights into the equilibrium strategies and the equilibrium price obtained in the first step, we will get an advice-based equilibrium with exogenous information.

As in the majority of the literature, we herein focus on *linear* equilibria, i.e., equilibria where strategies are linear functions of the signal and price and prices are linear in the signals and per-capital supply. Following the analysis in Hellwig (1980), Ozsoylev and Walden (2011), and Han and Yang (2013), we can infer the following convergence result as h increases to infinity. For any weights $(a_{z1}, ..., a_{zn})$ given to the strategies suggested by advisers with combination of signal precisions $\boldsymbol{\tau}_z^\diamond = (\tau_{z1}^\diamond, ..., \tau_{zn}^\diamond)$, $z = 1, ..., v$, the sequence of equilibrium prices of finite-agent economies converges in probability to[8]

$$p = \frac{1}{\Delta + \frac{\tau_\theta}{\Delta\tau_u + \rho}}(\Delta\theta - u), \tag{2}$$

where

$$\rho = \left(\sum_{k=1}^{m} \frac{\lambda_k}{\rho_k^\diamond}\right)^{-1} \tag{3}$$

and

$$\Delta = \sum_{k=1}^{m} \frac{\lambda_k}{\rho_k^\diamond}\left(\sum_{z=1}^{v} \omega_{kz} \sum_{j=1}^{n} a_{zj}\tau_{zj}^\diamond\right) \tag{4}$$

are the average risk aversion and the risk adjusted average signal precision in the economy. Interestingly, advisers' private signals enter prices in terms of averaged signal precision. All other things being equal, the larger the risk adjusted average signal precision, the greater is the weight of the fundamental in determining prices.

We further find that the adviser (i, j)'s suggested equilibrium strategy in the limit of a large economy equals to

$$x_{ij}(y_{ij}, p) = \frac{\mathbb{E}[\theta|y_{ij}, p] - p}{\rho_i \operatorname{Var}[\theta|y_{ij}, p]} = \rho_i^{-1}\left(\tau_{ij}y_{ij} - \left(\tau_{ij} + \frac{\rho\tau_\theta}{\Delta\tau_u + \rho}\right)p\right). \tag{5}$$

The first equality is the standard mean-variance portfolio strategy in the CARA-normality setting (see, e.g., Equations (6) and (11) in Grossman (1976)), and the second one follows from

[8]Recall that we assumed that all random variables have mean zero for notational convenience. Hence, there is no intercept in price function p.

(2) and the projection theorem for normal random variables. Additional computations yield the ex-ante welfare, i.e., the ex ante expected utility of the suggested strategy x_{ij} by the adviser (i, j):[9]

$$\begin{aligned}\mathbb{E}[U_i(W(x_{ij}))] &= \mathbb{E}[-\exp(-\rho_i x_{ij}(y_{ij}, p)(\theta - p))] \\ &= -\sqrt{\frac{\text{Var}[\theta|y_{ij}, p]}{\text{Var}(\theta - p)}} = -\left(\text{Var}(\theta - p)(\tau_\theta + \Delta^2\tau_u + \tau_{ij})\right)^{-\frac{1}{2}}. \end{aligned} \quad (6)$$

We next present an important result on the ex ante expected utility of the weighted average strategy taken by investors.

PROPOSITION 1. *Fix the weights $(a_{z1}, ..., a_{zn})_{z=1,...,v}$ and consider the resulting price p in (2), ρ in (3) and Δ in (4). Then for any weight and signal precision $(a_{ij}, \tau_{ij})_{j=1,...,n}$, the (ex-ante) expected utility of the weighted average strategy $x_i^* = \sum_{j=1}^n a_{ij} x_{ij}$ is given by*

$$\mathbb{E}[U_i(W(x_i^*))] = -\left(\text{Var}(\theta - p)(\tau_\theta + \Delta^2\tau_u + \tau_i^E)\right)^{-\frac{1}{2}} \quad (7)$$

$$= -\left(1 + \rho\alpha\beta + \tau_i^E\gamma\right)^{-\frac{1}{2}}, \quad (8)$$

where

$$\tau_i^E := \sum_{j=1}^n (2a_{ij} - a_{ij}^2)\tau_{ij}, \quad (9)$$

and

$$\alpha = \frac{\Delta\rho + \rho^2/\tau_u}{(\Delta^2\tau_u + \Delta\rho + \tau_\theta)^2}, \quad \beta = \frac{\tau_\theta}{\Delta\tau_u + \rho}, \quad \gamma = \frac{\tau_\theta + (\Delta\tau_u + \rho)^2/\tau_u}{(\Delta^2\tau_u + \Delta\rho + \tau_\theta)^2}. \quad (10)$$

Comparing (6), (7) and (9), we find that the expected utility of the weighted average x_i^* can be obtained by replacing the signal precision τ_{ij} in the expected utility of the strategy suggested by adviser (i, j) with what we term *equivalent signal precision* τ_i^E. While the strategy suggested by the adviser (i, j) enters the aggregated strategy with weight a_{ij}, the signal precision of adviser (i, j) enters into the equivalent signal precision with a weight $2a_{ij} - a_{ij}^2$. Clearly, this term is increasing in a_{ij}. Moreover, the higher the signal precision of advisers, the higher the equivalent signal precision τ_i^E. Surprisingly, although the weighted strategy x_i^* depends on the weights $(a_{ij})_{j=1,...,n}$, the signals $(y_{ij})_{j=1,...,n}$, the fundamental θ, and the potential price p,

[9] See also the proof of Lemma 2 in Rahi and Zigrand (2018).

the equivalent signal precision τ_i^E is only a function of the weights $(a_{ij})_{j=1,\ldots,n}$ and the signal precisions $(\tau_{ij})_{j=1,\ldots,n}$ of their advisers, independent of other model parameters.

Since this is a large economy, any particular investor's decision has no impact on the price p and amount Δ. These quantities are endogenously determined in equilibrium. This fact together with (7) and (9) imply that the optimal aggregation of suggested strategies can be determined by maximizing τ_i^E. Specifically, investors maximize the expected utility of the weighted strategies by choosing $(a_{ij}^*)_{j=1,\ldots,n}$ which solves the following maximization problem:

$$(a_{ij}^*)_{j=1,\ldots,n} \in \underset{a_{ij},j=1,\ldots,n}{\arg\max} \quad \sum_{j=1}^{n}(2a_{ij} - a_{ij}^2)\tau_{ij} \quad \text{s.t.} \sum_{j=1}^{n} a_{ij} = 1, a_{ij} \geq 0. \tag{11}$$

Next, we show how investors optimally aggregate the suggested strategies from their advisers.

Proposition **2.** *We have*

(i) Suppose $\tau_{i1} \geq \tau_{i2} \geq \cdots \geq \tau_{in}$, *and let*

$$t = \max\left\{ j \,\middle|\, 2 \leq j \leq n, 1 + \sum_{\ell=1}^{j-1} \frac{\tau_{ij}}{\tau_{i\ell}} > j-1 \right\}, \quad 2 \leq t \leq n.$$

Then the optimal solution to the optimization problem (11) is unique and given by

$$a_{ij}^* = \frac{a_{it}^*\tau_{it} + \tau_{ij} - \tau_{it}}{\tau_{ij}} = 1 - \frac{(t-1)\frac{\tau_{it}}{\tau_{ij}}}{1 + \sum_{\ell=1}^{t-1}\frac{\tau_{it}}{\tau_{i\ell}}}, \quad j = 1, \ldots, t-1;$$

$$a_{it}^* = \frac{1 + \sum_{\ell=1}^{t-1}\frac{\tau_{it}-\tau_{i\ell}}{\tau_{i\ell}}}{1 + \sum_{\ell=1}^{t-1}\frac{\tau_{it}}{\tau_{i\ell}}} = 1 - \frac{t-1}{1 + \sum_{\ell=1}^{t-1}\frac{\tau_{it}}{\tau_{i\ell}}};$$

$$a_{ij}^* = 0, j = t+1, \ldots, n.$$

The solution satisfies $a_{i1}^* \geq a_{i2}^* \geq \cdots \geq a_{it}^* > 0$, *where the inequality becomes an equality if and only if the corresponding two signal precisions are identical. In particular, when* $\tau_{i1} = \tau_{i2} = \cdots = \tau_{in}$, *it holds that* $a_{i1}^* = a_{i2}^* = \cdots = a_{in}^* = 1/n$.

(ii) The optimal value $\sum_{j=1}^{n}(2a_{ij}^*(\boldsymbol{\tau}_i) - (a_{ij}^*(\boldsymbol{\tau}_i))^2)\tau_{ij}$ *of (11) is increasing in* τ_{ij} *for any* i, j, *where* $\boldsymbol{\tau}_i = (\tau_{i1}, \ldots, \tau_{in})$. *Moreover, if* $a_{ij}^* > 0$, *it is strictly increasing in* τ_{ij}.

To better understand Proposition 2 it is useful to consider a fully rational investor that infers signals y_{ij} from suggested strategies x_{ij}. By the projection theorem, the optimal strategy of such a fully rational investor would then be given by

$$\frac{\mathbb{E}[\theta|y_{i1},...,y_{in},p]-p}{\rho_i \operatorname{Var}[\theta|y_{i1},...,y_{in},p]} = \rho_i^{-1}\left(\sum_{j=1}^{n}\tau_{ij}y_{ij} - \left(\frac{\rho\tau_\theta}{\Delta\tau_u+\rho}+\sum_{j=1}^{n}\tau_{ij}\right)p\right).$$

Using (5), we can infer that $a_{ij}=1$ for all j and $\varphi = -(n-1)\tau_\theta\rho/(\Delta\tau_u+\rho) < 0$ in (1). Hence, a fully rational investor gives unit weight to each of the suggested strategies. Note that the signal precision influences the strategy suggested by the advisers (5). The optimal strategy of a fully rational investor thus indirectly still depends on the signal precision of advisers. However, due to the sum-of-weights-equals-one heuristic and price information neglect, the investors of our model face the constraints $\sum_{j=1}^{n} a_{ij} = 1$ and $\varphi = 0$. These are clearly not satisfied by the allocation of the fully rational investor. Under the constraint of a boundedly rational investor, it is optimal to disregard some strategies completely, and to discriminate between the remaining strategies by giving higher weights to strategies suggested by advisers with higher signal precision.

Part (i) of Proposition 2 verifies our previous conjecture that for any investor i_1 and i_2 whose advisers have the same combination $\boldsymbol{\tau}_z^\diamond = (\tau_{z1}^\diamond,...,\tau_{zn}^\diamond)$ of signal precisions, the optimal weights of the two investors are identical, i.e., $(a_{i_11}^*,...,a_{i_1n}^*) = (a_{i_21}^*,...,a_{i_2n}^*)$, denoted as $(a_{z1}^*,...,a_{zn}^*)$. The analysis on the expressions (2)-(5) and Proposition 2 together lead to a unique advice-based equilibrium with exogenous information.

Proposition 3. *There exists a unique advice-based equilibrium with exogenous information.*

Some comments on the results in Proposition 2 are in order. First, for the special case of $n=2$, we can get the explicit solution $a_{ij}^* = \tau_{ij}/(\tau_{i1}+\tau_{i2})$, $j=1,2$. That is, the investor should consult both advisers no matter how large the difference in the precision between them, and the optimal weights are in the proportion of advisers' signal precisions.

Second, the optimal weights given to the strategies suggested by advisers with low signal precision is zero when their signal precision is low relative to the high-precision advisers. In other words, investors cannot always benefit from more consultations. Consulting with an additional adviser is only beneficial if his/her signal precision is in a similar range (or higher) than that of the advisers the investor is already consulting.

Third, a given investor i should consult all his/her advisers, if and only if $t = n$, i.e., $1+\sum_{\ell=1}^{n-1} \frac{\tau_{in}}{\tau_{i\ell}} > n-1$. Equivalently, $\tau_{in} > \frac{n-2}{n-1}\frac{n-1}{\sum_{\ell=1}^{n-1}\frac{1}{\tau_{i\ell}}}$, which holds when the difference between the highest precision and the other precisions is small. Specifically, when the signal precisions are identical, i.e., $\tau_{i1} = \cdots = \tau_{in}$, the optimal weight is uniform: $a^*_{i1} = \cdots = a^*_{in} = 1/n$. We remark that the threshold $\frac{n-2}{n-1}\frac{n-1}{\sum_{\ell=1}^{n-1}\frac{1}{\tau_{i\ell}}}$ is a multiple of the harmonic mean of the $(n-1)$ signal precisions.

Fourth, when $\tau_{i1} = \tau_{i2} = \cdots = \tau_{i(n-1)}$, $t = n$ if and only if $\tau_{in} > \frac{n-2}{n-1}\tau_{i(n-1)}$. This implies that for the weight a^*_{in} to be positive, τ_{in} needs to be close to $\tau_{i(n-1)}$, especially when n is large. Furthermore, when $\tau_{i2} = \tau_{i3} = \cdots = \tau_{in}$, it holds that $t = n$ for any $\tau_{i1} \geq \tau_{in}$. That is, when the low signal precisions are close to each other except the highest one, the investors should consult all the advisers no matter how low the signal precision is. In fact, in this case,

$$a^*_{i1} = \frac{(n-1)(\tau_{i1}-\tau_{in})+\tau_{in}}{(n-1)\tau_{i1}+\tau_{in}}, \quad a^*_{i2} = \cdots = a^*_{in} = \frac{\tau_{in}}{(n-1)\tau_{i1}+\tau_{in}}.$$

Interestingly, as $n \to \infty$ (signal precisions $\tau_{i1} > \tau_{in}$ are kept as constants), it holds that $a^*_{i1} \to \frac{\tau_{i1}-\tau_{in}}{\tau_{i1}} > 0$ and $a^*_{in} \to 0$ no matter how close τ_{in} is to τ_{i1}. That is, one should give a strictly positive weight to one's most trusted adviser even in the limit where one is able to consult with an arbitrary number of advisers.

Fifth, for any signal precision, there exists a threshold τ^* such that $a^*_{ij} > 0$ if and only if $\tau_{ij} \geq \tau^*$, and $a^*_{ij} = 0$ if and only if $\tau_{ij} < \tau^*$. Furthermore, for the positive components of the optimal solution, the higher signal precision, the larger optimal weight. This is intuitive and reasonable.

Sixth, since $t \geq 2$, there are at least two positive components in the optimal weight. In other words, investors should consult at least two advisers even if the signal precision of the best advisers is much larger than that of the second best adviser.

The marginal benefit from increasing the weight given to a suggested strategy converges to zero as the weight given to that strategy goes to one. Compared with the situation where all weight is given to the strategy suggested by the adviser with highest precision, an investor can thus increase his/her welfare by reducing this weight by a small number and correspondingly increasing the weight given to the strategy suggested by another adviser. This finding is due to the independence of signals observed by different advisers.

Finally, Part (ii) of Proposition 2 shows the intuitive result that the welfare of an investor increases in the precision of the advisers. Consider the situation where advisers are relatively homogenous such that the investor consults with all of them. Increasing the precision of a given adviser might lead to a situation where the investor now disregards some of the suggestions. The proposition shows that, in this scenario, the resulting welfare of the investor after the increase in precision of a single adviser is still superior than in the original setting where the investor consults with all advisers.

We next investigate how consultation with financial advisers affects the following market quality measures in equilibrium: *Price informativeness* is measured by $1/\operatorname{Var}[\theta|p] = \Delta^2\tau_u$ (Han and Yang 2013; Ozsoylev and Walden 2011) and refers to the degree with which market prices reflect information on fundamentals. *Market liquidity* is measured by $\frac{1}{\partial p/\partial(-u)} = \Delta + \frac{\tau_\theta}{\Delta\tau_u+\rho}$ (see equation (2)). High market liquidity implies that a shock in supply or noise trading is absorbed without moving the price much (Han and Yang (2013)). *Return volatility* is measured by $\sqrt{\operatorname{Var}(\theta-p)}$. Here $\operatorname{Var}(\theta-p) = \left(\frac{\beta^2}{\tau_\theta}+\frac{1}{\tau_u}\right)/(\Delta+\beta)^2$, where $\beta = \frac{\tau_\theta}{\Delta\tau_u+\rho}$. These expressions can be obtained from equation (2).

We compare the market quality measures implied by our model with the benchmark economy of Hellwig (1980). This model is identical to ours except that advisers directly invest themselves based on the signals they observe. This allows for a fair comparison between the two economies because the precision of observed signals is identical. We have the following proposition.

Proposition 4. *Comparing with the benchmark economy, consultation in our economy improves price informativeness. Furthermore, when all the advisers in our economy have the same signal precision, consultation increases investor welfare, but have no impact on market liquidity and return volatility.*

Intuitively, the optimal aggregation of the suggested strategies gives higher weight to signals observed with high precision, and this improves price informativeness compared with a benchmark where all strategies receive equal weight. When all advisers have the same signal precision, the optimal aggregation will exactly incorporate the averaged information of advisers and the price as well as price-related market quality measures are thus identical to the benchmark economy. However, the optimal aggregation will efficiently reduce noise contained in suggested strategies, so that the resulting investor welfare is higher.

The above discussion is based on the assumption that investors know the signal precisions of their advisers. In the reminder of this section, we discuss how to optimally aggregate suggested strategies when not knowing the precision of individual advisers but only the average precision of all advisers under a robust approach. In this situation, the optimal aggregation problem of an individual investor i in Definition 1 (ii) becomes

$$\begin{aligned} \sup_{a_{ij}, j=1,\ldots,n} \inf_{\tau_{ij}, j=1,\ldots,n} & \mathbb{E}\left[U_i\left(W\left(\sum_{j=1}^{n} a_{ij} x_{ij}(y_{ij}, p)\right)\right)\right], \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} = 1, a_{ij} \geq 0, \\ & \frac{1}{n}\sum_{j=1}^{n} \tau_{ij} = \bar{\tau}_n. \end{aligned} \tag{12}$$

The following result shows that it is optimal to adopt the simple average of suggested strategies when not knowing the signal precision of the advisers.

Proposition 5. *The optimal solution to optimization problem (12) is unique and given by $a_{ij}^* = 1/n$, $j = 1, \ldots, n$.*

We remark that the result of Proposition 5 still holds when replacing the constraint on the average signal precision in (12) by

$$\frac{1}{n}\sum_{j=1}^{n} \frac{1}{\tau_{ij}} = K$$

for some $K > 0$. A more general formulation of the robust aggregation problem that allows the investor to differentiate between advisers is discussed in Appendix A.3.

To close this section, we consider the question of whether investors taking the simple average strategy benefit from more advisers. Suppose that investor i is consulting n advisers and let $\bar{\tau}_n$ denote the average precision of their signals. This investor benefits from consulting with an additional adviser and integrating the additionally suggested strategy into the simple average if and only if the signal precision τ of the additional adviser satisfies

$$\left(2 - \frac{1}{n+1}\right)\frac{n\bar{\tau}_n + \tau}{n+1} > \left(2 - \frac{1}{n}\right)\bar{\tau}_n,$$

or equivalently, $\tau > \frac{2n^2-1}{2n^2+n}\bar{\tau}_n$. Note that $\frac{2n^2-1}{2n^2+n}$ is increasing in n, takes the value 1/3 when $n = 1$, the value 7/10 when $n = 2$, and converges to 1 when n goes to infinity. Therefore, for investors who consult only one adviser, it is worthwhile to consult a second adviser and average the two suggested strategies unless the signal precision of the first adviser is more than three times larger than that of the second. When already consulting with a large number of advisers, adding an additional adviser and integrating the suggested strategy into the simple average is worthwhile if and only if the signal precision of the additional adviser is larger than the average of the existing advisers.

4 Endogenous Information

In this section, we consider the case where signal precisions are determined endogenously. The investors face a cost depending on the signal precisions of their advisers and aim to optimally balance informativeness of the signal and resulting cost. We assume that the information acquisition cost function $c : [0, \infty) \to [0, \infty)$ is strictly convex, strictly increasing, twice continuously differentiable, and satisfies the conditions $c(0) = 0$, $\lim_{\tau\to 0} c'(\tau) = 0$ and $\lim_{\tau\to\infty} c'(\tau) = \infty$. Our goal is to explore how many advisers the investors should optimally consult and how much to spend on each adviser.[10]

We study endogenization of both information and the number of advisers in two steps: First, we consider the case where only information acquisition is endogenous and the number of advisers is exogenously given. This is the mirror situation of the analysis in Section 3, where information acquisition was exogenous. In the second step, we study the case where both information acquisition and the number of advisers are endogenous.

Recall that from Assumption 1 there is a finite number of coefficients of risk aversion in the economy, i.e., $\rho_i \in \{\rho_1^\diamond, ..., \rho_m^\diamond\}$ for all $i \in \mathbb{N}$ and $\{\rho_1^\diamond, ..., \rho_m^\diamond\} \in \mathbb{R}^m_{>0}$. For a given investor i with coefficient of risk aversion $\rho_k^\diamond$ we let $r_k \in \mathbb{N}$ be the number of advisers that the investor consults. We only consider the case where all investors with a given coefficient of risk aversion consult with the same number of advisers.

[10]The results in this section also hold for a more general cost structure where $c(\cdot)$ also includes a nonnegative consultation fee charged by advisers from the investors, besides the information acquisition cost.

An *exogenously imposed consultation structure* can be described by a vector $\mathbf{r} = (r_1, \ldots, r_m) \in \mathbb{N}^m$ stipulating that investors with risk aversion coefficient $\rho_k^\diamond$ consults with r_k advisers. For a given exogenous consultation structure $\mathbf{r} = (r_1, \ldots, r_m) \in \mathbb{N}^m$, we denote by $\mathcal{N}_i$ the set of r_k advisers who provide suggestions to an investor i with the risk aversion coefficient $\rho_k^\diamond$.

Investors choose the signal precision for each of their advisers to maximize their expected utility. According to (7) and taking into account the information acquisition cost, the expected utility of an adviser i with risk aversion coefficient $\rho_k^\diamond$ choosing signal precisions τ_{ij}, $j \in \mathcal{N}_i$ for his/her advisers is given by

$$-\left(\mathrm{Var}(\theta - p)\exp\left(-2\rho_k^\diamond \sum_{j\in\mathcal{N}_i} c(\tau_{ij})\right)\left(\tau_\theta + \Delta^2\tau_u + \sum_{j\in\mathcal{N}_i}(2a_{ij} - a_{ij}^2)\tau_{ij}\right)\right)^{-\frac{1}{2}}. \tag{13}$$

Since this is a large economy and any particular investor's decision has no impact on the price p and the amount Δ, the maximization problem (13) faced by investor i can be reduced to

$$\max_{\tau_{ij}, j\in\mathcal{N}_i} \exp\left(-2\rho_k^\diamond \sum_{j\in\mathcal{N}_i} c(\tau_{ij})\right)\left(\tau_\theta + \Delta^2\tau_u + \sum_{j\in\mathcal{N}_i}(2a_{ij}^* - (a_{ij}^*)^2)\tau_{ij}\right), \tag{14}$$

where $(a_{ij}^*)_{j\in\mathcal{N}_i}$ are functions of $(\tau_{ij})_{j\in\mathcal{N}_i}$ given in Proposition 2. The following proposition shows that the investor spends an equal amount on each adviser if the cost function is sufficiently convex.

PROPOSITION **6.** *Suppose that $c''(\cdot)$ is increasing and $\inf_{\tau>0} \frac{c''(\tau)\tau}{c'(\tau)} \geq r_k - 1$. Then any optimal solution $(\tau_{ij}^*)_{j\in\mathcal{N}_i}$ to the optimization problem (14) satisfies that $\tau_{i1}^* = \cdots = \tau_{ir_k}^*$.*

Condition $\inf_{\tau>0} \frac{c''(\tau)\tau}{c'(\tau)} \geq r_k - 1$ requires that the cost function is sufficiently convex. For example, the condition is satisfied when the cost function takes the form of $\kappa\tau^\nu$ with $\nu \geq r_k$. Without this condition Proposition 6 may not hold. For instance, when the cost function is linear and an equilibrium exists, we can show that any optimal solution $(\tau_{ij}^*)_{j=1,\ldots,r_k}$ to the optimization problem (14) must be a corner solution, i.e., it is optimal to spend everything on a single adviser and then disregard the suggestions of all others.

In view of Proposition 6, we from now on consider only equilibria in which investors spend equal amounts on all advisers, i.e., $\tau_{ij_1} = \tau_{ij_2}$ for any $j_1, j_2 \in \mathcal{N}_i$. This assumption can for example be enforced by considering a setting where there is an upper bound n on the number of advisers any investor can consult as in the previous Section 3 and assuming that the cost function is sufficiently convex. When all advisers $(i, j), j \in \mathcal{N}_i$ acquire a signal with precision τ, the cost faced by the investor i is $r_k c(\tau)$. Moreover, in the case that all advisers consulting a given investor have identical signal precision, it is optimal for the investor to take the simple average of the suggested strategies (see Proposition 2). We thus consider the simple average of suggested strategies in the remaining part of this section without further loss of generality.

Definition 2. *An equilibrium with endogenous information but exogenously imposed consultation structure* $\mathbf{r} \in \mathbb{N}^m$ *is a tuple* $\left((\tau_i^*)_{i=1,\ldots,\infty}, p\right)$ *such that*

(i) for each $i = 1, \ldots, \infty$, τ_i^* *is the optimal precision given the precisions of other investors and resulting optimal ex ante suggested strategies by their advisers, i.e.,*

$$\tau_i^* \in \arg\max_{\tau_i > 0} \mathbb{E}\left[U_i\left(W(x^*_{\mathcal{N}_i}(\tau_i)) - r_k c(\tau_i)\right)\right],$$

where

$$x^*_{\mathcal{N}_i}(\tau_i) = \frac{1}{r_k}\left(\sum_{j \in \mathcal{N}_i} x_{ij}(y_{ij}(\tau_i), p)\right)$$

is the simple average of suggested strategies $x_{ij}(y_{ij}(\tau), p)$, *where* $y_{ij}(\tau)$ *is the signal with precision* τ *received by adviser* $(i, j), j \in \mathcal{N}_i$.

(ii) the market clears, i.e.,

$$\lim_{h \to \infty} \frac{1}{h} \sum_{i=1}^{h} \left(\frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} x_{ij}\right) = u.$$

In Condition (i) of Definition 2, the strategy adviser $(i, j), j \in \mathcal{N}_i$ suggests when the precision is τ is

$$x_{ij}(y_{ij}(\tau), p) = \frac{\mathbb{E}[\theta | y_{ij}(\tau), p] - p}{\rho_k^\diamond \operatorname{Var}[\theta | y_{ij}(\tau), p]}, \tag{15}$$

where $\rho_k^\diamond$ is the risk aversion coefficient of the investor i and p is the endogenous equilibrium price defined by (2) when Δ is replaced with $\sum_{k=1}^m \lambda_k \tau_{i,k}^* / \rho_k^\diamond$, $\tau_{i,k}^*$ being the optimal signal precision of investors with risk aversion coefficient $\rho_k^\diamond$. This condition states that each investor spends optimally on financial advisers given the expenses of other investors in the network. Condition (ii) of Definition 2 is the usual market clearing condition stating that, in equilibrium, supply must equal demand.

The following proposition shows that an equilibrium with endogenous information but exogenous consultation structure exists, is unique, and leads to a situation where investors with the same risk aversion spend identical amounts on each financial adviser.

PROPOSITION **7.** *For any exogenously given* $\mathbf{r} \in \mathbb{N}^m$,

(i) there exists a unique equilibrium with endogenous information but exogenously imposed consultation structure $\mathbf{r}$, $\left((\tau_i^*)_{i=1,\ldots,\infty}, p_{\mathbf{r}}\right)$, *which satisfies* $\tau_i^* = \tau_j^* =: \tau_k^*(\mathbf{r})$ *for any* i, j *when the coefficient of risk aversion of investors* i, j *is* $\rho_k^\diamond$, *and* $p_{\mathbf{r}} = \frac{1}{\Delta_{\mathbf{r}} + \frac{\tau_\theta}{\Delta_{\mathbf{r}}\tau_u + \rho}}(\Delta_{\mathbf{r}}\theta - u)$, *where* $\Delta_{\mathbf{r}} = \sum_{k=1}^m \lambda_k \tau_k^*(\mathbf{r})/\rho_k^\diamond$.

(ii) $(\tau_k^*(\mathbf{r}))_{k=1,\ldots,m}$ *are jointly determined by the system of equations:*

$$2\rho_k^\diamond c'(\tau_k^*(\mathbf{r}))\left(\frac{r_k}{2-\frac{1}{r_k}}(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u) + r_k\tau_k^*(\mathbf{r})\right) = 1, \ k = 1, ..., m.$$

(iii) the welfare (taking into account the cost of information acquisition) of the investors with risk aversion $\rho_k^\diamond$ *and* r_k *advisers is given by*

$$V_k(\mathbf{r}) := -\left(\exp\left(-2\rho_k^\diamond r_k c(\tau_k^*(\mathbf{r}))\right)\mathrm{Var}(\theta - p_{\mathbf{r}})(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + (2 - 1/r_k)\tau_k^*(\mathbf{r}))\right)^{-\frac{1}{2}}.$$

As an immediate corollary to Proposition 7 (i), we obtain that the assumption of a finite number of profiles of coefficients of risk aversion and signal precisions throughout the economy we imposed in Assumption 1 when information is exogenous automatically holds also for the endogenous case as long as the number of coefficients of risk aversion is finite. Proposition 7 (ii) and (iii) characterizes the unique equilibrium and the resulting welfare of each investor with endogenous information but exogenously imposed consultation structure.

A fully endogenous equilibrium, where both information acquisition and consultation size are determined endogenously, is defined as follows.

Definition 3. *A fully endogenous equilibrium is a tuple* $\left(\mathbf{r}, (\tau_k^*(\mathbf{r}))_{k=1,\ldots,m}, p\right)$ *such that*

(i) setting $\tau_i^* = \tau_k^*(\mathbf{r})$ *for any investor* i *with the coefficient of risk aversion* $\rho_k^\diamond$, $\left((\tau_i^*)_{i=1,\ldots,\infty}, p\right)$ *is an equilibrium with endogenous information but exogenous consultation structure* $\mathbf{r}$, *and*

(ii) for any investor i with the risk aversion coefficient $\rho_k^\diamond$, and any possible consultation set $\mathcal{N}_i'$ with r_k' advisers, it holds that

$$\max_{\tau>0} \mathbb{E}\left[U_i\left(W\left(\frac{1}{r_k'}\left(\sum_{j\in\mathcal{N}_i'} x_{ij}(y_{ij}(\tau),p)\right)\right) - r_k' c(\tau)\right)\right] \le V_k(\mathbf{r}),$$

where $x_{ij}(y_{ij}(\tau),p)$ is the suggested optimal strategy by adviser (i,j) with signal precision τ as given in (15).

Definition 3 makes two requirements on fully endogenous equilibria. First, given the consultation structure, the signal structure together with the price constructs an endogenous equilibrium. Second, there is no incentive for investors to deviate from their current number of advisers.

The following proposition identifies the optimal number of advisers within the framework of fully endogenous equilibria. Together with Proposition 7, this will give a fully characterization of equilibria with endogenous information acquisition and consultation structure. Denote

$$A_+ = \sup_{\tau>0} \frac{\tau c'(\tau)}{c(\tau)}, \quad A_- = \inf_{\tau>0} \frac{\tau c'(\tau)}{c(\tau)}.$$

Let $\lfloor b \rfloor$ and $\lceil b \rceil$ denote the maximum integer not greater than b and the minimum integer not smaller than b, respectively.

Proposition 8. *The consultation structure $\mathbf{r} = (r_1, \ldots, r_m)$ in any fully endogenous equilibrium satisfies that $\lfloor \frac{A_-+1}{2} \rfloor \le r_k \le \lceil \frac{A_++1}{2} \rceil$ for every $1 \le k \le m$. In particular,*

- *if $A_+ = A_-$ is an odd number, then $r_1 = r_2 = \cdots = r_m = (A_+ + 1)/2$;*
- *if $A_+ = A_-$ is an even number, then $r_k \in \{\frac{A_+}{2}, \frac{A_+}{2} + 1\}$ for every $1 \le k \le m$.*

Furthermore, if the cost function is of the form $c(\tau) = \kappa\tau^2$ with $\kappa > 0$,[11] *then the fully endogenous equilibrium is unique and $r_k = 2$ for every $1 \le k \le m$.*

[11] The assumption of a quadratic cost function is common in the literature, see for example, Gao and Liang (2013), He et al. (2021) and Goldstein and Yang (2017). In our case, when the cost function takes a more general form of $c(\tau) = \kappa_\ell \tau^\ell + \kappa_{\ell-1}\tau^{\ell-1} + \cdots + \kappa_1 \tau$, $\ell \ge 2$, we have $r_k \le \lceil \frac{\ell+1}{2} \rceil$, i.e., both endogenous information acquisition and consultation structure will lead to small consultation size not greater than $\lceil \frac{\ell+1}{2} \rceil$.

The first part of Proposition 8 gives a lower and upper bound on the endogenous consultation size, which depend only on the structure of information acquisition cost function. When the cost function $c(\tau)$ takes the form of $\kappa\tau^{\nu}$ (ν being a positive integer), it holds that $A_{+} = A_{-} = \nu$. If ν is an odd number, then $r_1 = r_2 = \cdots = r_m = (\nu + 1)/2$, while if ν is an even number, then $r_k = \nu/2$ or $r_k = \nu/2 + 1$. Recall that for a cost function of the form $c(\tau) = \kappa\tau^{\nu}$, we showed in Proposition 6 that the components of the optimal solution must be identical if $\nu \geq r_k$, which automatically holds as indicated by Proposition 8.

For quadratic cost functions, a common choice in the literature, the optimal consultation size emerging in a fully endogenous equilibrium is uniform across investors, $r_k = r_s = 2$ for all $k, s = 1, \ldots, m$. In particular, consultation size in a fully endogenous equilibrium does not depend on the risk-aversion of investors. Proposition 8 shows that fully endogenous equilibria typically lead to few advisers. The intuition behind this is that investors generally reduce information acquisition when consulting more advisers as the utility benefit is a concave function of the consultation size, but the utility loss (due to information acquisition cost) is a convex function of the consultation size, for any fixed signal precision. Although reducing information acquisition leads to a reduction in the total cost for advisers to acquire information, the utility benefit from cost saving is off-set by the loss in information available in the economy. Anticipating this, investors have an incentive to consult few advisers where investors would then increase information acquisition.

At the end of this section, we analyze the effects of consultation on market outcomes of the fully endogenous equilibrium. To avoid the trivial case, we assume that the endogenous number of consultation size is greater or equal to two. We have the following result.

Proposition **9.** *Comparing with the benchmark economy, consultation impairs price informativeness and increases return volatility. Furthermore, consultation improves investor welfare and market liquidity in informationally inefficient markets.*

Compared with the benchmark economy as in Section 4, consulting with advisers reduces the investor's incentive to acquire information. This resulting reduction in information acquisition decreases price informativeness. Low level of information in the economy implies that prices are not very indicative of the fundamental value and that uncertainty about the final payoff is consequently higher. This results in a more volatile asset return. Furthermore, this also results in risk averse agents facing higher trading risks and thus experiencing low expected utilities. But on the other hand, an increase in risk will also lead to higher expected returns (Kurlat and Veldkamp (2015)), called *return effect*, a term coined by He et al. (2021), especially in informationally inefficient or noisier markets. In summary, the risky asset in an economy with low information acquisition contains high risk, but also offers a higher expected return. But the second effect dominates the first one, and hence consultation improves investor welfare.

5 Conclusions

We consider a classical rational expectations equilibrium economy populated by two types of agents: Investors and their financial advisers. Investors cannot construct their own investment strategies but instead rely on their advisers to do so. Financial advisers observe a private signal about the fundamental of the risky asset and communicate an investment strategy that takes into account the investor. Investors then optimally aggregate all strategies that were suggested to them under two constraints of bounded rationality: the sum-of-weights-equals-one heuristic and price information neglect.

Our main research question is to study how many financial advisers investors should consult with. We do so in two separate settings, first with information being exogenous and second with endogenous information. In the case of endogenous information investors control the precision of the signals of their advisers at a cost, and we are further interested in how much investors should pay to each of their advisers.

We find that the investor should always consult with at least two advisers, even if there is a large difference in the precision of their signals if information is exogenous. However, it is not optimal to consult with all possible advisers unless their signal precision is relatively homogeneous.

When information is endogenous, how much to spend on each adviser depends on the function mapping expenses on a given adviser to the precision of the signal this adviser receives. If this cost function is sufficiently convex, it is optimal to spend an equal amount to all advisers one is consulting with and then to equal weight their suggested strategies. For example, under quadratic information acquisition costs, it is optimal to consult with exactly two advisers, spend an equal amount on both of them, and equal-weight their suggested strategies.

There are several interesting directions for future research. First, in the model of this paper, we assume that each financial adviser suggests a strategy to a single investor and the strategy is in the best interest of the investor taking into account their risk preferences. It would be interesting to study a setting where advisers can suggest the same strategy to multiple investors and/or have objectives other than the welfare of the investors.

Second, we herein consider an economy consisting of a risk-free and a single risky asset. It would be interesting to consider an economy containing multiple risky assets and allow for a more general interpretation of strategies as investment portfolios. When information acquisition is costly, a setup with multiple risky assets typically leads to under-diversification in the optimal strategy of a single investor (van Nieuwerburgh and Veldkamp, 2010). Relying on suggested portfolios of multiple advisers could thus lead to additional benefits in terms of diversification.

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Appendix

A Discussions

This appendix contains further discussions on the modelling assumptions in the main body of the paper. Bounded rationality of investors is modelled through two behavioral features, the sum-of-weights-equals-one heuristic and price information neglect. We discuss model predictions if investors exhibit only one of them, the sum-of-weights-equals-one heuristic in Appendix A.1 and price information neglect in Appendix A.2. In Appendix A.3, we study a robust approach to the setting where investors do not know the signal precision of advisers that still allows investors to differentiate between advisers.

A.1 Implications of the sum-of-weights-equals-one heuristic

We consider the setting where investors only exhibit the sum-of-weights-equals-one heuristic and are not affected by price information neglect. Surprisingly, Proposition 10 shows that when investors adopt the sum-of-weights-equals-one heuristic, giving an additional degree of freedom by allowing for aggregate strategies to depend on prices does not improve investors' expected utility. This holds when information is exogenous and when information is endogenous. When adopting the sum-of-weights-equals-one heuristic, the slope parameter of the aggregated demand curve in the price is already optimal.

PROPOSITION **10.** *We have*

(i) Suppose that the information is exogenous. Fix the weights $(a_{z1}, ..., a_{zn})_{z=1,...,v}$ *and consider the resulting price* p *in (2),* ρ *in (3), and* Δ *in (4). When investors adopt the sum-of–weights-equals-one heuristic, it is optimal for investors to exhibit price information neglect. That is,*

$$\max_{a_{ij}\geq 0, \sum_{j=1}^n a_{ij}=1, \varphi\in\mathbb{R}} \mathbb{E}\left[U_i\left(W\left(\sum_{j=1}^n a_{ij}x_{ij} - \varphi p\right)\right)\right]$$
$$= \max_{a_{ij}\geq 0, \sum_{j=1}^n a_{ij}=1} \mathbb{E}\left[U_i\left(W\left(\sum_{j=1}^n a_{ij}x_{ij}\right)\right)\right].$$

(ii) Suppose the information is endogenous. When investors adopt the sum-of–weights-equals-one heuristic, it is optimal for investors to exhibit price information neglect. That is,

$$\max_{a_{ij}\geq 0, \sum_{j\in\mathcal{N}_i} a_{ij}=1, \tau_{ij}>0, \varphi\in\mathbb{R}} \mathbb{E}\left[U_i\left(W\left(\sum_{j\in\mathcal{N}_i} a_{ij}x_{ij} - \varphi p\right) - \sum_{j\in\mathcal{N}_i} c(\tau_{ij})\right)\right]$$
$$= \max_{a_{ij}\geq 0, \sum_{j\in\mathcal{N}_i} a_{ij}=1, \tau_{ij}>0} \mathbb{E}\left[U_i\left(W\left(\sum_{j\in\mathcal{N}_i} a_{ij}x_{ij}\right)\right) - \sum_{j\in\mathcal{N}_i} c(\tau_{ij})\right],$$

where $\mathcal{N}_i$ *is the set of advisers who provide suggestions to the investor* i*.*

As a consequence of Proposition 10, all results in the main body of the paper hold true when investors are of bounded rationality only through the sum-of-weights-equals-one heuristic.

A.2 Implications of price information neglect

We here consider the mirror situation to the above, namely the setting where investors only exhibit price information neglect but are not affected by the sum-of-weights-equals-one heuristic. While the analysis is more intricate, most of our results remain qualitatively similar. While it wasn't possible to provide analytical results, we provide intuition based on numerical examples. We first consider the case where information is exogenous and then move on the setting with endogenous information.

PROPOSITION **11.** *Suppose the information is exogenous. Fix the weights* $(a_{z1}, ..., a_{zn})_{z=1,...,v}$ *and consider the resulting price* p *in (2),* ρ *in (3), and* Δ *in (4). When investors only exhibit price information neglect, then*

(i) For any weight and signal precision $(a_{ij}, \tau_{ij})_{j=1,...,n}$*, the (ex-ante) expected utility of the weighted average strategy* $\sum_{j=1}^{n} a_{ij}x_{ij}$ *is given by*

$$\mathbb{E}\left[U_i\left(W\left(\sum_{j=1}^{n} a_{ij}x_{ij}\right)\right)\right] = -\left(1 + (2a_i - a_i^2)\rho\alpha\beta + \sum_{j=1}^{n}(2a_{ij} - a_{ij}^2)\tau_{ij}\gamma\right)^{-\frac{1}{2}}, \quad (16)$$

where $a_i = \sum_{j=1}^{n} a_{ij}$*,* α*,* β*, and* γ *are given in (10).*

(ii) Let $\{a_{ij}^*\}_{j=1,...,n}$ *be the optimal solution of* $\max_{a_{ij}\geq 0, \sum_{j=1}^{n} a_{ij}=1} \mathbb{E}\left[U_i\left(W\left(\sum_{j=1}^{n} a_{ij}x_{ij}\right)\right)\right]$ *and denote* $a_i^* = \sum_{j=1}^{n} a_{ij}^*$*. Then, it holds that* $1 \leq a_i^* \leq n$ *for every* i*. Furthermore,* a_i^* *is close to one if* $\rho\alpha\beta/\gamma$ *is sufficiently large.*

(iii) Suppose $\tau_{i1} \geq \tau_{i2} \geq \cdots \geq \tau_{in}$*, and let*

$$t = \max\left\{j \,\middle|\, 2 \leq j \leq n, a_i^* + \sum_{\ell=1}^{j-1} \frac{\tau_{ij} - \tau_{i\ell}}{\tau_{i\ell}} > 0\right\}, \quad 2 \leq t \leq n.$$

Then the optimal solution a_{ij}^**'s is unique and given by*

$$a_{ij}^* = \frac{a_{it}^*\tau_{it} + \tau_{ij} - \tau_{it}}{\tau_{ij}}, \quad j = 1, ..., t-1;$$

$$a_{it}^* = \frac{a_i^* + \sum_{\ell=1}^{t-1} \frac{\tau_{it} - \tau_{i\ell}}{\tau_{i\ell}}}{1 + \sum_{\ell=1}^{t-1} \frac{\tau_{it}}{\tau_{i\ell}}};$$

$$a_{ij}^* = 0, j = t+1, ..., n.$$

The solution satisfies $1 \geq a_{i1}^* \geq a_{i2}^* \geq \cdots \geq a_{it}^* > 0$*, where the inequality becomes an equality if and only if the corresponding two signal precisions are identical. In particular, when* $\tau_{i1} = \tau_{i2} = \cdots = \tau_{in}$*, it holds that* $a_{i1}^* = a_{i2}^* = \cdots = a_{in}^* = a_i^*/n$*.*

(iv) There exists a unique advice-based equilibrium with exogenous information.

Part (i) of Proposition 11 presents an analytical expression of the expected utility of investors who only exhibit price information neglect without adopting the sum-of-weights-equals-one heuristic. The expected utility in (16) reduces to the one in (8) when adopting the sum-of-weights-equals-one heuristic, i.e., when $a_i = 1$. Part (ii) gives an estimate on the sum of optimal weights. The sum is close to one when $\rho\alpha\beta/\gamma$ is large. This holds true, for example, when τ_u is sufficiently small and τ_θ is sufficiently large. If this holds true, model predictions when investors only exhibit price information neglect are very close to results discussed in the main body of the paper. Part (iii) tells that the results in Proposition 2 in the main paper also hold when not adopting the sum-of-weights-equals-one heuristic. In particular, the implications following Proposition 2 also apply here.

Proposition 11 discusses the exogenous information case. Now we proceed to consider the endogenous information case. First, we claim that Proposition 6 holds when investors are only affected by price information neglect. The key step in the proof is to ensure the following inequality holds (still omit the superscript $*$):

$$(a_i - a_{i1})\frac{\tau_{i1}}{\tau_{i2}} \geq (a_{i1} - a_{i2})\frac{\tau_{i1}}{\tau_{i1} - \tau_{i2}}.$$

Since $a_{i1} = \frac{(a_{it}-1)\tau_{it}}{\tau_{i1}} + 1$, $a_{i2} = \frac{(a_{it}-1)\tau_{it}}{\tau_{i2}} + 1$, the above inequality is equivalent to

$$\left(a_i - 1 - \frac{(a_{it}-1)\tau_{it}}{\tau_{i1}}\right)\frac{\tau_{i1}}{\tau_{i2}} \geq (a_{it}-1)\tau_{it}\left(\frac{1}{\tau_{i1}} - \frac{1}{\tau_{i2}}\right)\frac{\tau_{i1}}{\tau_{i1} - \tau_{i2}},$$

which is true due to $a_i \geq 1$.

We now move on to revisit Proposition 8. For tractability, here we consider a homogeneous case that all investors in the economy have the same risk aversion $\rho > 0$, and we assume that the condition in Proposition 6 holds so that all investors spend an equal amount on their advisers. Suppose that all investors are consulting $s \geq 1$ advisers with signal precision τ. Now we let a

denote the weight sum. From (41) in the Appendix and using the fact $a_{ij} = a/s$, we have

$$\mathbb{E}\left[U_i\left(W\left(\sum_{j=1}^{n} a_{ij}x_{ij}\right)\right)\right] = -\left(1 + (2a - a^2)\rho\alpha\beta + (2a - a^2/s)\tau\gamma\right)^{-\frac{1}{2}},$$

where α, β, and γ are given in (10). The first-order condition with respect to a leads to the following optimal weight sum

$$a^* = a^*(s,\tau) = \frac{\tau[\tau_\theta\tau_u + (\Delta\tau_u + \rho)^2] + \rho^2\tau_\theta}{\frac{\tau}{s}[\tau_\theta\tau_u + (\Delta\tau_u + \rho)^2] + \rho^2\tau_\theta}. \tag{17}$$

Observe that $1 \le a^* \le s$, which is consistent with the result in Proposition 11.

Moreover, the endogenous equilibrium signal precision $\tau^*_{\mathcal{N}}(s)$ satisfies

$$\tau^*_{\mathcal{N}}(s) \in \arg\max_{\tau>0}\Bigg[\exp\left(-2\rho s c(\tau)\right) \cdot \left(1 + (2a^*(s,\tau) - (a^*(s,\tau))^2)\rho\alpha\beta + \left(2a^*(s,\tau) - \frac{(a^*(s,\tau))^2}{s}\right)\tau\gamma\right)\Bigg],$$

i.e., $\tau^*_{\mathcal{N}}(s)$ satisfies

$$\begin{aligned}&2\rho s c'(\tau)\left(1 + (2a^*(s,\tau) - (a^*(s,\tau))^2)\rho\alpha\beta + \left(2a^*(s,\tau) - \frac{(a^*(s,\tau))^2}{s}\right)\tau\gamma\right)\\ &= 2(1 - a^*(s,\tau))\frac{\partial a^*(s,\tau)}{\partial\tau}\rho\alpha\beta + \left[2a^*(s,\tau) - \frac{(a^*(s,\tau))^2}{s} + 2\left(1 - \frac{a^*(s,\tau)}{s}\right)\frac{\partial a^*(s,\tau)}{\partial\tau}\tau\right]\gamma.\end{aligned} \tag{18}$$

With some simple calculations, we see that

$$\frac{\partial a^*(s,\tau)}{\partial\tau} = \frac{[\tau_\theta\tau_u + (\Delta\tau_u + \rho)^2]\rho^2\tau_\theta(1 - 1/s)}{\left(\frac{\tau}{s}[\tau_\theta\tau_u + (\Delta\tau_u + \rho)^2] + \rho^2\tau_\theta\right)^2}. \tag{19}$$

Observing that

$$\begin{aligned}&\lim_{\tau\to0} c'(\tau) = 0, \lim_{\tau\to0} a^*(s,\tau) = 1, \lim_{\tau\to0}\frac{\partial a^*(s,\tau)}{\partial\tau} = [\tau_\theta\tau_u + (\Delta\tau_u + \rho)^2](1 - 1/s)/(\rho^2\tau_\theta),\\ &\lim_{\tau\to\infty} c'(\tau) = \infty, \lim_{\tau\to\infty} a^*(s,\tau) = s, \lim_{\tau\to\infty}\frac{\partial a^*(s,\tau)}{\partial\tau} = 0,\end{aligned}$$

the optimal solution $\tau^*_{\mathcal{N}}(s)$ must exist. Substituting (17) and (19) into (18), we see that $\tau^*_{\mathcal{N}}(s)$ is the solution to the equation

$$2\rho s c'(\tau)\left[1 + \frac{\tau[\tau_\theta\tau_u + (\Delta\tau_u + \rho)^2] + \rho^2\tau_\theta}{\frac{\tau}{s}[\tau_\theta\tau_u + (\Delta\tau_u + \rho)^2] + \rho^2\tau_\theta}\right.$$

$$\cdot\left(\frac{(2/s-1)\tau[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+\rho^2\tau_\theta}{\frac{\tau}{s}[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+\rho^2\tau_\theta}\frac{\rho\tau_\theta}{\Delta\tau_u+\rho}\frac{\Delta\rho+\rho^2/\tau_u}{(\Delta^2\tau_u+\Delta\rho+\tau_\theta)^2}\right.$$
$$\left.+\frac{\tau[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+(2s-1)\rho^2\tau_\theta}{\tau[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+s\rho^2\tau_\theta}\tau\frac{\tau_\theta+(\Delta\tau_u+\rho)^2/\tau_u}{(\Delta^2\tau_u+\Delta\rho+\tau_\theta)^2}\right)\Bigg]$$
$$=\left(2\frac{(1/s-1)\tau[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]}{\frac{\tau}{s}[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+\rho^2\tau_\theta}\frac{\rho\tau_\theta}{\Delta\tau_u+\rho}\frac{\Delta\rho+\rho^2/\tau_u}{(\Delta^2\tau_u+\Delta\rho+\tau_\theta)^2}\right.$$
$$\left.+2\frac{(s-1)\rho^2\tau_\theta}{\tau[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+s\rho^2\tau_\theta}\tau\frac{\tau_\theta+(\Delta\tau_u+\rho)^2/\tau_u}{(\Delta^2\tau_u+\Delta\rho+\tau_\theta)^2}\right)\frac{[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]\rho^2\tau_\theta(1-1/s)}{\left(\frac{\tau}{s}[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+\rho^2\tau_\theta\right)^2}$$
$$+\frac{\tau[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+(2s-1)\rho^2\tau_\theta}{\tau[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+s\rho^2\tau_\theta}\frac{\tau[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+\rho^2\tau_\theta}{\frac{\tau}{s}[\tau_\theta\tau_u+(\Delta\tau_u+\rho)^2]+\rho^2\tau_\theta}\frac{\tau_\theta+(\Delta\tau_u+\rho)^2/\tau_u}{(\Delta^2\tau_u+\Delta\rho+\tau_\theta)^2}.$$

For any given positive integer $s \geq 1$, substituting the relation $\Delta = \tau/\rho$ into the above equation, we can solve $\tau^* = \tau^*(s;\rho,\tau_\theta,\tau_u)$ which is a function of variable s as well as the model parameters ρ, τ_θ, and τ_u. Now, we have shown that there exists an equilibrium with endogenous information but exogenously imposed consultation structure characterized by the consultation size s.

We now discuss the fully endogenous equilibrium. Let $\Delta = \tau^*(s;\rho,\tau_\theta,\tau_u)/\rho$ and fix this Δ, we investigate the effect of t on the welfare

$$-\left[\exp\left(-2\rho t c(\tau^*_{\mathcal{N}}(t))\right)\left(1+a^*(t,\tau^*_{\mathcal{N}}(t))\left((2-a^*(t,\tau^*_{\mathcal{N}}(t)))\rho\alpha\beta+\left(2-\frac{a^*(t,\tau^*_{\mathcal{N}}(t))}{t}\right)\tau^*_{\mathcal{N}}(t)\gamma\right)\right)\right]^{-\frac{1}{2}}.$$

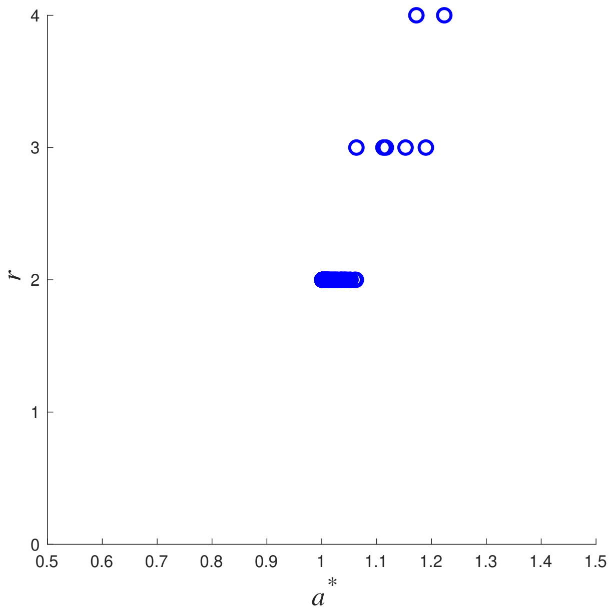
If the welfare achieves its maximum at s, then we say that s is the stable consultation size in the fully endogenous equilibrium.

Here we take the cost function as $c(\tau) = 0.5\tau^2$. The following Figure 1 presents the stable consultation size r and the corresponding values of $a^*(r,\tau^*_{\mathcal{N}}(r))$ for 100 different model parameter values of ρ, τ_u and τ_θ which are randomly and independently generated from the interval $[0.1, 3]$. Figure 1 shows that the stable consultation size is still small even if investors do not adopt the sum-of-weights-equals-one heuristic (still exhibit price information neglect), which is consistent with Proposition 8.

A.3 Knowledge of signal precisions of advisers

We revisit the discussion of the economy with exogenous information where investors do not have full knowledge of the signal precisions of their advisers. In the main body of the paper,

Figure 1: The stable consultation size and optimal aggregation weight sum



we considered the (12) and found that investors aggregate suggested strategies by giving an equal weight of $1/n$ to each of the n suggested strategies. This finding is because, in (12), the average investor precision is restricted in a symmetric manner. We herein discuss a generalized version of (12) that allows the investor to differentiate between advisers.

$$\begin{aligned} \sup_{a_{ij},j=1,\dots,n} \inf_{\tau_{ij},j=1,\dots,n} \quad & \sum_{j=1}^{n}(2a_{ij} - a_{ij}^2)\tau_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} = 1, a_{ij} \geq 0, \\ & \sum_{j=1}^{n} w_{ij}\tau_{ij} = K, \end{aligned} \tag{20}$$

where $w_{ij} > 0$ and $\sum_{j=1}^{n} w_{ij} = 1$. A larger $w_{ij} > 0$ reflects greater relative confidence of investor i in the suggestions of adviser (i, j).

With the change of the variable $\tau'_{ij} = w_{ij}\tau_{ij}$, the robust optimization problem (20) can be transferred into:

$$\begin{aligned}\sup_{a_{ij},j=1,\ldots,n}\ \inf_{\tau'_{ij},j=1,\ldots,n}\ & \sum_{j=1}^{n}\frac{2a_{ij}-a_{ij}^2}{w_{ij}}\tau'_{ij}\\ \text{s.t. } & \sum_{j=1}^{n}a_{ij}=1, a_{ij}\geq 0,\\ & \sum_{j=1}^{n}\tau'_{ij}=K.\end{aligned}\tag{21}$$

Fix $\{a_{ij}\}_{j=1,\ldots,n}$ and consider the following optimization problem: $\inf_{\tau'_{ij},j=1,\ldots,n}\sum_{j=1}^{n}\frac{2a_{ij}-a_{ij}^2}{w_{ij}}\tau'_{ij}$. The lowest value of $(2a_{ij}-a_{ij}^2)/w_{ij}$ will be given all the precision K while other values receive zero precision. That is, $\tau'_{ij_1}=K$ for $j_1\in\arg\min_{1\leq j\leq n}(2a_{ij}-a_{ij}^2)/w_{ij}$, and $\tau'_{ij}=0$ for $j\neq j_1$. Then, the robust optimization problem (21) reduces to $\sup_{a_{ij},j=1,\ldots,n}\min_{1\leq j\leq n}\ (2a_{ij}-a_{ij}^2)/w_{ij}$, which we next solve.

We claim that the optimal solution (still denoted as a_{ij}'s) must satisfy $a_{ij}>0$ for any j and that

$$\frac{2a_{i1}-a_{i1}^2}{w_{i1}}=\frac{2a_{i2}-a_{i2}^2}{w_{i2}}=\cdots=\frac{2a_{in}-a_{in}^2}{w_{in}}.\tag{22}$$

Otherwise, we can increase a_{ij_1} by a small ε for $j_1\in\arg\min_{1\leq j\leq n}\frac{2a_{ij}-a_{ij}^2}{w_{ij}}$ and decrease another a_{ij_2} with $\frac{2a_{ij_2}-a_{ij_2}^2}{w_{ij_2}}>\frac{2a_{ij_1}-a_{ij_1}^2}{w_{ij_1}}$ by ε to increase the lowest value of $\left\{\frac{2a_{ij}-a_{ij}^2}{w_{ij}}\right\}_{j=1,\ldots,n}$. Without loss of generality, we assume $w_{i1}\geq w_{i2}\geq\cdots\geq w_{in}$. It then follows from (22) that $a_{i1}\geq a_{i2}\geq\cdots\geq a_{in}$. From (22), we have

$$a_{ij}=1-\sqrt{1+\frac{w_{ij}}{w_{i1}}(a_{i1}^2-2a_{i1})}, j=2,\ldots,n.\tag{23}$$

This is natural, recalling that a larger $w_{ij}>0$ reflects greater relative confidence of investor i in the suggestions of adviser (i,j).

Finally, we can solve a_{i1} from the condition $\sum_{j=1}^{n}a_{ij}=1$, i.e.,

$$\sum_{j=1}^{n}\left(1-\sqrt{1+\frac{w_{ij}}{w_{i1}}(a_{i1}^2-2a_{i1})}\right)=1,$$

from which we can first determine a unique $0<a_{i1}<1$, and then $a_{ij},j\geq 2$ by the equation (23).

B Proofs

The following lemma is used to compute the expected utility of a quadratic function (see the result on page 382 in Vives (2008) or Lemma A.1 in the Appendix in Marín and Rahi (1999)).

LEMMA **1.** *Suppose that z is an n-dimensional normal random vector with mean 0 and positive definite variance-covariance matrix Σ, and B is a symmetric $n \times n$ matrix. If the matrix $(\Sigma^{-1} + 2B)$ is positive definite, then $\mathbb{E}[\exp(-z'Bz)] = (\det(I_n + 2\Sigma B))^{-\frac{1}{2}}$, where I_n denotes the identity matrix in $\mathbb{R}^n$ and $\det(\cdot)$ is the determinant operator.*

Proof of Proposition 1

The weighted average of the suggested strategies by the advisers of the investor i can be expressed as

$$x_i^* = \sum_{j=1}^{n} a_{ij} x_{ij} = \rho_i^{-1} \left(\bar{\tau}_i \theta + \xi_i - \left(\bar{\tau}_i + \frac{\tau_\theta}{1 + \rho^{-1}\Delta\tau_u} \right) p \right),$$

where $\bar{\tau}_i = \sum_{j=1}^{n} a_{ij}\tau_{ij}$, $\xi_i = \sum_{j=1}^{n} a_{ij}\tau_{ij}\epsilon_{ij}$.

We intend to apply Lemma 1 for $z = (\theta - p, \xi_i, p)'$. The variance-covariance matrix Σ and B are given by

$$\Sigma = \begin{pmatrix} \gamma & 0 & -\alpha \\ 0 & \mathrm{Var}(\xi_i) & 0 \\ -\alpha & 0 & \frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2} \end{pmatrix}, \quad B = \begin{pmatrix} \bar{\tau}_i & \frac{1}{2} & -\frac{\rho\beta}{2} \\ \frac{1}{2} & 0 & 0 \\ -\frac{\rho\beta}{2} & 0 & 0 \end{pmatrix},$$

where $\beta = \frac{\tau_\theta}{\Delta\tau_u + \rho}$,

$$\gamma = \frac{\beta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2} = \frac{\tau_\theta + (\Delta\tau_u + \rho)^2/\tau_u}{(\Delta^2\tau_u + \Delta\rho + \tau_\theta)^2}, \quad \alpha = \frac{-\Delta\beta/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2} = \frac{\Delta\rho + \rho^2/\tau_u}{(\Delta^2\tau_u + \Delta\rho + \tau_\theta)^2}.$$

In order to apply Lemma 1, we need that $\Sigma^{-1} + 2B$ is positive definite. We first show the following claim. Suppose $z'Bz = \hat{z}'\hat{B}\hat{z}$, for some symmetric matrix $\hat{B}$, where z, $\hat{z}$ are two normal random vectors. Let Γ be invertible such that $\hat{z} = \Gamma z$ holds and let Σ and $\hat{\Sigma}$ denote the respective positive definite variance-covariance matrices of z and $\hat{z}$, respectively. Clearly, we have $\hat{\Sigma} = \Gamma\Sigma\Gamma'$. We claim that $\Sigma^{-1} + 2B$ is positive definite if and only if $\hat{\Sigma}^{-1} + 2\hat{B}$ is positive definite. First, from $\hat{z}'\hat{B}\hat{z} = z'\Gamma'\hat{B}\Gamma z = z'Bz$, we have $B = \Gamma'\hat{B}\Gamma$. Then it follows that $\Sigma^{-1} + 2B = \Gamma'\hat{\Sigma}^{-1}\Gamma + 2\Gamma'\hat{B}\Gamma = \Gamma'(\hat{\Sigma}^{-1} + 2\hat{B})\Gamma$, which implies the claim.

Observe that we can alternatively write $\rho_i x_i^*(\theta - p)$ as $\hat{z}'\hat{B}\hat{z}$ for some normal random vector $\hat{z}$ and symmetric matrix $\hat{B}$. In fact, from the expressions $p = (\Delta\theta - u)/(\Delta + \beta)$ (see Equation (2)) and $\theta - p = (\beta\theta + u)/(\Delta + \beta)$, we have

$$\begin{aligned}\rho_i x_i^*(\theta - p) &= (\bar{\tau}_i(\theta - p) + \xi_i - \rho\beta p)(\theta - p)\\ &= \frac{1}{\Delta + \beta}\left(\bar{\tau}_i \frac{\beta\theta + u}{\Delta + \beta} + \xi_i - \rho\beta\frac{\Delta\theta - u}{\Delta + \beta}\right)(\beta\theta + u)\\ &= \frac{1}{\Delta + \beta}\left(\frac{(\bar{\tau}_i - \rho\Delta)\beta}{\Delta + \beta}\theta + \frac{\bar{\tau}_i + \rho\beta}{\Delta + \beta}u + \xi_i\right)(\beta\theta + u),\end{aligned}$$

which can be written as $\hat{z}'\hat{B}\hat{z}$ with $\hat{z} = (\theta, u, \xi_i)$ and

$$\hat{B} = \frac{1}{\Delta + \beta}\begin{pmatrix} \frac{(\bar{\tau}_i - \rho\Delta)\beta^2}{\Delta+\beta} & \frac{(\bar{\tau}_i + \rho(\beta - \Delta)/2)\beta}{\Delta+\beta} & \frac{\beta}{2} \\ \frac{(\bar{\tau}_i + \rho(\beta - \Delta)/2)\beta}{\Delta+\beta} & \frac{\bar{\tau}_i + \rho\beta}{\Delta+\beta} & \frac{1}{2} \\ \frac{\beta}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$

Let $\hat{\Sigma}$ denote the variance-covariance matrix of random vector $\hat{z}$. Some simple calculations give

$$\hat{\Sigma}^{-1} + 2\hat{B} = \begin{pmatrix} \tau_\theta + \frac{2(\bar{\tau}_i - \rho\Delta)\beta^2}{(\Delta+\beta)^2} & \frac{(2\bar{\tau}_i + \rho(\beta - \Delta))\beta}{(\Delta+\beta)^2} & \frac{\beta}{\Delta+\beta} \\ \frac{(2\bar{\tau}_i + \rho(\beta - \Delta))\beta}{(\Delta+\beta)^2} & \tau_u + \frac{2(\bar{\tau}_i + \rho\beta)}{(\Delta+\beta)^2} & \frac{1}{\Delta+\beta} \\ \frac{\beta}{\Delta+\beta} & \frac{1}{\Delta+\beta} & \frac{1}{\bar{\tau}_i} \end{pmatrix}.$$

By some simple but tedious derivations, we can show that $\hat{\Sigma}^{-1} + 2\hat{B}$ is positive definite. We omit the details here.

From the expressions $p = (\Delta\theta - u)/(\Delta + \beta)$ and $\theta - p = (\beta\theta + u)/(\Delta + \beta)$ again, we see that

$$(\theta - p, \xi_i, p)' = \begin{pmatrix} \frac{\beta}{\Delta+\beta} & \frac{1}{\Delta+\beta} & 0 \\ 0 & 0 & 1 \\ \frac{\Delta}{\Delta+\beta} & -\frac{1}{\Delta+\beta} & 0 \end{pmatrix}(\theta, u, \xi_i)'$$

is an invertible transformation, by the above claim, we know that matrix $\Sigma^{-1} + 2B$ is positive definite.

Using Lemma 1 with $z = (\theta - p, \xi_i, p)'$, and the matrices Σ, B, we obtain

$$\mathbb{E}[-\exp(-\rho_i x_i^*(\theta - p))] = -(\det(I_3 + 2\Sigma B))^{-\frac{1}{2}} = -(\det(I_3 + 2B\Sigma))^{-\frac{1}{2}},$$

where

$$I_3 + 2B\Sigma = \begin{pmatrix} 1 + 2(\bar{\tau}_i\gamma + \frac{\rho\alpha\beta}{2}) & \mathrm{Var}(\xi_i) & 2\phi \\ \gamma & 1 & -\alpha \\ -\rho\beta\gamma & 0 & 1 + \rho\alpha\beta \end{pmatrix}$$

with $\phi = -\bar{\tau}_i\alpha - \frac{\rho\beta}{2}\frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2}$. Expanding the determinant $\det(I_3 + 2B\Sigma)$ along the first row yields

$$\begin{aligned} \det(I_3 + 2B\Sigma) &= (1 + 2\bar{\tau}_i\gamma + \rho\alpha\beta)(1 + \rho\alpha\beta) - \mathrm{Var}(\xi_i)\gamma + 2\phi\rho\beta\gamma \\ &= (1 + \rho\alpha\beta)^2 - (\rho\beta)^2\gamma\frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta + \beta)^2} + 2\bar{\tau}_i\gamma(1 + \rho\alpha\beta) - \mathrm{Var}(\xi_i)\gamma - 2\bar{\tau}_i\alpha\rho\beta\gamma \\ &= 1 + \rho\alpha\beta + (2\bar{\tau}_i - \mathrm{Var}(\xi_i))\gamma \\ &= 1 + \rho\alpha\beta + \sum_{j=1}^{n}(2a_{ij} - a_{ij}^2)\tau_{ij}\gamma, \end{aligned}$$

where we use the relation

$$\alpha + \rho\beta\left(\alpha^2 - \gamma\frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta + \beta)^2}\right) = 0. \tag{24}$$

Thus, the expression (8) follows.

Similarly, we can show that the expected utility at x_{ij} is given by

$$\mathbb{E}[-\exp(-\rho_i x_{ij}(\theta - p))] = -\left(1 + \rho\alpha\beta + \tau_{ij}\gamma\right)^{-\frac{1}{2}}.$$

Therefore, each investor's welfare by adopting the weighted average x_i^* will be the same as that by directly following the suggested strategy by adviser (i,j) if $\tau_{ij} = \sum_{j=1}^{n}(2a_{ij} - a_{ij}^2)\tau_{ij}$. The first expression (7) then follows from the alternative expression (6) of the expected utility at x_{ij}.

Proof of Proposition 2

We first show Part (i). To economize the notation, here instead we consider the following constrained optimization problem:

$$\max_{a_j, j=1,\ldots,n} \quad \sum_{j=1}^{n}(2a_j\tau_j - a_j^2\tau_j) \quad \text{s.t.} \quad \sum_{j=1}^{n} a_j = 1, a_j \geq 0. \tag{25}$$

and show the following result.

Suppose $\tau_1 \geq \tau_2 \geq \cdots \geq \tau_n > 0$, *and let* $t = \max\{j|2 \leq j \leq n, 1 + \sum_{\ell=1}^{j-1} \frac{\tau_j - \tau_\ell}{\tau_\ell} > 0\}$, $2 \leq t \leq n$. *Then the unique optimal solution to the optimization problem (25) is given by*

$$a_t^* = 1 - \frac{t-1}{1 + \sum_{\ell=1}^{t-1} \frac{\tau_t}{\tau_\ell}};$$

$$a_j^* = \frac{a_t^* \tau_t + \tau_j - \tau_t}{\tau_j} = 1 - \frac{(t-1)\frac{\tau_t}{\tau_j}}{1 + \sum_{\ell=1}^{t-1} \frac{\tau_t}{\tau_\ell}}, j = 1, ..., t-1;$$

$$a_j^* = 0, j = t+1, ..., n,$$

and the solution satisfies that $a_1^* \geq a_2^* \geq \cdots \geq a_t^* > 0$, *where the inequality becomes equality if and only if the corresponding two signal precisions are identical.*

Proof. There exists an optimal solution $(a_j^*)_{j=1,\ldots,n}$ with $0 \leq a_j^* \leq 1$ to this constrained optimization problem since the constraint set is a bounded, closed set and the objective function is continuous. Moreover, the optimal solution is unique since the objective function is strictly convex.

We now derive the necessary conditions that the optimal solution satisfies. We claim that for any i and j with $a_j^* > 0$, it must hold that $a_i^* \tau_i - a_j^* \tau_j = \tau_i - \tau_j$. Let $0 < \varepsilon < a_j^*$ and consider the feasible solution where the i-th component is $a_i^* + \varepsilon$, the j-th component is $a_j^* - \varepsilon$ and the other components equal $a_\ell^*, \ell \neq i, j$. The function value with the feasible solution is given by

$$2(a_i^* + \varepsilon)\tau_i - (a_i^* + \varepsilon)^2 \tau_i + 2(a_j^* - \varepsilon)\tau_j - (a_j^* - \varepsilon)^2 \tau_j + \sum_{\ell \neq i,j} (2a_\ell^* \tau_\ell - (a_\ell^*)^2 \tau_\ell),$$

which achieves its maximum at $\varepsilon = 0$. Taking derivative at $\varepsilon = 0$ leads to the claim. The claim implies that $a_i^* \tau_i \geq a_j^* \tau_j > 0$ whenever $\tau_i \geq \tau_j$ and $a_j^* > 0$, and further that if $a_j^* > 0$, then $a_i^* \geq a_j^* > 0$ for all $i \leq j$ (otherwise, if $a_i^* < a_j^*$, then $a_i^* \tau_i - a_j^* \tau_j < a_j^* (\tau_i - \tau_j) \leq \tau_i - \tau_j$, a contradiction), and $a_i^* = a_j^*$ if and only if $\tau_i = \tau_j$. That is, if the optimal weight given to one low precision is positive, then the optimal weight given to one high precision is larger.

According to the definition of t, we have $2 \leq t \leq n$. We claim that $a_j^* = 0$ for all $j \geq t+1$. Otherwise, let $s = \max\{j|t+1 \leq j \leq n, a_j^* > 0\}$, then from the relation $a_j^* \tau_j - a_s^* \tau_s = \tau_j - \tau_s$ for $j \leq s$, we have $a_j^* = \frac{a_s^* \tau_s + \tau_j - \tau_s}{\tau_j}, j = 1, ..., s$. By the definition of s, $a_j^* = 0$ for $j \geq s+1$. Thus,

$$\sum_{\ell=1}^{n} a_\ell^* = \sum_{\ell=1}^{s} a_\ell^* = \sum_{\ell=1}^{s} \frac{a_s^* \tau_s + \tau_\ell - \tau_s}{\tau_\ell} = 1.$$

We can solve $a_s^* = \frac{1+\sum_{\ell=1}^{s-1}\frac{\tau_s-\tau_\ell}{\tau_\ell}}{1+\sum_{\ell=1}^{s-1}\frac{\tau_s}{\tau_\ell}}$, which is nonpositive from the definition of t, but positive by the definition of s, a contradiction. Thus, $a_j^* = 0$ for all $j \geq t+1$. Similar to the above arguments, we can solve $a_t^* = \frac{1+\sum_{\ell=1}^{t-1}\frac{\tau_t-\tau_\ell}{\tau_\ell}}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}} = 1 - \frac{t-1}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}}$, which is positive by the definition of t, and $a_j^* = \frac{a_t^*\tau_t+\tau_j-\tau_t}{\tau_j} = 1-\frac{(t-1)\frac{\tau_t}{\tau_j}}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}}$ for $j = 1, ..., t-1$ as given in the lemma based on the established relation $a_j^*\tau_j - a_t^*\tau_t = \tau_j - \tau_t$. Moreover, the relation $a_1^* \geq a_2^* \geq \cdots \geq a_t^*$ follows from the result that if $a_j^* > 0$, then $a_i^* \geq a_j^* > 0$ for all $i \leq j$ we have shown in the first paragraph. The last part is straightforward.

Taking the notations in Part (i), we now show Part (ii). In the rest of the proof, we omit the superscript $*$ for simplicity. We have

$$\frac{\partial \sum_{j=1}^n (2a_j - a_j^2)\tau_j}{\partial \tau_1} = 2a_1 - a_1^2 + (2-2a_1)\frac{\partial a_1}{\partial \tau_1}\tau_1 + \sum_{j\neq 1}(2-2a_j)\frac{\partial a_j}{\partial \tau_1}\tau_j. \tag{26}$$

By Proposition 2 Part (i), we have

$$\begin{aligned}
\frac{\partial a_t}{\partial \tau_1} &= -\frac{t-1}{(1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell})^2}\frac{\tau_t}{\tau_1^2},\\
\frac{\partial a_j}{\partial \tau_1} &= \frac{\partial a_t}{\partial \tau_1}\frac{\tau_t}{\tau_j}, j \neq 1,\\
\frac{\partial a_1}{\partial \tau_1} &= \frac{\partial[(a_t-1)\frac{\tau_t}{\tau_1}]}{\partial \tau_1} = \frac{\partial a_t}{\partial \tau_1}\frac{\tau_t}{\tau_1} - (a_t-1)\frac{\tau_t}{\tau_1^2}.
\end{aligned}$$

As a result,

$$\begin{aligned}
(2-2a_1)\frac{\partial a_1}{\partial \tau_1}\tau_1 + \sum_{j\neq 1}(2-2a_j)\frac{\partial a_j}{\partial \tau_1}\tau_j &= \sum_{j=1}^{t}(2-2a_j)\frac{\partial a_t}{\partial \tau_1}\tau_t + (2-2a_1)(1-a_t)\frac{\tau_t}{\tau_1}\\
&= -\frac{(2t-2)(t-1)}{(1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell})^2}\frac{\tau_t^2}{\tau_1^2} + 2\frac{(t-1)\frac{\tau_t}{\tau_1}}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}}\frac{t-1}{1+\sum_{\ell=1}^{t-1}\frac{\tau_t}{\tau_\ell}}\frac{\tau_t}{\tau_1}\\
&= 0.
\end{aligned} \tag{27}$$

From (26) and (27) we obtain

$$\frac{\partial \sum_{j=1}^n (2a_j^* - (a_j^*)^2)\tau_j}{\partial \tau_1} = 2a_1^* - (a_1^*)^2 > 0. \tag{28}$$

The sensitivity analysis with respect to other τ_j is similar and omitted. □

Proof of Proposition 3

The existence and uniqueness of advice-based equilibrium with exogenous information follows from replacing the weights $(a_{zj})_{z=1,\ldots,v,j=1,\ldots,n}$ in the expressions (2), (4) and (5) with the optimal solution $(a^*_{zj})_{z=1,\ldots,v,j=1,\ldots,n}$ given in Proposition 2. □

Proof of Proposition 4

We first show that consultation increases Δ and then improves the price informativeness. From the expression of a^*_{ij} in Proposition 2 and (4), it suffices to show that

$$\sum_{j=1}^{n} a^*_{ij}\tau_{ij} = \sum_{j=1}^{t}\left(1 - \frac{(t-1)\frac{\tau_{it}}{\tau_{ij}}}{1+\sum_{\ell=1}^{t-1}\frac{\tau_{it}}{\tau_{i\ell}}}\right)\tau_{ij} \geq \sum_{j=1}^{n}\tau_{ij}/n,$$

which is equivalent to

$$(n-1)(\tau_{i1}+\tau_{i2}+\cdots+\tau_{it}) \geq \frac{(t-1)\tau_{it}}{\sum_{\ell=1}^{t}\frac{\tau_{it}}{\tau_{i\ell}}}tn + \tau_{i(t+1)} + \tau_{i(t+2)} + \cdots + \tau_{in}.$$

The above is indeed true due to the relations $(\tau_{i1}+\tau_{i2}+\cdots+\tau_{it})\sum_{\ell=1}^{t}\frac{\tau_{it}}{\tau_{i\ell}} \geq t^2\tau_{it}$, $\tau_{i(t+1)} + \tau_{i(t+2)} + \cdots + \tau_{in} \leq (n-t)\tau_{it}$, and $\sum_{\ell=1}^{t}\frac{\tau_{it}}{\tau_{i\ell}} \leq t$. The claim follows.

Now, consider the case where all the advisers in our economy have the same signal precision. From (4) and the definition of the benchmark economy, we know that consultation does not impact Δ, and then not p, and consequently improves equilibrium welfare by (7), (9) and the following relation

$$\sum_{j=1}^{n}(2a^*_{ij} - (a^*_{ij})^2)\tau_{ij} \geq \sum_{j=1}^{n}(2/n - 1/n^2)\tau_{ij} > \sum_{j=1}^{n}\tau_{ij}/n.$$

The proof is completed. □

Proof of Proposition 5

By virtue of Proposition 1, optimization problem (12) is equivalent to

$$\begin{aligned} \sup_{a_{ij},j=1,\dots,n} \inf_{\tau_{ij},j=1,\dots,n} \quad & \sum_{j=1}^{n} (2a_{ij} - a_{ij}^2)\tau_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij} = 1, a_{ij} \geq 0, \\ & \frac{1}{n}\sum_{j=1}^{n} \tau_{ij} = \bar{\tau}_n \end{aligned} \tag{29}$$

Fix $\{a_{ij}\}_{j=1,\dots,n}$ and consider optimization problem $\inf_{\tau_{ij},j=1,\dots,n} \sum_{j=1}^{n}(2a_{ij} - a_{ij}^2)\tau_{ij}$. The lowest value of $(2a_{ij} - a_{ij}^2)$ will be given all the precision $n\bar{\tau}_n$ while other values receive zero precision. That is, $\tau_{ij_1} = n\bar{\tau}_n$ for $j_1 \in \arg\min_{1\leq j\leq n}(2a_{ij} - a_{ij}^2)$, and $\tau_{ij} = 0$ for $j \neq j_1$. Then the robust optimization problem (29) reduces to $\sup_{a_{ij},j=1,\dots,n} \min_{1\leq j\leq n} \ (2a_{ij} - a_{ij}^2)$, which clearly has solution $a_{ij} = 1/n$ for all j. □

Proof of Proposition 6

The conclusion is obvious when $r_k = 1$. We next assume that $r_k \geq 2$. Suppose $\tau_{i1}^* \geq \tau_{i2}^* \geq \cdots \geq \tau_{ir_k}^*$ is an optimal solution to the optimization problem (14) and let the corresponding optimal weights be $a_{i1}^* \geq a_{i2}^* \geq \cdots \geq a_{it}^* > a_{i(t+1)}^* = \cdots = a_{ir_k}^* = 0$. Then taking partial derivative with respect to τ_{i2} over (14) and let $\tau_{i2} = \tau_{i2}^*$ leads to

$$\begin{aligned} & -2\rho_k^{\diamond} c'(\tau_{i2}^*) \left(\tau_\theta + \Delta^2\tau_u + \sum_{j=1}^{r_k} (2a_{ij}^* - (a_{ij}^*)^2)\tau_{ij}^* \right) \\ & \qquad + \left. \frac{\partial[\sum_{j\neq 2}^{r_k}(2a_{ij}^* - (a_{ij}^*)^2)\tau_{ij}^* + (2a_{i2}^* - (a_{i2}^*)^2)\tau_{i2}]}{\partial \tau_{i2}} \right|_{\tau_{i2}=\tau_{i2}^*} = 0. \end{aligned} \tag{30}$$

We next remove the superscript $*$ for simplifying notations. It follows the relation (28) that

$$\frac{\partial \sum_{j=1}^{r_k}(2a_{ij} - a_{ij}^2)\tau_{ij}}{\partial \tau_{i2}} = 2a_{i2} - a_{i2}^2, \tag{31}$$

and consequently, from (30)

$$-2\rho_k^{\diamond} c'(\tau_{i2}) \left(\tau_\theta + \Delta^2\tau_u + \sum_{j=1}^{r_k} (2a_{ij} - a_{ij}^2)\tau_{ij} \right) + 2a_{i2} - a_{i2}^2 = 0. \tag{32}$$

Similarly, we can also show that

$$\frac{\partial \sum_{j=1}^{r_k}(2a_{ij}-a_{ij}^2)\tau_{ij}}{\partial \tau_{i1}} = 2a_{i1}-a_{i1}^2. \tag{33}$$

We show the conclusion by contradiction. Without loss of generality, we assume that $\tau_{i1} > \tau_{i2}$ (otherwise if $\tau_{i1} = \tau_{i2}$, then we consider $\{\tau_{ij_1}, \tau_{i(j_1+1)}\}$ instead of $\{\tau_{i1}, \tau_{i2}\}$ and a similar contradiction can be obtained, where j_1 is the smallest index such that $\tau_{ij_1} \neq \tau_{i(j_1+1)}$). Let $0 < \varepsilon < \tau_{i1}$ and let us consider the feasible solution $(\tau_{i1}-\varepsilon, \tau_{i2}+\varepsilon, \tau_{i3}, ..., \tau_{ir_k})$. The corresponding function value of the objective in (14) at this feasible solution is given by

$$\begin{aligned} g(\varepsilon) &:= \exp\left(-2\rho_k^\diamond \sum_{j\geq 3} c(\tau_{ij})\right) \exp\left(-2\rho_k^\diamond (c(\tau_{i1}-\varepsilon)+c(\tau_{i2}+\varepsilon))\right) \\ &\quad\times \left(\tau_\theta + \Delta^2\tau_u + \sum_{j\geq 3}^{r_k}(2a_{ij}-a_{ij}^2)\tau_{ij} + (2a_{i1}-a_{i1}^2)(\tau_{i1}-\varepsilon) + (2a_{i2}-a_{i2}^2)(\tau_{i2}+\varepsilon)\right). \end{aligned} \tag{34}$$

Note that in (34), $(a_{ij})_{j=1,\ldots,r_k}$ are the optimal weights corresponding to $(\tau_{i1}-\varepsilon, \tau_{i2}+\varepsilon, \tau_{i3}, ..., \tau_{ir_k})$ and depend on ε. It is clear that $g(0) \geq g(\varepsilon)$ for any small ε by the optimality of $(\tau_{i1}, ..., \tau_{ir_k})$.

From (34), we have

$$\begin{aligned} &\frac{\partial g(\varepsilon)}{\partial \varepsilon}\Big|_{\varepsilon=0} \\ &\propto 2\rho_k^\diamond(c'(\tau_{i1})-c'(\tau_{i2}))\left(\tau_\theta+\Delta^2\tau_u+\sum_{j=1}^{r_k}(2a_{ij}-a_{ij}^2)\tau_{ij}\right) \\ &\qquad - \frac{\partial \sum_{j=1}^{r_k}(2a_{ij}-a_{ij}^2)\tau_{ij}}{\partial \tau_{i1}} + \frac{\partial \sum_{j=1}^{r_k}(2a_{ij}-a_{ij}^2)\tau_{ij}}{\partial \tau_{i2}} \\ &\propto 2\rho_k^\diamond(c'(\tau_{i1})-c'(\tau_{i2}))\left(\tau_\theta+\Delta^2\tau_u+\sum_{j=1}^{r_k}(2a_{ij}-a_{ij}^2)\tau_{ij}\right) - (2-a_{i1}-a_{i2})(a_{i1}-a_{i2}) \\ &\geq 2\rho_k^\diamond c''(\tau_{i2})(\tau_{i1}-\tau_{i2})\left(\tau_\theta+\Delta^2\tau_u+\sum_{j=1}^{r_k}(2a_{ij}-a_{ij}^2)\tau_{ij}\right) - (2-a_{i1}-a_{i2})(1-a_{i2})\frac{\tau_{i1}-\tau_{i2}}{\tau_{i1}} \\ &\propto 2\rho_k^\diamond c''(\tau_{i2})\tau_{i1}\left(\tau_\theta+\Delta^2\tau_u+\sum_{j=1}^{r_k}(2a_{ij}-a_{ij}^2)\tau_{ij}\right) - (2-a_{i1}-a_{i2})(1-a_{i2}) \\ &= 2\rho_k^\diamond c''(\tau_{i2})\tau_{i1}\frac{2a_{i2}-a_{i2}^2}{2\rho_k^\diamond c'(\tau_{i2})} - (2-a_{i1}-a_{i2})(1-a_{i2}) \\ &= a_{i2}(2-a_{i2})\frac{c''(\tau_{i2})\tau_{i1}}{c'(\tau_{i2})} - (2-a_{i1}-a_{i2})(1-a_{i2}) \\ &> \frac{(2-a_{i2})(1-a_{i1})\frac{\tau_{i1}}{\tau_{i2}}}{r_k-1}\frac{c''(\tau_{i2})\tau_{i2}}{c'(\tau_{i2})} - (2-a_{i2})(1-a_{i2}) \end{aligned}$$

$$\propto \frac{1}{r_k - 1} \frac{c''(\tau_{i2})\tau_{i2}}{c'(\tau_{i2})} - 1,$$

which is positive under the condition in the proposition, where the second $\propto$ follows from (31) and (33), the first inequality follows from the relations

$$c'(\tau_{i1}) - c'(\tau_{i2}) = c''(\tau)(\tau_{i1} - \tau_{i2}) \geq c''(\tau_{i2})(\tau_{i1} - \tau_{i2})$$

for some $\tau_{i2} < \tau < \tau_{i1}$ (using the increasingness of $c''(\cdot)$ assumed in the proposition) and

$$a_{i1} - a_{i2} = (1 - a_{it})\tau_{it} \left(\frac{1}{\tau_{i2}} - \frac{1}{\tau_{i1}} \right) = (1 - a_{i2})\tau_{i2} \left(\frac{1}{\tau_{i2}} - \frac{1}{\tau_{i1}} \right) = (1 - a_{i2}) \frac{\tau_{i1} - \tau_{i2}}{\tau_{i1}},$$

the first equality from (32), the second inequality from the relation $a_{i2} \geq \cdots \geq a_{it}$ and then $a_{i2} \geq \sum_{j=2}^{t} a_{ij}/(t-1) \geq (1 - a_{i1})/(r_k - 1)$, the last $\propto$ from the relation

$$(1 - a_{i1}) \frac{\tau_{i1}}{\tau_{i2}} = \frac{(t-1)\frac{\tau_{it}}{\tau_{i1}}}{1 + \sum_{\ell=1}^{t-1} \frac{\tau_{it}}{\tau_{i\ell}}} \frac{\tau_{i1}}{\tau_{i2}} = \frac{(t-1)\frac{\tau_{it}}{\tau_{i2}}}{1 + \sum_{\ell=1}^{t-1} \frac{\tau_{it}}{\tau_{i\ell}}} = 1 - a_{i2}.$$

Thus, $g(\varepsilon) > g(0)$ for sufficiently small ε. This contradicts the optimality of $(\tau_{ij})_{j=1,\ldots,r_k}$. This completes the proof. □

Proof of Proposition 7

It follows from (7) that

$$\begin{aligned}
&\mathbb{E}[U_i(W(x^*_{\mathcal{N}_i}(\tau_i)) - r_k c(\tau_i))] \\
&= -\exp\left(\rho^\diamond_k r_k c(\tau_i)\right) \left(\mathrm{Var}(\theta - p_{\mathbf{r}}) \left(\tau_\theta + \Delta^2_{\mathbf{r}} \tau_u + \left(2 - \frac{1}{r_k}\right) \tau_i \right) \right)^{-\frac{1}{2}} \\
&= -\left(\mathrm{Var}(\theta - p_{\mathbf{r}}) \exp\left(-2\rho^\diamond_k r_k c(\tau_i)\right) \left(\tau_\theta + \Delta^2_{\mathbf{r}} \tau_u + \left(2 - \frac{1}{r_k}\right) \tau_i \right) \right)^{-\frac{1}{2}}. \qquad (35)
\end{aligned}$$

By taking derivative with respect to τ^*_i for both sides of (35), we see that τ^*_i is determined by

$$r_k c'(\tau^*_i) = \frac{2 - 1/r_k}{2\rho^\diamond_k \left(\tau_\theta + \Delta^2_{\mathbf{r}} \tau_u + (2 - 1/r_k)\tau^*_i \right)}, \qquad (36)$$

from which we conclude that $\tau^*_i = \tau^*_j$, denoted as $\tau^*_k(\mathbf{r})$, for any i, j with the same risk aversion coefficient $\rho^\diamond_k$. It then follows from (36) that $\{\tau^*_k(\mathbf{r})\}_{k=1}^m$ satisfies

$$c'(\tau^*_k(\mathbf{r})) = \frac{\frac{2}{r_k} - \frac{1}{r_k^2}}{2\rho^\diamond_k \left(\tau_\theta + \Delta^2_{\mathbf{r}} \tau_u + (2 - 1/r_k)\tau^*_k(\mathbf{r}) \right)}$$

$$= \frac{1}{2\rho_k^\diamond \left(\frac{r_k}{2-\frac{1}{r_k}} (\tau_\theta + \Delta_{\mathbf{r}}^2 \tau_u) + r_k \tau_k^*(\mathbf{r}) \right)}, \qquad k = 1, ..., m. \tag{37}$$

We next show existence of an equilibrium by showing that the system of equations (37) has a solution. Recall that for each $k \in \{1, ..., m\}$, λ_k denotes the non-negative fraction of groups with risk aversion coefficient $\rho_k^\diamond$ in the limit economy. Observing (37), we define the mapping $\boldsymbol{f} = (f_1, f_2, ..., f_m)$, $f_k : (0, \infty)^m \to (0, \infty)$ as follows:

$$f_k(\tau) = (c')^{-1} \left(\frac{1}{2\rho_k^\diamond \left(\frac{r_k}{2-\frac{1}{r_k}} (\tau_\theta + (\Delta(\boldsymbol{\tau}))^2 \tau_u) + r_k \tau_k \right)} \right),$$

where $\boldsymbol{\tau} = (\tau_1, ..., \tau_m)$, $\Delta(\boldsymbol{\tau}) = \sum_{k=1}^m \lambda_k \tau_k / \rho_k^\diamond$. Let $\delta_{max} = 1/\left(2\tau_\theta \min_{1\le k\le m} \frac{\rho_k^\diamond r_k}{2-\frac{1}{r_k}}\right)$ and

$$\delta_{min} = \frac{1}{2} \left(\max_{1\le k\le m} \left(\rho_k^\diamond \frac{r_k}{2-\frac{1}{r_k}} \right) \left(\tau_\theta + \left((c')^{-1}(\delta_{max}) \sum_{k=1}^m \frac{\lambda_k}{\rho_k^\diamond} \right)^2 \tau_u \right) + \max_{1\le k\le m} (\rho_k^\diamond r_k)(c')^{-1}(\delta_{max}) \right)^{-1}.$$

We then can see that

$$(c')^{-1}(\delta_{min}) \le f_k(\tau) \le (c')^{-1}(\delta_{max})$$

for any $\tau \in (0, \infty)^m$ with $|\tau_k| \le (c')^{-1}(\delta_{max})$, $k = 1, ..., m$. Hence, the mapping $\boldsymbol{f}$ maps the convex, compact set $[(c')^{-1}(\delta_{min}), (c')^{-1}(\delta_{max})]^m$ into itself. Note that the two numbers of $(c')^{-1}(\delta_{min})$ and $(c')^{-1}(\delta_{max})$ are well-defined due to the conditions $\lim_{\tau\to 0} c'(\tau) = 0$ and $\lim_{\tau\to\infty} c'(\tau) = \infty$. In addition, the mapping $\boldsymbol{f}$ is also continuous over $(0, \infty)^m$. Hence applying Brouwer's Fixed Point Theorem to the mapping $\boldsymbol{f}(\cdot)$ will lead to a fixed point, or the solution to (37).

Finally, we show uniqueness. Suppose that both $\{\tau_k^*(\mathbf{r})\}_{k=1}^m$ and $\{\hat{\tau}_k^*(\mathbf{r})\}_{k=1}^m$ are solutions to (37). Then we first claim that $\hat{\Delta}_{\mathbf{r}} := \sum_{k=1}^m \lambda_k \hat{\tau}_k^*(\mathbf{r})/\rho_k^\diamond = \Delta_{\mathbf{r}}$. Otherwise, if $\hat{\Delta}_{\mathbf{r}} > \Delta_{\mathbf{r}}$, then $\hat{\tau}_k^*(\mathbf{r}) < \tau_k^*(\mathbf{r})$ for every $k = 1, ..., m$ from (37), and consequently, $\hat{\Delta}_{\mathbf{r}} < \Delta_{\mathbf{r}}$, a contradiction. A similar contradiction also arises if $\hat{\Delta}_{\mathbf{r}} < \Delta_{\mathbf{r}}$. Hence the claim $\hat{\Delta}_{\mathbf{r}} = \Delta_{\mathbf{r}}$ follows and then $\{\tau_k^*(\mathbf{r})\}_{k=1}^m$ is uniquely determined by (37). We complete the proof of Part (i).

Parts (ii) and (iii) follow directly from (35) with a replacement of τ_i with $\tau_k^*(\mathbf{r})$ and (37). $\square$

Proof of Proposition 8

Consider any investor i with risk aversion coefficient $\rho_k^\diamond$ and his/her any possible consultation set $\mathcal{N}$ with $|\mathcal{N}| = s \le n$, and denote $D_s = \mathbb{E}\left[U_i\left(W\left(x_{\mathcal{N}}^*(\tau_{\mathcal{N}}^*)\right) - sc\left(\tau_{\mathcal{N}}^*\right)\right)\right]$. From (35) and (36), the expected utility of investor i is given by

$$D_s = -\left[\mathrm{Var}(\theta - p_{\mathbf{r}})\exp\left(-2\rho_k^\diamond sc(\tau_{\mathcal{N}}^*(s))\right)\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2 - \frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)\right)\right]^{-\frac{1}{2}},$$

where $\tau_{\mathcal{N}}^*(s)$ satisfies

$$2\rho_k^\diamond c'(\tau_{\mathcal{N}}^*(s))\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2 - \frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)\right) = \frac{2}{s} - \frac{1}{s^2}. \tag{38}$$

By letting s be a fictitious, continuous variable taking values in $[1, n]$, we first analyze the monotonicity of D_s, or equivalently,

$$\hat{D}_s = \exp\left(-2\rho_k^\diamond sc(\tau_{\mathcal{N}}^*(s))\right)\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2 - \frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)\right).$$

We have

$$\begin{aligned}
\frac{\partial \hat{D}_s}{\partial s} &\propto -\left(2\rho_k^\diamond sc'(\tau_{\mathcal{N}}^*(s))\frac{\partial \tau_{\mathcal{N}}^*(s)}{\partial s} + 2\rho_k^\diamond c(\tau_{\mathcal{N}}^*(s))\right)\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2 - \frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)\right) \\
&\quad + \frac{\tau_{\mathcal{N}}^*(s)}{s^2} + \left(2 - \frac{1}{s}\right)\frac{\partial \tau_{\mathcal{N}}^*(s)}{\partial s} \\
&= \frac{\tau_{\mathcal{N}}^*(s)}{s^2} - 2\rho_k^\diamond c(\tau_{\mathcal{N}}^*(s))\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2 - \frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)\right) \\
&= \frac{\tau_{\mathcal{N}}^*(s)}{s^2} - \frac{c(\tau_{\mathcal{N}}^*(s))}{c'(\tau_{\mathcal{N}}^*(s))}\left(\frac{2}{s} - \frac{1}{s^2}\right) \\
&\propto \tau_{\mathcal{N}}^*(s) - \frac{c(\tau_{\mathcal{N}}^*(s))}{c'(\tau_{\mathcal{N}}^*(s))}(2s - 1),
\end{aligned} \tag{39}$$

where the two equalities follow from (38). Thus, the lower and upper bound in the proposition follow from (39).

We now consider the case of quadratic cost function $c(\tau) = \kappa\tau^2$. From (39), we have $\frac{\partial \hat{D}_s}{\partial s} \propto 3 - 2s$, which is negative when $s \ge 2$. Hence $\hat{D}_s$ is strictly decreasing in s for $s \ge 2$. We next show that $\hat{D}_2 > \hat{D}_1$. Let $\bar{D}_s = \log(\hat{D}_s)$, $s = 1, 2$. We have

$$\bar{D}_s = \log\left(\bar{\tau}_\theta + \bar{\tau}_{\mathcal{N}}^*(s)\right) - 2\rho_k^\diamond \kappa s(\tau_{\mathcal{N}}^*(s))^2,$$

where $\bar{\tau}_\theta = \tau_\theta + \Delta_{\mathbf{r}}^2\tau_u$, $\bar{\tau}_{\mathcal{N}}^*(s) = (2 - 1/s)\tau_{\mathcal{N}}^*(s)$. From (38), we have

$$4\rho_k^\diamond \kappa \bar{\tau}_{\mathcal{N}}^*(s)(\bar{\tau}_\theta + \bar{\tau}_{\mathcal{N}}^*(s)) = \frac{1}{s}\left(2 - \frac{1}{s}\right)^2,$$

so that

$$\bar{\tau}^*_{\mathcal{N}}(s) = \frac{1}{2}\left(-\bar{\tau}_\theta + \sqrt{(\bar{\tau}_\theta)^2 + \frac{\frac{1}{s}(2-\frac{1}{s})^2}{\rho_k^\diamond \kappa}}\right).$$

Hence,

$$\begin{aligned}
\bar{D}_s &= \log\left(\bar{\tau}_\theta + \bar{\tau}^*_{\mathcal{N}}(s)\right) - 2\rho_k^\diamond \kappa s (\tau^*_{\mathcal{N}}(s))^2 \\
&= \log\left(\bar{\tau}_\theta + \bar{\tau}^*_{\mathcal{N}}(s)\right) - \frac{2\rho_k^\diamond \kappa s}{\left(2-\frac{1}{s}\right)^2}(\bar{\tau}^*_{\mathcal{N}}(s))^2 \\
&= \log\left(\frac{\bar{\tau}_\theta + \sqrt{(\bar{\tau}_\theta)^2 + \frac{\frac{1}{s}\left(2-\frac{1}{s}\right)^2}{\rho_k^\diamond\kappa}}}{2}\right) - \frac{2\rho_k^\diamond \kappa s}{\left(2-\frac{1}{s}\right)^2}\frac{2(\bar{\tau}_\theta)^2 + \frac{\frac{1}{s}\left(2-\frac{1}{s}\right)^2}{\rho_k^\diamond\kappa} - 2\bar{\tau}_\theta\sqrt{(\bar{\tau}_\theta)^2 + \frac{\frac{1}{s}\left(2-\frac{1}{s}\right)^2}{\rho_k^\diamond\kappa}}}{4} \\
&= \log\left(1 + \sqrt{1 + \frac{\frac{1}{s}\left(2-\frac{1}{s}\right)^2}{\rho_k^\diamond\kappa(\bar{\tau}_\theta)^2}}\right) - \log\left(2/\bar{\tau}_\theta\right) - 1/2 \\
&\quad - \frac{s}{2\left(2-\frac{1}{s}\right)^2}\left(2\rho_k^\diamond\kappa(\bar{\tau}_\theta)^2 - 2\sqrt{(\rho_k^\diamond\kappa(\bar{\tau}_\theta)^2)^2 + \frac{1}{s}\left(2-\frac{1}{s}\right)^2\rho_k^\diamond\kappa(\bar{\tau}_\theta)^2}\right) \\
&=: q(s,b) - \log(2/\bar{\tau}_\theta) - 1/2,
\end{aligned}$$

where

$$q(s,b) = \log\left(1 + \sqrt{1 + \frac{1}{s}\frac{(2-\frac{1}{s})^2}{b}}\right) - \frac{1 - \sqrt{1 + \frac{1}{s}\frac{(2-\frac{1}{s})^2}{b}}}{\left(2-\frac{1}{s}\right)^2} bs$$

with $b = \rho_k^\diamond\kappa(\bar{\tau}_\theta)^2$. As a result,

$$q(2,b) - q(1,b) = \log\left(\frac{1 + \sqrt{1 + \frac{9}{8b}}}{1 + \sqrt{1 + \frac{1}{b}}}\right) - \left(\frac{8}{9} - 1 - \frac{8}{9}\sqrt{1 + \frac{9}{8b}} + \sqrt{1 + \frac{1}{b}}\right) b.$$

We have

$$\begin{aligned}
&\frac{d(q(2,b) - q(1,b))}{db} \\
&= -\frac{\frac{1}{2}\frac{\frac{9}{8b^2}}{\sqrt{1+\frac{9}{8b}}}}{1 + \sqrt{1 + \frac{9}{8b}}} + \frac{\frac{1}{2}\frac{\frac{1}{b^2}}{\sqrt{1+\frac{1}{b}}}}{1 + \sqrt{1 + \frac{1}{b}}} + \frac{1}{9} + \frac{4}{9}\frac{2b + \frac{9}{8}}{\sqrt{b^2 + \frac{9}{8}b}} - \frac{1}{2}\frac{2b+1}{\sqrt{b^2+b}} \\
&\propto \frac{8}{9}\frac{2b + \frac{9}{8}}{\sqrt{b^2 + \frac{9}{8}b}} - \frac{9}{8}\frac{1}{b^2 + \frac{9}{8}b + b\sqrt{b^2 + \frac{9}{8}b}} + \frac{1}{b^2 + b + b\sqrt{b^2+b}} - \frac{2b+1}{\sqrt{b^2+b}} + \frac{2}{9}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\frac{8}{9}(2b+\frac{9}{8})\left(b+\sqrt{b^2+\frac{9}{8}b}\right)-\frac{9}{8}}{b^2+\frac{9}{8}b+b\sqrt{b^2+\frac{9}{8}b}} + \frac{1-(2b+1)(b+\sqrt{b^2+b})}{b^2+b+b\sqrt{b^2+b}} + \frac{2}{9} \\
&\propto \frac{8}{9}\left(2b+\frac{9}{8}\right)\left(b+\sqrt{b^2+\frac{9}{8}b}\right)(\sqrt{b^2+b}+b+1) \\
&\quad -(2b+1)\left(b+\sqrt{b^2+b}\right)\left(\sqrt{b^2+\frac{9}{8}b}+b+\frac{9}{8}\right)-\frac{9}{8}\left(\sqrt{b^2+b}+b+1\right) \\
&\quad +\left(\sqrt{b^2+\frac{9}{8}b}+b+\frac{9}{8}\right)+\frac{2}{9}\sqrt{b^2+b}\left(b+\sqrt{b^2+b}\right)\left(\sqrt{b^2+\frac{9}{8}b}+b+\frac{9}{8}\right).
\end{aligned}
$$

Then

$$
\begin{aligned}
&\frac{d(q(2,b)-q(1,b))}{db} \\
&\propto \frac{8}{9}\left(2b+\frac{9}{8}\right)\left(b+\sqrt{b^2+\frac{9}{8}b}\right)(\sqrt{b^2+b}+b+1) \\
&\quad -(2b+1)\left(b+\sqrt{b^2+b}\right)\left(\sqrt{b^2+\frac{9}{8}b}+b\right)-\frac{9}{4}(b+1)\left(\sqrt{b^2+b}+b\right) \\
&\quad +\sqrt{b^2+\frac{9}{8}b}+b+\frac{2b}{9}\left(\sqrt{b^2+\frac{9}{8}b}+b+\frac{9}{8}\right)\left(\sqrt{b^2+b}+b+1\right) \\
&= \left(\frac{16b}{9}+1-\frac{2b}{9}\left(b+\sqrt{b^2+b}\right)+1+\frac{2b}{9}\left(b+\sqrt{b^2+b}\right)+\frac{2b}{9}\right)\left(b+\sqrt{b^2+\frac{9}{8}b}\right) \\
&\quad +\frac{b}{4}\left(\sqrt{b^2+b}+b+1\right)-\frac{9}{4}(b+1)\left(\sqrt{b^2+b}+b\right) \\
&= 2(b+1)\left(b+\sqrt{b^2+\frac{9}{8}b}\right)-\left(2b+\frac{9}{4}\right)\left(\sqrt{b^2+b}+b\right)+\frac{b}{4} \\
&= 2(b+1)\sqrt{b^2+\frac{9}{8}b}-\left(2b+\frac{9}{4}\right)\sqrt{b^2+b} \\
&\propto (b+1)-\left(b+\frac{9}{8}\right) \\
&= -\frac{1}{8} < 0.
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
\lim_{b\to\infty}(q(2,b)-q(1,b)) &= \lim_{b\to\infty}\left(b\left(1-\sqrt{1+\frac{1}{b}}\right)-\frac{8b}{9}\left(1-\sqrt{1+\frac{9}{8b}}\right)\right) \\
&= \lim_{z\to 0}\left(\frac{1-\sqrt{1+z}}{z}-\frac{8}{9}\frac{1-\sqrt{1+\frac{9z}{8}}}{z}\right)
\end{aligned}
$$

$$= -\frac{1}{2} + \frac{1}{2} = 0,$$

and

$$\lim_{b \to 0} (q(2,b) - q(1,b)) = \sqrt{\frac{9}{8}} > 0.$$

Thus, $\hat{D}_2 > \hat{D}_1$. Moreover, it is clear from the above arguments that the order of the D_j's does not depend on $\rho_k^{\diamond}$ and $\Delta_{\mathbf{r}}$. The above arguments show that the fully endogenous equilibrium is unique and satisfies that $r_k = 2$ for every $1 \le k \le m$. The existence of fully endogenous equilibrium follows from Proposition 7 by setting $r_k = 2$ for every $1 \le k \le m$. □

Proof of Proposition 9

We omit the subscript $\mathbf{r}$ from the notations $p_{\mathbf{r}}$ and $\Delta_{\mathbf{r}}$ for this proof. First, from (38) we can show by contradiction that consultation reduces Δ and then impairs the price informativeness.

Second, recall that $\mathrm{Var}(\theta - p) = \left(\frac{\beta^2}{\tau_\theta} + \frac{1}{\tau_u}\right) / (\Delta + \beta)^2$, where $\beta = \frac{\tau_\theta}{\Delta\tau_u + \rho}$. Direct computations lead to

$$\begin{aligned}
\frac{\partial \,\mathrm{Var}(\theta - p)}{\partial \Delta} &= \frac{\frac{2\beta}{\tau_\theta}\frac{\partial\beta}{\partial\Delta}(\Delta+\beta)^2 - 2(\frac{\beta^2}{\tau_\theta} + \frac{1}{\tau_u})(\Delta+\beta)(1+\frac{\partial\beta}{\partial\Delta})}{(\Delta+\beta)^4} \\
&= \frac{2}{(\Delta+\beta)^3}\left(\frac{\beta}{\tau_\theta}\frac{\partial\beta}{\partial\Delta}(\Delta+\beta) - \left(\frac{\beta^2}{\tau_\theta} + \frac{1}{\tau_u}\right)\left(1 + \frac{\partial\beta}{\partial\Delta}\right)\right) \\
&= \frac{2}{(\Delta+\beta)^3}\left(\frac{\beta}{\tau_\theta}\frac{\partial\beta}{\partial\Delta}\Delta - \frac{\beta^2}{\tau_\theta} - \frac{1}{\tau_u} - \frac{1}{\tau_u}\frac{\partial\beta}{\partial\Delta}\right) \\
&= \frac{2}{(\Delta+\beta)^3}\left(\left(\frac{\Delta}{\Delta\tau_u+\rho} - \frac{1}{\tau_u}\right)\frac{\partial\beta}{\partial\Delta} - \frac{1}{\tau_u} - \frac{\tau_\theta}{(\Delta\tau_u+\rho)^2}\right) \\
&= -\frac{2}{(\Delta+\beta)^3}\left(\frac{\Delta}{\Delta\tau_u+\rho}\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2} + \frac{1}{\tau_u}\right) \qquad (40) \\
&< 0,
\end{aligned}$$

where we use the relation $\frac{\partial\beta}{\partial\Delta} = -\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2}$. Since we already know that consultation decreases Δ, we can conclude that consultation increases return volatility.

Third, direct calculations show that

$$\begin{aligned}
&\frac{\partial[\mathrm{Var}(\theta-p)\exp\left(-2\rho_k^{\diamond} sc(\tau_{\mathcal{N}}^*(s))\right)\left(\tau_\theta + \Delta^2\tau_u + \left(2 - \frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)\right)]}{\partial s} \\
&\propto \mathrm{Var}(\theta - p)\left(\tau_{\mathcal{N}}^*(s) - \frac{c(\tau_{\mathcal{N}}^*(s))}{c'(\tau_{\mathcal{N}}^*(s))}(2s-1) + 2\Delta\tau_u\frac{\partial\Delta}{\partial s}\right)
\end{aligned}$$

$$+\frac{\partial \operatorname{Var}(\theta-p)}{\partial \Delta}\frac{\partial \Delta}{\partial s}\left(\tau_\theta+\Delta^2\tau_u+\left(2-\frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)\right),$$

where we use the relation (39). Since $\tau_{\mathcal{N}}^*(s)-\frac{c(\tau_{\mathcal{N}}^*(s))}{c'(\tau_{\mathcal{N}}^*(s))}(2s-1)>0$ and $\frac{\partial\Delta}{\partial s}<0$, in order to show that the derivative is positive it suffices to show that $2\Delta\tau_u \operatorname{Var}(\theta-p)+\frac{\partial \operatorname{Var}(\theta-p)}{\partial\Delta}(\tau_\theta+\Delta^2\tau_u)<0$. From (40), we have

$$\begin{aligned}
&2\Delta\tau_u \operatorname{Var}(\theta-p)+\frac{\partial \operatorname{Var}(\theta-p)}{\partial\Delta}(\tau_\theta+\Delta^2\tau_u)\\
&=\frac{2\Delta\tau_u}{(\Delta+\beta)^2}\left(\frac{\beta^2}{\tau_\theta}+\frac{1}{\tau_u}\right)-\frac{2}{(\Delta+\beta)^3}\left(\frac{\Delta}{\Delta\tau_u+\rho}\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2}+\frac{1}{\tau_u}\right)(\tau_\theta+\Delta^2\tau_u)\\
&\propto \Delta\tau_u\left(\frac{\beta^2}{\tau_\theta}+\frac{1}{\tau_u}\right)-\frac{1}{\Delta+\beta}\left(\frac{\Delta}{\Delta\tau_u+\rho}\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2}+\frac{1}{\tau_u}\right)(\tau_\theta+\Delta^2\tau_u)\\
&=\frac{\Delta\tau_u\tau_\theta}{(\Delta\tau_u+\rho)^2}\frac{\Delta\rho}{\Delta^2\tau_u+\Delta\rho+\tau_\theta}+\Delta-\frac{\Delta^2\tau_u+\tau_\theta}{\tau_u\left(\Delta+\frac{\tau_\theta}{\Delta\tau_u+\rho}\right)},
\end{aligned}$$

which is negative when Δ is small. That is, consultation improves equilibrium welfare in informationally inefficient markets.

Finally, direct calculations show that

$$\frac{\partial\left(\Delta+\frac{\tau_\theta}{\Delta\tau_u+\rho}\right)}{\partial\Delta}=1-\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2},$$

which is positive if and only if $\Delta>(\sqrt{\tau_\theta\tau_u}-\rho)/\tau_u$. Thus, we can conclude that consultation improves market liquidity in informationally inefficient markets. □

Proof of Proposition 10

We first show Part (i). Following the similar arguments in the proof of Proposition 1, we can show that the expected utility of the strategy $\sum_{j=1}^n a_{ij}x_{ij}-\varphi p$ is given by

$$\begin{aligned}
&\mathbb{E}\left[U_i\left(W\left(\sum_{j=1}^n a_{ij}x_{ij}-\varphi p\right)\right)\right]\\
&=-\left[(1+(\rho\beta+\rho_i\varphi)\alpha)^2-(\rho\beta+\rho_i\varphi)^2\gamma\frac{\Delta^2/\tau_\theta+1/\tau_u}{(\Delta+\beta)^2}+\sum_{j=1}^n(2a_{ij}-a_{ij}^2)\tau_{ij}\gamma\right]^{-\frac{1}{2}}\\
&=:-\left[g(\varphi)+\sum_{j=1}^n(2a_{ij}-a_{ij}^2)\tau_{ij}\gamma\right]^{-\frac{1}{2}},
\end{aligned}$$

where

$$
\begin{aligned}
g(\varphi) &= \rho_i^2\left(\alpha^2-\gamma\frac{\Delta^2/\tau_\theta+1/\tau_u}{(\Delta+\beta)^2}\right)\varphi^2+\left(2\rho_i\alpha(1+\rho\alpha\beta)-2\rho\beta\rho_i\gamma\frac{\Delta^2/\tau_\theta+1/\tau_u}{(\Delta+\beta)^2}\right)\varphi \\
&\quad +(1+\rho\alpha\beta)^2-(\rho\beta)^2\gamma\frac{\Delta^2/\tau_\theta+1/\tau_u}{(\Delta+\beta)^2} \\
&= -\frac{\rho_i^2\alpha}{\rho\beta}\varphi^2+2\rho_i\left(\alpha(1+\rho\alpha\beta)-\rho\beta\gamma\frac{\Delta^2/\tau_\theta+1/\tau_u}{(\Delta+\beta)^2}\right)\varphi+1+\rho\alpha\beta \\
&= -\frac{\rho_i^2\alpha}{\rho\beta}\varphi^2+1+\rho\alpha\beta,
\end{aligned}
$$

where we use the relation (24). Thus, the expected utility of the strategy $\sum_{j=1}^n a_{ij}x_{ij}-\varphi p$ achieves its maximum at $\varphi=0$. Part (ii) follows immediately from the result in Part (i). □

Proof of Proposition 11

First, following the similar arguments in the proof of Proposition 1, we can show that the expected utility of the weighted strategy $x_i^*=\sum_{j=1}^n a_{ij}x_{ij}$ is given by

$$
\begin{aligned}
\mathbb{E}\left[U_i\left(W\left(x_i^*\right)\right)\right] &= -\left[(1+a_i\rho\alpha\beta)^2-(a_i\rho\beta)^2\gamma\frac{\Delta^2/\tau_\theta+1/\tau_u}{(\Delta+\beta)^2}+\sum_{j=1}^n(2a_{ij}-a_{ij}^2)\tau_{ij}\gamma\right]^{-\frac{1}{2}} \\
&= -\left[1+(2a_i-a_i^2)\rho\alpha\beta+\sum_{j=1}^n(2a_{ij}-a_{ij}^2)\tau_{ij}\gamma\right]^{-\frac{1}{2}}, \qquad (41)
\end{aligned}
$$

where we use the relation (24). Thus, Part (i) follows.

We can first conclude from (41) that a_i^* must be finite and positive. We now show that for the optimal solution a_{ij}^*'s, it must hold that $a_{ij}^*\le 1$ for any $1\le j\le n$. It is easy to see that $2a_i-a_i^2$ strictly increases in $a_i\in(0,1]$ and strictly decreases in $[1,\infty)$. Moreover, also note the fact that $2a_{ij}-a_{ij}^2$ increases in $a_{ij}\in(0,1]$ and decreases in $a_{ij}\in[1,\infty)$. Thus, we can first conclude from (41) that it is impossible that $a_{ij}^*\ge 1$ for any j and the inequality is strict for some j, since otherwise a reduction by a small $\varepsilon>0$ to some a_{ij}^* with $a_{ij}^*>1$ while keeping other weights unchanged will lead to a higher welfare (noting that $2a_i-a_i^2$ is decreasing when $a_i>1$), a contradiction. Second, we can also conclude that it is impossible that $a_{ij}^*>1$ for some j and $a_{ij}^*<1$ for the other j. For example, if $a_{ij_1}^*>1$ and $a_{ij_2}^*<1$, then the weight

$(a^*_{ij}, j \neq j_1, j_2, a^*_{ij_1} - \varepsilon, a^*_{ij_2} + \varepsilon)$ will also lead to a higher welfare, a contradiction again. Based on the two conclusions, we can see that $a^*_{ij} \leq 1$ for any j.

We now show by contradiction that $a^*_i \geq 1$. Otherwise, multiplying the optimal weights $(a^*_{ij})_j$ with a multiple $1 + \varepsilon$ with a sufficiently small $\varepsilon > 0$ will result in a higher welfare, a contradiction. Hence $a^*_i \geq 1$. Together with $a^*_{ij} \leq 1$ for any j that we just have shown, we have $1 \leq a^*_i \leq n$. The remaining part of Part (ii) follows from (16).

The expression of the optimal solution of a^*_{ij}'s in Part (iii) can be obtained by using the similar arguments in the proof of Proposition 2. The quadratic structure of (41) implies the uniqueness of the optimal solution. Finally, Part (iv) follows from similar arguments in the proof of Proposition 3. The proof is completed. □

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