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Information linkages in a financial market with imperfect competition

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ABSTRACT

This study proposes a tractable imperfectly competitive economy where traders are socially connected with each other via an information network. We investigate the impact of information linkages on market equilibrium outcomes. In the linear-quadratic-normal framework, we first establish the existence and uniqueness of symmetric linear Bayesian Nash equilibrium. We then show that an increase of the level of information linkages decreases price impact, belief disagreement, return volatility and trading volume. Additionally, unlike the non-monotonicity results in large economies, we also show that increasing the level of information linkages always improves liquidity, increases price volatility, and decreases trading profits in our imperfectly competitive economy.

1. Introduction

There is substantial evidence suggesting that professional fund managers transmit investment ideas socially, either through direct word-of-mouth communication of information (Hong et al., 2005) or sharing of profitable ideas (Crawford et al. 2017; Pool et al. 2015), or indirect access to information through the same channels (e.g., local TV stations, newsletters, advisory services, etc.) Shiller and Pound (1989) also established that institutional investors' portfolio choices are driven in part by interpersonal communication.[1] While these works focus on the effect of social interactions on investment decisions, some important unanswered questions on the effect of social interactions on the market aggregate outcomes remain. For example, do information linkages improve market efficiency? Do information linkages make traders better off? Do information linkages enhance market liquidity? This study addresses these questions.

Some related research examines the influence of information networks on market equilibrium outcomes under the framework of REE, for example, Ozsoylev and Walden (2011), Han and Yang (2013) and Walden (2019). However, these often consider large, perfectly competitive economies to obtain closed-form solutions. Very few studies consider imperfectly competitive economies that capture the significant price impact by large traders. An exception is Colla and Antonio (2010), who

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[1] Additionally, David Einhorn (one of most famous hedge fund managers) acknowledges that "*sometimes an analyst generates the idea, sometimes other fund managers, a conference, or an idea dinner.*" (https://www.marketfolly.com/2012/03/david-einhorns-extensive-q-session-from.html)

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mainly limit their investigation to numerical analysis due to the complexity of the combination of information networks, price impact, and endogenous prices. There is a lack of quantitative analysis on the effects of information networks on equilibrium variables for imperfectly competitive markets. This study contributes to the literature by filling this gap.

We consider a tractable finite-agent economy, which is in essence similar to Kyle (1989) with imperfect competition. We quantitatively analyze the effect of information linkages among investors on trading behavior and market quality parameters, such as market efficiency, price volatility, liquidity, trading volume, and so on. In the economy, there is a single risky asset in zero net supply. There are finitely many speculators (professional fund managers) who compete in demand schedules and have linear-quadratic preferences over their holdings of the risky asset. There are also noise traders, who trade for non-speculative motives, for instance, hedging or liquidity needs, to prevent the price from being fully revealed. Speculators are strategic in the sense that they realize that their demands have an impact on the asset price and take such an effect into account when choosing optimal demand schedules. Based on the market-clearing condition, the equilibrium price is set by an auctioneer who aggregates the demand schedules from all speculators and the noise demand.

A key feature of our model is that speculators are informationally connected and their information sets are overlapped. For example, two geographically close speculators exchange information through word-of-mouth communication (Hong et al., 2005) or acquire their information through the same local newspaper or TV station (Colla and Antonio, 2010). Following the network in Colla and Antonio (2010) and also for mathematical tractability, we assume that the information network is a k-cyclical graph (which is modellingly equivalent to a k-regular graph); more specifically, the signal available at any speculator's location can be observed by $(k-1)/2$ clockwise neighbors and $(k-1)/2$ counterclockwise neighbors of any given speculator, where k is an odd integer. We refer to k as *the number of information linkages*, which plays a similar role as the parameter of "network connectedness" in Han and Yang (2013) and Ozsoylev and Walden (2011). A higher number of information linkages means that there are more common information sources among speculators and the degree of information overlap is higher.

We first establish the existence and uniqueness of symmetric linear Bayesian Nash equilibria. For the case where speculators are risk neutral, we theoretically demonstrate that price impact, belief disagreement, return volatility, and trading profits strictly decrease with the number of information linkages, while liquidity, signal sensitivity, and price volatility strictly increase with the number of information linkages. We then numerically show that the above implications of information linkages on market equilibrium outcomes also hold for the case where speculators are risk neutral but with a quadratic holding cost. Intuitively, when the level of information linkages increases, speculators trade on more similar signals, which reduces speculators' belief disagreement, and the trading volume and trading profits due to a disappearance of the potential trading opportunities. An increase of the level of information linkages increases the competition between speculators on the information, and then reduces each speculator's monopolistic power. An increase of the level of information linkages also induces speculators to be more likely to trade in the same direction so that the price becomes more volatile. As an average of speculators' beliefs which are more accurate to predict the asset payoff, the price gets closer to the asset payoff so that the return volatility decreases. Furthermore, as the level of information linkages increases, speculators' beliefs depend less on the price so that their demands are potentially more elastic. In addition, a decrease of the price impact further increases the price elasticity of demand. As a result, the demand of one more unit of the risky asset by noise traders will move the price less, that is, the market liquidity improves.

Additionally, we demonstrate that the equilibrium price reveals more than one-half the private precision of speculators, and almost all that when the number of speculators is large and the number of information linkages takes the maximum value, as noise trading vanishes or as speculators become risk neutral. This differs from the classical result in Kyle (1989) without information linkages. Intuitively, when there are more information linkages, speculators trade more aggressively on their information so that speculators' information is incorporated into prices more efficiently. We also consider two extensions of the baseline model to show the robustness of the implications of information linkages on market equilibrium outcomes for more general information networks and imperfectly shared signals between speculators.

The rest of this paper is organized as follows. In the next section, we discuss related literature. We introduce the model in Section 3 and characterize the equilibrium in Section 4. In Section 5, we discuss how the distance between two speculators affect correlated trading and how the information linkages affect market efficiency. In Section 6, we analyze the impact of the information linkages on market equilibrium outcomes including liquidity, price impact, return volatility, trading volume, and trading profits, and so on, for linear utility and linear utility with a quadratic holding cost, respectively. Section 7 presents some empirical predictions. Finally, concluding remarks are presented in Section 8. The proofs of all propositions and the two extensions can be found in the Appendix.

2. Related literature

Our study contributes to the recent literature that theoretically investigates the impact of social networks on market equilibrium outcomes in perfectly competitive markets (Han and Yang 2013; Lou and Yang 2022; Manela 2014; Ozsoylev and Walden 2011; Walden 2019) and imperfectly competitive markets (Colla and Antonio, 2010).[2] Ozsoylev and Walden (2011) consider a large REE model where information is shared among agents over a social network with a power

[2] There is also extensive empirical research on the effects of social interactions on investment decisions by retail investors (Brown et al. 2008; Feng and Seasholes 2004; Hong et al. 2004; Ivkovic and Weisbenner 2007; Kaustia and Knupfer 2012; Ouimet and Tate 2020; Ozsoylev and Walden 2014), and

law degree distribution and find that market efficiency and correlation of trading among traders strictly increase with network connectedness (which plays a similar role to the number of information linkages in our study), but liquidity, price volatility, and ex-ante trading profits are non-monotonic functions of network connectedness in general. Han and Yang (2013) consider a large REE economy consisting of infinitely many disconnected groups where all (informed and uninformed) traders can receive a noisy version of the signals of informed traders within the same group. The authors demonstrate that in the case of exogenous information, social communication improves market efficiency, improves liquidity if and only if the market is sufficiently informationally efficient, reduces welfare and increases trading volume, and the implications in the case of endogenous information are in contrast to that in the case of exogenous information. Lou and Yang (2022) further reveal that the opposite implications hold in the case of endogenous information when there are no uninformed traders in the economy of Han and Yang (2013). Walden (2019) considers a dynamic large REE model with decentralized information diffusion through a general network, and establishes that more central agents make higher profits, and agents that are close to each other have more positively correlated trades. Manela (2014) considers a continuum-agent economy and finds that the value of information is hump-shaped in the speed of information diffusion.

Different from the assumption of CARA utility in the above research, we assume that traders have linear utility with a quadratic holding cost for tractability. Furthermore, when traders in the perfectly competitive economies do not enjoy market power and act as price takers, traders in our finite-agent economy have price impact and take such an impact on the asset equilibrium price into account when choosing optimal demand schedules.

It is worth noting that the implications of information networks on market equilibrium outcomes in our finite-agent economy may differ from that in large economies. For example, liquidity,[3] price volatility and trading profits are shown to be non-monotonic functions of network connectedness in the large economy of Ozsoylev and Walden (2011), but they are monotonic in our finite-agent economy. The main reason for the non-monotonicity of γ_k in Ozsoylev and Walden (2011) is that speculators' demands first rely more (less) on the price as an information source as traders have less (sufficient) information and learn more (less) additional information for a low (high) network connectedness. In our finite-agent economy, any increase in the number of information linkages leads to a non-negligible influence on the finite-agent economy, which corresponds to a setup with a relatively large network connectedness in Ozsoylev and Walden (2011). In fact, our results on liquidity as well as price volatility and trading profits are consistent with those in Ozsoylev and Walden (2011) with a large value of network connectedness. Moreover, in Ozsoylev and Walden (2011), trading volume is shown to be increasing, but is decreasing in our economy. The main reason is that the force of the decreasing price impact, which plays a similar role to a conditional-variance adjusted term in the expression of trading volume in Ozsoylev and Walden (2011), is not strong so that speculators reduce their trade in our economy due to imperfect competition.

Our work is also closely related to that of Colla and Antonio (2010), who consider a dynamic finite-agent model with information linkages described by a cyclical graph. To the best of our knowledge, this study is the second to analyze the impact of the number of information linkages on market equilibrium outcomes in an imperfectly competitive economy following Colla and Antonio (2010). There are three main differences that distinguish our work from Colla and Antonio (2010). First, the model in Colla and Antonio (2010) is dynamic and the equilibrium price is set by a market maker based on the semi-strong efficiency rule in Kyle (1985). Our model is static and the equilibrium price is set by an auctioneer according to market-clearing conditions. Second, the focus of Colla and Antonio (2010) is on the comparison of the equilibrium outcomes when there are information linkages or not. Our focus is on the quantitative analysis of the number of information linkages taking odd values in $[1, n-2]$ on equilibrium outcomes.[4] Finally, Colla and Antonio (2010) mainly adopt the numerical method, our model, especially in the case of linear utility, is theoretically tractable. The work of Colla and Antonio (2010) and ours therefore complement each other.

Our theoretical model also adds to the vast REE literature on the large economies, including of Hellwig (1980), Grossman and Stiglitz (1980), Lou et al. (2019), and so on, and especially on the finite-agent economies in Kyle (1989) and Kyle (1985) with imperfect competition. The main feature that distinguishes our model from these studies is the presence of information linkages, that is, speculators are informationally connected with their peers in our model. In fact, our model is similar to Kyle (1989) in that we introduce an information network into the economy, but we assume that there are no uninformed speculators and informed speculators have linear utility function (with a quadratic holding cost) for tractability. The introduction of information linkages quantitatively changes the market equilibrium statistics. For example, Kyle (1989) demonstrates that prices never reveal more than one-half the private precision of speculators as noise trading vanishes or as speculators become risk neutral. The reason is that as noise trading becomes small, speculators' price impact becomes large and speculators trade less proportionally, which prevents the information from being incorporated into the price. In our model, the degree of prevention is alleviated due to the information linkages so that the equilibrium price reveals more than one-half the private precision of speculators as the noise trading becomes small or as speculators become risk neutral.

professional investors (Cohen et al. 2008; Hong et al. 2005; Pool et al. 2015; Shiller and Pound 1989); see Kuchler and Stroebel (2021) for a review, and see Enward et al. (2019), who investigate the endogenous effects of social communication on trading behavior and market performance, for experimental work.

[3] Proposition 3 (b) in Ozsoylev and Walden (2011) shows that the noise-trading driven volatility (equivalently γ_k) is a non-monotonic function of network connectedness. Note that we use $1/\gamma_k$ to measure the liquidity, which is also non-monotonic in the setup of Ozsoylev and Walden (2011).

[4] Colla and Antonio (2010) consider only the case where traders have no neighbors, or have two or four neighbors.

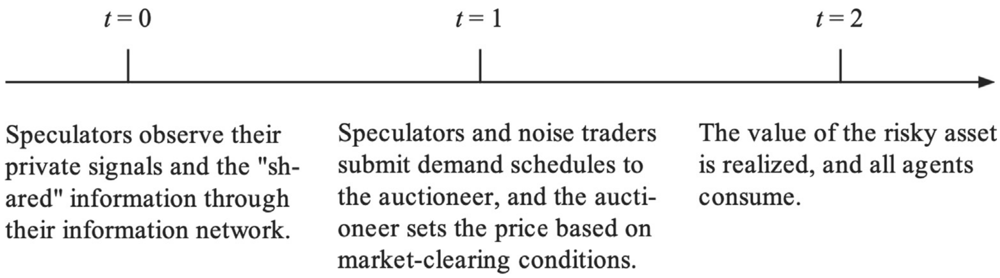


Fig. 1. The timeline of the model.

3. The model

We consider a Kyle-type model (Kyle 1989) with imperfect competition and extend to a setting of information linkages among traders. The economy has three dates, $t = 0, 1, 2$. The timeline of the economy is described in Fig. 1. In the economy, there are $n \geq 3$ speculators who are engaged in a simultaneous game and the market is organized as a uniform-price double auction. All random variables are normally distributed with mean normalized to zero for simplicity. There is a single risky asset that pays $\theta \sim N(0, 1/\tau_\theta)$, $\tau_\theta > 0$, at $t = 2$, in zero net supply. Trading occurs on $t = 1$. Let p denote the date-1 price of the risky asset. Speculators are risk neutral with a quadratic holding cost (Lou and Rahi 2021; Manzano and Vives 2021; Rostek and Weretka 2012; 2015; Vives 2011; 2017). Then, the utility of speculator i who buys $x_i \in \mathbb{R}$ units of the risky asset at date-1 price $p \in \mathbb{R}$ is given by

$$\pi_i = x_i(\theta - p) - \frac{\xi}{2} x_i^2,$$

where $\xi \geq 0$ can be interpreted as a parameter of holding cost or proxy for risk aversion, and the marginal benefit of buying x_i units of the risky asset is $\theta - \xi x_i$. Speculators have linear utility when $\xi = 0$, and linear utility with a quadratic holding cost when $\xi > 0$. The nonrandom initial wealth of speculators is normalized to zero (without loss of generality with linear-quadratic preferences). To prevent the price from being fully revealing, there is also noise demand $u \sim N(0, 1/\tau_u)$, $\tau_u > 0$, in the economy, where u is independent of other random variables.

On $t = 0$, each speculator i can receive a private signal $y_i = \theta + \epsilon_i$, where $\epsilon_i \sim N(0, 1/\tau_\epsilon)$ is a noise term, $\tau_\epsilon > 0$, $i = 1, \ldots, n$. A key difference between our model and Kyle (1989) is that speculators are locally connected with each other via an information network.[5] We follow Colla and Antonio (2010) to use a cyclical graph to describe the structure of information linkages among speculators through word-of-communication.[6] The initial private signal y_i can also be alternatively interpreted as the information broadcast to speculator i's location from a local channel, for example, newspaper or TV station.[7] Speculators positioned geographically close will gain access to some common sources of information; specifically, under the assumption of the cyclical graph, the signal available at any speculator's location can be observed by $(k-1)/2$ clockwise neighbors and $(k-1)/2$ counterclockwise neighbors of any given speculator. To have a well-defined structure of cyclical graphs, n and $1 \leq k \leq n-2$ are assumed to be odd integers following Colla and Antonio (2010).[8] Let $\mathcal{N}_i$ denote the neighbor set of agent i including themselves. According to the cyclical network assumption, $|\mathcal{N}_i| = k$ for all i. Consequently, the information set of agent i is[9]

$$\mathcal{F}_i = \{y_j, j \in \mathcal{N}_i, p\}.$$

Speculators behave strategically in the finite-agent economy. Speculators realize that their demands have an impact on the equilibrium price, and take such an impact into account when choosing optimal demand schedules.

As is standard in the literature, we restrict our attention to linear equilibria. Moreover, since the signal structure and holding cost function are symmetric across speculators, and the network structure is symmetric, we are interested in sym-

[5] There are two further differences between our model and Kyle (1989). To facilitate analysis, we assume that there are no uninformed speculators and speculators are risk neutral with a quadratic holding cost. When there are uninformed speculators in the economy, there will be an additional price impact parameter for these speculators, which couples together with the price impact parameter for informed speculators. Combined with the information linkages among speculators, this will considerably complicate the analysis.

[6] Here we assume cyclical graphs mainly for the purpose of mathematical tractability. In Appendix B, we numerically show that the implications for cyclical graphs also continue to hold for more general information networks. Moreover, we can also use regular graphs (which only require that nodes have the same degree, but not the symmetricity of nodes) to describe the information network since k-cyclical graphs and k-regular graphs lead to the same market equilibrium although they might be not isomorphic.

[7] Although the two modelings of information linkages are different, they are modelingly equivalent.

[8] When $k = n$, each speculator can gain access to all the signals in the economy. In this case, a unique symmetric linear equilibrium exists in which $\lambda = \xi/(n-2)$, $p = \mathbb{E}[\theta | y_1, \ldots, y_n] + (n-1)\xi u/(n(n-2))$ and $x_i^* = -u/n$ for every i. In the equilibrium, the private signal of each speculator does not affect their optimal demand and speculators behave like noise traders. We exclude the uninteresting case of $k = n$.

[9] In Appendix C, we also consider a more general setting that the signals shared between speculators are not perfect, but polluted by noises. We numerically show that the implication results in the baseline model still hold.

metric equilibria with speculators using the same trading strategy, that is, the linearity coefficients on signals and the price are the same across speculators' demand strategies. Let the symmetric linear demand schedules of speculators be

$$x_i^*(y_i, y_j, j \in \mathcal{N}_i, p) = \phi \sum_{j \in \mathcal{N}_i} y_j - \varphi p, i = 1, \ldots, n, \tag{1}$$

where ϕ and φ are two constants to be determined in equilibrium. Hence, aggregate demand $D(p, y_1, \ldots, y_n, u)$ by all traders is given by

$$\begin{aligned} D(p, y_1, \ldots, y_n, u) &= \sum_{i=1}^{n} x_i^*(y_j, j \in \mathcal{N}_i, p) + u \\ &= \phi \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} y_j - n\varphi p + u. \end{aligned} \tag{2}$$

Given the market-clearing condition, speculator i understands that if they buy x_i units of the asset, the equilibrium price is determined by the equation

$$x_i + D(p, y_1, \ldots, y_n, u) - \left(\phi \sum_{j \in \mathcal{N}_i} y_j - \varphi p \right) = 0.$$

From the previous equation and (2), the residual supply function p_i that speculator i faces is

$$p_i(x_i) = \frac{1}{(n-1)\varphi} x_i + \frac{1}{(n-1)\varphi} \left[\phi \sum_{r \neq i} \sum_{j \in \mathcal{N}_r} y_j + u \right] =: \frac{1}{(n-1)\varphi} x_i + p_i^{rs}, \tag{3}$$

where $\frac{1}{(n-1)\varphi}$ is the slope parameter, p_i^{rs} is the intercept of the residual supply curve. Trader i's objective is to choose x_i to maximize their expected utility conditional on observing $\{y_j, j \in \mathcal{N}_i, p_i^{rs}\}$, which is informationally equivalent to $\{y_j, j \in \mathcal{N}_i, p\}$.

Next, we formally introduce the definition of symmetric linear equilibria. A *symmetric linear Bayesian Nash equilibrium* is defined as a collection $\{x_i^*(y_j, j \in \mathcal{N}_i, p), i = 1, \ldots, n, p\}$ of linear demand schedules (1) and a price function p, which is linear in speculators' signals and noise trading such that

(i) the maximum of the expected utility of speculator i is achieved at x_i^*, that is,

$$x_i^* \in \arg\max_{x_i} \mathbb{E}\left[(\theta - p_i(x_i))x_i - \frac{\xi}{2} x_i^2 \,\middle|\, y_j, j \in \mathcal{N}_i, p_i^{rs} \right], \quad i = 1, \ldots, n, \tag{4}$$

where $p_i(x_i)$ is given by (3), and

(ii) the market clears, that is,

$$\sum_{i=1}^{n} x_i^*(y_j, j \in \mathcal{N}_i, p) + u = 0, \tag{5}$$

or equivalently, $p_i(x_i^*(y_j, j \in \mathcal{N}_i, p)) = p$ for all i.

In a Bayesian Nash equilibrium, each speculator first optimally chooses a demand strategy to maximize his/her conditional expected utility taking as given strategies of other speculators and then submits his/her demand schedule to an auctioneer. Speculator i exercises market power by taking how his/her quantity x_i affects the price $p_i(x_i)$ on his/her residual supply schedule (3) into account. The auctioneer then collects the demand schedules from all speculators and the noise demand to find the equilibrium price. Finally, the auctioneer assigns speculators their positions at the equilibrium price according to their submitted demand schedules.

4. Equilibrium characterization

In this section, we characterize the linear Bayesian Nash equilibrium. To find the linear equilibrium, we next follow the standard procedure. First, a linear price function and the price impact parameter are conjectured. Second, the beliefs of speculators are updated using this conjecture, and speculators' optimal demand schedules are computed according to their utility. Third, the market-clearing condition leads to an actual relation between the price and signals and the noise trading, and an equation that the price impact parameter satisfies. Finally, the conjectured price and the price impact parameter must be self-fulfilling.

First, suppose the conjectured linear price function is $p = \pi \sum_{i=1}^{n} y_i + \gamma u$,[10] where π and γ are two endogenous positive constants to be determined later. From the projection theorem for normal random variables[11] and using some simple computations, we have

$$\begin{aligned}
\mathbb{E}[\theta | y_j, j \in \mathcal{N}_i, p] &= \mathbb{E}\left[\theta \middle| y_j, j \in \mathcal{N}_i, \pi \sum_{r \notin \mathcal{N}_i} y_r + \gamma u\right] \\
&= \mathbb{E}\left[\theta \middle| y_j, j \in \mathcal{N}_i, \sum_{r \notin \mathcal{N}_i} y_r + \frac{\gamma}{\pi} u\right] \\
&= \frac{\tau_\epsilon \sum_{j \in \mathcal{N}_i} y_j + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}} \left(\theta + \frac{\sum_{r \notin \mathcal{N}_i} \epsilon_r}{n-k} + \frac{\gamma}{(n-k)\pi} u\right)}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}}} \\
&= \frac{\tau_\epsilon \sum_{j \in \mathcal{N}_i} y_j + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}} \frac{1}{(n-k)\pi} \left(p - \pi \sum_{j \in \mathcal{N}_i} y_j\right)}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}}} \\
&=: \alpha \sum_{j \in \mathcal{N}_i} y_j + \beta p,
\end{aligned} \tag{6}$$

where

$$\alpha = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon} + \frac{\gamma^2}{(n-k)\pi^2\tau_u}}}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}}}, \tag{7}$$

$$\beta = \frac{\frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}} \frac{1}{(n-k)\pi}}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}}}. \tag{8}$$

By (4), the first-order condition of the utility of speculator i yields

$$\theta - p_i^{rs} - 2\lambda x_i - \xi x_i = 0,$$

and the optimal demand by speculator i is thus given by

$$x_i^* = \frac{\mathbb{E}[\theta | y_j, j \in \mathcal{N}_i, p_i^{rs}] - p_i^{rs}}{2\lambda + \xi} = \frac{\mathbb{E}[\theta | y_j, j \in \mathcal{N}_i, p] - p}{\lambda + \xi}, \tag{9}$$

where λ is the conjectured slope of inverse supply schedule facing the individual speculator and referred to as the price impact parameter, and we use the fact that $\{y_j, j \in \mathcal{N}_i, p_i^{rs}\}$ is informationally equivalent to $\{y_j, j \in \mathcal{N}_i, p\}$ and the relation $p = \lambda x_i^* + p_i^{rs}$ (see the condition (ii) in the definition of symmetric linear Bayesian Nash equilibrium). The second-order condition is $-(2\lambda + \xi)$, which is negative (since $\lambda > 0$, which will be shown later). It then follows from (6), (9) and the market-clearing condition that

$$\sum_{i=1}^{n} x_i^* + u = \sum_{i=1}^{n} \frac{\mathbb{E}[\theta | y_j, j \in \mathcal{N}_i, p] - p}{\lambda + \xi} + u = \sum_{i=1}^{n} \frac{\alpha \sum_{j \in \mathcal{N}_i} y_j + \beta p - p}{\lambda + \xi} + u = 0,$$

from which we have

$$p = \frac{1}{n(1-\beta)} \left[k\alpha \sum_{i=1}^{n} y_i + (\lambda + \xi) u\right].$$

[10] Here, the conjectured linear equilibrium price is symmetric in the sense that the signal coefficients in the price are the same across speculators. The symmetry can be observed from the market-clearing condition and is a consequence of the facts that speculators have the same level of signal precision, holding cost and price impact, and the information-sharing network is symmetric. Moreover, since we assume that all random variables have mean zero for notational convenience, there is no intercept in price function p.

[11] Please refer to Appendix A in Kyle (1989) for the projection theorem.

Identifying coefficients of the previous price function and the conjectured one yields

$$\pi = \frac{1}{n(1-\beta)} k\alpha, \tag{10}$$

$$\gamma = \frac{1}{n(1-\beta)} (\lambda + \xi). \tag{11}$$

Moreover, from (6), (9) and (11), we have

$$\begin{aligned} \lambda &= \left[(n-1) \frac{1-\beta}{\lambda+\xi} \right]^{-1} \\ &= \frac{n}{n-1} \gamma. \end{aligned} \tag{12}$$

In conclusion, a symmetric linear equilibrium exists if and only if the system of equations (10), (11) and (12) has a positive solution (π, γ, λ). The next proposition demonstrates that a unique symmetric linear equilibrium exists.

Proposition 1. *A unique symmetric linear Bayesian Nash equilibrium exists in which the equilibrium price is given by* $p = \pi \sum_{i=1}^{n} y_i + \gamma u$, *where* $\pi > 0, \gamma > 0$ *and* γ/π *is the unique positive solution of the following cubic equation*

$$f(z) := a_3 z^3 - a_2 z^2 - a_1 z - a_0 = 0, \tag{13}$$

where

$$a_3 = \frac{k\tau_\epsilon}{(n-k)\tau_u}, \ a_2 = \frac{n-1}{n-2} \frac{\tau_\theta + k\tau_\epsilon}{(n-k)\tau_u} \xi, \ a_1 = \frac{n}{n-2}, \ a_0 = \frac{n-1}{n-2} \left(\frac{\tau_\theta}{\tau_\epsilon} + n \right) \xi.$$

To highlight the functional dependence of the variables on the number of information linkages k, throughout the rest of the paper, we denote α, β, p, π, γ, λ, z, f, a_i, x_i^*, $\mathcal{N}_i$ as α_k, β_k, p_k, π_k, γ_k, λ_k, z_k, f_k, $a_i(k)$, $x_{i,k}^*$, and $\mathcal{N}_{i,k}$, respectively.

5. Correlated trading and market efficiency

In this section, we investigate the effect of information networks on correlated trading and market efficiency. Correlated trading is defined as $\mathrm{Corr}(x_{i,k}^*, x_{i+d,k}^*)$, which measures the correlation between the demands of any two speculators, where $d \in \{1, \ldots, \frac{n-1}{2}\}$ is the distance between the two speculators. We have the following proposition.

Proposition 2. *The correlated trading* $\mathrm{Corr}(x_{i,k}^*, x_{i+d,k}^*)$ *decreases with d.*

The above proposition reveals that as two speculators have more common signals, their tradings are more correlated. This result is consistent with the empirical findings in Hong et al. (2004) that the portfolio choices of fund managers in the same city are influenced by word-of-mouth communication, and in Feng and Seasholes (2004) that geographically close speculators have highly correlated trading, and also with the theoretical finding in Ozsoylev and Walden (2011) and Walden (2019) of large economies.

Next, we analyze the effects of information linkages on market efficiency. We first discuss the monotonicity of the solution $z_k := \gamma_k/\pi_k$ to the Eq. (13) over k. z_k measures how sensitive the equilibrium price is to the noise trading. It follows from the definition of z_k that $f_k(z_k) = 0$, or equivalently,

$$g_k(z_k) := \frac{a_3(k)}{a_2(k)} z_k^3 - z_k^2 - \frac{a_1 z_k + a_0}{a_2(k)} = \frac{k\tau_\epsilon}{\frac{n-1}{n-2}\xi(\tau_\theta + k\tau_\epsilon)} z_k^3 - z_k^2 - \frac{a_1 z_k + a_0}{a_2(k)} = 0.$$

Since $a_2(k+1) > a_2(k)$, it holds that $g_{k+1}(z_k) > 0$, implying that $z_{k+1} < z_k$. That is, z_k strictly decreases with k.

Following Ozsoylev and Walden (2011) and Han and Yang (2013), we first consider the measure of market efficiency $1/\mathrm{Var}[\theta|p_k]$. From Proposition 1, we have

$$\begin{aligned} \frac{1}{\mathrm{Var}[\theta|p_k]} &= \frac{1}{\mathrm{Var}\left[\theta \middle| \pi_k \sum_{i=1}^{n} y_i + \gamma_k u\right]} \\ &= \frac{1}{\mathrm{Var}\left[\theta | \theta + \frac{\sum_{i=1}^{n} \epsilon_i}{n} + \frac{\gamma_k}{n\pi_k} u\right]} \\ &= \tau_\theta + \frac{1}{\frac{1}{n\tau_\epsilon} + \frac{z_k^2}{n^2\tau_u}}, \end{aligned} \tag{14}$$

where the last equality follows from the projection theorem for normal random variables. It is clear from (14) that the market efficiency $1/\mathrm{Var}[\theta|p_k]$ is strictly decreasing in z_k. Moreover, we have shown that z_k strictly decreases with k, hence we can conclude that the market efficiency $1/\mathrm{Var}[\theta|p_k]$ strictly increases with k. This is consistent with the theoretical results in Ozsoylev and Walden (2011) (Proposition 4 therein), and Han and Yang (2013) (Proposition 2 therein), and also consistent with the numerical results in Colla and Antonio (2010). The intuition is as follows. When agents have more common information, their demands will compound more information into the economy and consequently, the asset price will be more precise to predict the asset payoff.

We now consider the other measure of market efficiency used in Kyle (1989), which is defined by the term ψ_k in the following expression:

$$\begin{aligned}\frac{1}{\mathrm{Var}[\theta|y_j, j\in\mathcal{N}_{i,k}, p_k]} &= \tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{\gamma_k^2}{(n-k)^2\pi_k^2\tau_u}} \\ &=: \tau_\theta + k\tau_\epsilon + \psi_k(n-k)\tau_\epsilon,\end{aligned} \tag{15}$$

where the first equality follows from the projection theorem for normal random variables, and

$$\psi_k = \frac{1}{1+\frac{z_k^2\tau_\epsilon}{(n-k)\tau_u}}.$$

Different from the measure of market efficiency $1/\mathrm{Var}[\theta|p_k]$, which summarizes all the information contained in the price, the measure of market efficiency ψ_k in Kyle (1989) captures the additional information contained in the price except the information that has been contained in their own private information and the common information with their neighbors.

We first consider the case of $\xi > 0$; that is, speculators are risk neutral with a quadratic holding cost. For comparison with the result in Kyle (1989), we consider the limit case of $\tau_u \to \infty$. It follows from (13) that $a_1 z_k \le a_3(k) z_k^3$, that is, $a_1 \le a_3(k) z_k^2$, from which we obtain

$$\frac{z_k^2}{\tau_u} \ge \frac{n(n-k)}{(n-2)k\tau_\epsilon},$$

implying that $z_k \to \infty$. We also have

$$\frac{k\tau_\epsilon}{n-k} z_k\left(\frac{z_k^2}{\tau_u}\right) - \frac{n-1}{n-2}\xi\frac{\tau_\theta+k\tau_\epsilon}{n-k}\left(\frac{z_k^2}{\tau_u}\right) - \frac{n}{n-2}z_k - \frac{n-1}{n-2}\xi\left(\frac{\tau_\theta}{\tau_\epsilon}+n\right) = 0,$$

from which we obtain $z_k^2/\tau_u \to \frac{n(n-k)}{(n-2)k\tau_\epsilon}$. Hence,

$$\psi_k = \frac{1}{1+\frac{z_k^2\tau_\epsilon}{(n-k)\tau_u}} \to \frac{1}{1+\frac{n}{(n-2)k}} =: \hat{\psi}_k.$$

We can easily see that $\hat{\psi}_k$ strictly increases with k, $\hat{\psi}_1 = \frac{n-2}{2(n-1)}$, and $\hat{\psi}_{n-2} = \frac{1}{1+\frac{n}{(n-2)(n-1)}}$, which is close to one when n is large.

We now consider the case of $\xi = 0$, that is, speculators are risk neutral. In this case,

$$\frac{z_k^2}{\tau_u} = \frac{n(n-k)}{(n-2)k\tau_\epsilon}$$

for any $\tau_u > 0$, which is the same as the limit of $\tau_u \to \infty$. We then see that the above results under the limit of $\tau_u \to \infty$ also hold for the case of $\xi = 0$. The following proposition summarizes the results.

Proposition 3. *As noise trading vanishes or speculators become risk neutral, the measure of market efficiency ψ_k increases with the number of information linkages k, and achieves its maximum value $\frac{n^2-4n+4}{n^2-3n+4}$ at $k = n-2$.*[12]

In the absence of information linkages, Theorem 7.2 in Kyle (1989) indicates that prices never reveal more than one-half the private precision of informed speculators, which is consistent with our result that $\hat{\psi}_1 < 1/2$. This is because as noise trading becomes small, speculators' price impact becomes large and speculators trade less proportionally, which prevents the information from being incorporated into the price. However, Proposition 3 shows that when there are information linkages, the degree of prevention is alleviated and the equilibrium price reveals more than one-half the private precision of speculators as the noise trading becomes small. Furthermore, the market becomes more efficient when there are more information linkages. The underlying intuition is as follows. When there are more information linkages, speculators trade more aggressively on their information so that speculators' information is incorporated into prices more efficiently. Here speculators' trading intensities on information can be measured by the coefficient $k\alpha_k/(\lambda_k+\xi)$ on $\sum_{i=1}^n y_i$ in the aggregate demand $\sum_{i=1}^n x_i^*$ by all speculators, which equals $1/z_k$ and increases with k. When there are more information linkages, on the one hand, speculators' beliefs depend more on their signals, i.e., $k\alpha_k$ increases with k, which potentially increases the

[12] Note that n and k are assumed to be odd numbers. Moreover, we do not consider the uninteresting case of $k = n$; please refer to Footnote 8 for more detailed discussions.

trading intensities. On the other hand, more signal sharing implies that speculators trade on more similar signals, which intuitively increases the competition among speculators and then reduces speculators' price impact λ_k,[13] which further increases the trading intensities.

6. Implications of information linkages

In this section, we examine the implications of information linkages on the market equilibrium outcomes including price impact, liquidity, signal sensitivity, belief disagreement, price volatility, return volatility, trading volume, and (ex-ante) trading profits. Before proceeding to the analysis, we first present the closed-form expressions for these outcomes under the general case of $\xi \geq 0$.

Price impact is measured by $\lambda_k := \partial p_k/\partial x_i$. A high price impact means that a demand shock will drive the price higher. In the literature, this parameter is referred to as "Kyle's lambda" following the seminal work of Kyle (1985). From (7), (10), and (11), we have

$$\begin{aligned}\lambda_k + \xi = \frac{k\alpha_k}{\frac{\pi_k}{\gamma_k}} &= z_k \frac{k\left(\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{z_k^2}{(n-k)\tau_u}}\right)}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}} \\ &\propto z_k^2\left[\frac{k\left(\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{z_k^2}{(n-k)\tau_u}}\right)}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}}\right]^2 \\ &= z_k^2\left[\frac{k\tau_\epsilon - \frac{k}{\frac{1}{\tau_\epsilon}+\frac{z_k^2}{(n-k)\tau_u}}}{\tau_\theta + k\tau_\epsilon + \frac{n-k}{\frac{1}{\tau_\epsilon}+\frac{z_k^2}{(n-k)\tau_u}}}\right]^2.\end{aligned} \tag{16}$$

We refer to π_k as *signal sensitivity*, which reflects the sensitivity of the equilibrium price to signals. From (8) and (10), we have

$$n\pi_k\left(1 - \frac{\frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}}\frac{1}{(n-k)\pi_k}\right) = k\alpha_k.$$

Combining the above equation with (7), we further obtain

$$\pi_k = \frac{\frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}}\frac{1}{n-k} + \frac{k\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{z_k^2}{(n-k)\tau_u}}}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}}}{n}. \tag{17}$$

Liquidity is measured by $1/\gamma_k$ (Han and Yang, 2013), which is inversely related to price impact λ_k by the relation (12). A lower γ_k means a liquid/deep market in which a noise trading shock is absorbed without moving the asset price much.

Belief disagreement is defined as the expected absolute value of the difference between two traders' conditional expectations on the asset payoff. The belief disagreement between trader i and trader j is given by

$$\mathbb{E}|\mathbb{E}[\theta|y_r, r \in \mathcal{N}_{i,k}, p_k] - \mathbb{E}[\theta|y_r, r \in \mathcal{N}_{j,k}, p_k]| = \alpha_k\mathbb{E}\left|\sum_{r\in\mathcal{N}_{i,k}} y_r - \sum_{r\in\mathcal{N}_{j,k}} y_r\right|.$$

Price volatility is defined as $\sqrt{\text{Var}(p_k)}$. Price volatility measures the ex-ante uncertainty about the equilibrium price. We have

$$\begin{aligned}\text{Var}(p_k) = \text{Var}\left(\pi_k\sum_{i=1}^n y_i + \gamma_k u\right) &= \pi_k^2\left(\frac{n^2}{\tau_\theta} + \frac{n}{\tau_\epsilon}\right) + \frac{\gamma_k^2}{\tau_u} \\ &= \pi_k^2\left(\frac{n^2}{\tau_\theta} + \frac{n}{\tau_\epsilon} + \frac{z_k^2}{\tau_u}\right).\end{aligned} \tag{18}$$

[13] This is true when $\xi = 0$, since in this case, $k\alpha_k = (n-1)\pi_k/n$ from (D.1), which is increasing in k by Proposition 4 in Section 6. Our simulation shows that it is also true when $\xi > 0$ for the model parameters in Section 6.

Return volatility is defined as $\sqrt{\mathrm{Var}(\theta - p_k)}$. Return volatility measures the ex-ante uncertainty about the return of the risky asset. We have

$$\mathrm{Var}(\theta - p_k) = \frac{(1 - n\pi_k)^2}{\tau_\theta} + \frac{n\pi_k^2}{\tau_\epsilon} + \frac{z_k^2}{\tau_u}\pi_k^2. \tag{19}$$

Trading volume is defined as the expected absolute value of the asset holdings by all speculators at the equilibrium price:

$$\begin{aligned}\sum_{i=1}^{n} \mathbb{E}|x_{i,k}^*| &= n\mathbb{E}|x_{i,k}^*| = n\mathbb{E}|\frac{\mathbb{E}\big[\theta\big|y_j, j \in \mathcal{N}_{i,k}, p_k\big] - p_k}{\lambda_k + \xi}| \\ &= \frac{n\sqrt{2/\pi}}{\lambda_k + \xi}\sqrt{\mathrm{Var}\Big(\mathbb{E}\big[\theta - p_k\big|y_j, j \in \mathcal{N}_{i,k}, p_k\big]\Big)} \\ &= \frac{n\sqrt{2/\pi}}{\lambda_k + \xi}\sqrt{\mathrm{Var}(\theta - p_k) - \mathrm{Var}\big[\theta\big|y_j, j \in \mathcal{N}_{i,k}, p_k\big]},\end{aligned} \tag{20}$$

where the second equality follows from (9), the third from the formula that $\mathbb{E}|y| = \sigma\sqrt{2/\pi}$ if $y \sim N(0, \sigma^2)$, and the fourth from the law of total variance.

The *(expected) trading profit* of speculator i buying $x_{i,k}^*$ units of the risky asset at price p_k is given by

$$\begin{aligned}\mathbb{E}[(\theta - p_k)x_{i,k}^*] &= \mathbb{E}\left[(\theta - p_k)\frac{\mathbb{E}[\theta|y_j, j \in \mathcal{N}_{i,k}, p_k] - p_k}{\lambda_k + \xi}\right] \\ &= \frac{1}{\lambda_k + \xi}\mathbb{E}[\mathbb{E}[\theta - p_k|y_j, j \in \mathcal{N}_{i,k}, p_k]^2] \\ &= \frac{1}{\lambda_k + \xi}\mathrm{Var}[\mathbb{E}[\theta - p_k|y_j, j \in \mathcal{N}_{i,k}, p_k]] \\ &= \frac{1}{\lambda_k + \xi}\Big(\mathrm{Var}(\theta - p_k) - \mathrm{Var}[\theta|y_j, j \in \mathcal{N}_{i,k}, p_k]\Big),\end{aligned} \tag{21}$$

where the first equality follows from (9) and the last one from the law of total variance.

As we will see, there are highly tractable closed-form expressions for the equilibrium outcomes when $\xi = 0$, but this is not the case when $\xi > 0$ since the closed-form expression for z_k is too complicated to analyze; see the cubic Eq. (13) that z_k satisfies. Next, we first focus on the case of $\xi = 0$, that is, speculators are risk neutral, in Section 6.1, and shift our attention to the more general case of $\xi > 0$ in Section 6.2 with the help of numerical examples.

6.1. Linear utility

In this subsection, we consider the case where speculators are risk neutral, that is, $\xi = 0$. In this case, from the Eq. (13) with the setting of $\xi = 0$, we have the closed-form expression

$$z_k = \sqrt{\frac{n(n-k)\tau_u}{(n-2)k\tau_\epsilon}}. \tag{22}$$

Note that in Section 5 we have demonstrated that $z_k^2/\tau_u \to \frac{n(n-k)}{(n-2)k\tau_\epsilon}$ as $\tau_u \to \infty$, and the limit is the same as that in the limit case of $\xi \to 0$. We then find that the following implications also apply to the case of $\tau_u \to \infty$, that is, as noise trading becomes small. The following proposition summarizes the implications of information linkages on market equilibrium outcomes.

Proposition 4. *Suppose speculators are risk neutral, that is, $\xi = 0$. Then*

(i) *Price impact strictly decreases with the number of information linkages, that is, $\partial\lambda_k/\partial k < 0$.*
(ii) *Belief disagreement between any two traders strictly decreases with the number of information linkages, that is, $\partial(\mathbb{E}|\mathbb{E}[\theta|y_r, r \in \mathcal{N}_{i,k}, p_k] - \mathbb{E}[\theta|y_r, r \in \mathcal{N}_{j,k}, p_k]|)/\partial k < 0$.*
(iii) *The signal sensitivity strictly increases with the number of information linkages, that is, $\partial\pi_k/\partial k > 0$.*
(iv) *Liquidity strictly increases with the number of information linkages, that is, $\partial(1/\gamma_k)/\partial k > 0$.*
(v) *Price volatility strictly increases with the number of information linkages, that is, $\partial\sqrt{\mathrm{Var}(p_k)}/\partial k > 0$.*
(vi) *Return volatility strictly decreases with the number of information linkages, that is, $\partial\sqrt{\mathrm{Var}(\theta - p_k)}/\partial k < 0$.*
(vii) *Trading volume is independent of the number of information linkages, that is, $\partial(\sum_{i=1}^{n}\mathbb{E}|x_{i,k}^*|)/\partial k = 0$.*
(viii) *Trading profits strictly decrease with the number of information linkages, that is, $\partial\mathbb{E}[(\theta - p_k)x_{i,k}^*]/\partial k < 0$.*

6.2. Linear utility with a quadratic holding cost

In this subsection, we consider the case where speculators are risk neutral with a quadratic holding cost, that is, $\xi > 0$. As seen in the last section, to theoretically analyze the implications, we need the closed-form expression for z_k. Although there is also a closed-form expression for z_k in the case of $\xi > 0$, z_k is the root of a cubic equation,[14] and hence, a fully tractable analysis is not available. Here, we investigate the implications of the number of information linkages numerically.

The effects of the number of information linkages on the price impact, signal sensitivity, liquidity, belief disagreement, price volatility, return volatility, trading volume, and trading profits are respectively displayed in Fig. 2 with the parameter values of $n = 23$, $\tau_\theta = 25$, $\tau_u = 10$ and $\tau_\epsilon = 5$ (Han and Yang (2013)), and $\xi = 2$. In Panel (b) of Fig. 2, we take $d = 7$ as the distance parameter of two speculators. According to Fig. 2, the implications of the number of information linkages on market quality in the case of $\xi = 0$ (speculators are risk neutral) also hold in the case of $\xi > 0$ (speculators are risk neutral with a quadratic holding cost) except that the trading volume is independent of the number of information linkages in the case of $\xi = 0$, but strictly decreases with the number of information linkages in the case of $\xi > 0$.

As the number of information linkages k increases, speculators' signals become informationally closer and speculators trade on more similar signals, which increases the competition among speculators and then reduces each speculator's monopolistic power, that is, the price impact of each speculator decreases with k. This is the result in Part (i) of Proposition 4 and Panel (a) of Fig. 2. Moreover, it is quite intuitive that the belief disagreement between speculators decreases with k since speculators' beliefs are generated by their signals and their signals contain more common information as k increases. This is the result in Part (ii) of Proposition 4 and Panel (b) of Fig. 2.

As the number of information linkages k increases, speculators' signals are more informative compared with the price so that speculators trade more aggressively relatively on their signals than on the price, leading to that more information is incorporated into the price, and consequently, the signal sensitivity π_k increases with k. This is shown in Part (iii) of Proposition 4 and Panel (c) of Fig. 2. Furthermore, as k increases, speculators' beliefs depend less on the price so that their demands are potentially more elastic. In addition, a decrease of the price impact further increases the price elasticity of demand. As a result, buying one more unit of the risky asset by noise traders will increase the price less, that is, the market liquidity improves with k. This is shown in Part (iv) of Proposition 4 and Panel (d) of Fig. 2. Moreover, this is consistent with the implication of a decreasing price impact (Part (i)), i.e., the demand of one more unit of the risky asset by speculators will move the price less.

Following Ozsoylev and Walden (2011), the price volatility can be decomposed into an information-driven volatility component, $\pi_k^2(n^2/\tau_\theta + n/\tau_\epsilon)$, and a noise-trading driven volatility γ_k^2/τ_u. Part (iii) reveals that the first component increases (i.e., π_k increases with k), and Part (iv) demonstrates that the second component decreases (i.e., γ_k decreases with k). The information-driven volatility component dominates the noise-trading driven volatility component, and consequently, their sum, the price volatility, increases with k. This is the result in Part (v) of Proposition 4 and Panel (e) of Fig. 2. Intuitively, as speculators have more common signals, they have more similar beliefs/expectations on the asset payoff (Part (ii)) and are then more likely to trade in the same direction (against the trading direction by noise traders to satisfy the market-clearing condition) so that the competition between speculators becomes greater and the price p consequently becomes more volatile. This is consistent with the phenomenon in financial markets that asset prices fluctuate more when speculators have similar expectations on the future payoff of stocks due to possessing similar information, for example, the public monetary policy released by central banks, company announcements–for example, company merger, earnings announcements, and so on, or even popular rumors in the trader population (Ahern and Sosyura, 2015; Pound and Zeckhauser, 1990).

The return volatility $\sqrt{\mathrm{Var}(\theta - p_k)} = \sqrt{\mathrm{Var}(\theta) + \mathrm{Var}(p_k) - 2\mathrm{Cov}(\theta, p_k)}$ can be decomposed into a positive price volatility component, $\mathrm{Var}(p_k)$, and a negative correlation component, $2\mathrm{Cov}(\theta, p_k)$. Although price volatility increases with k (Part (v)), the price is more correlated with the asset payoff ($\mathrm{Cov}(\theta, p_k) = n\pi_k\sigma_\theta^2$, see Part (iii)) and the latter component dominates the first so that return volatility decreases with k. Moreover, from the optimal demand strategies (9) and the market-clearing condition (5), we can see that the price can alternatively be expressed as

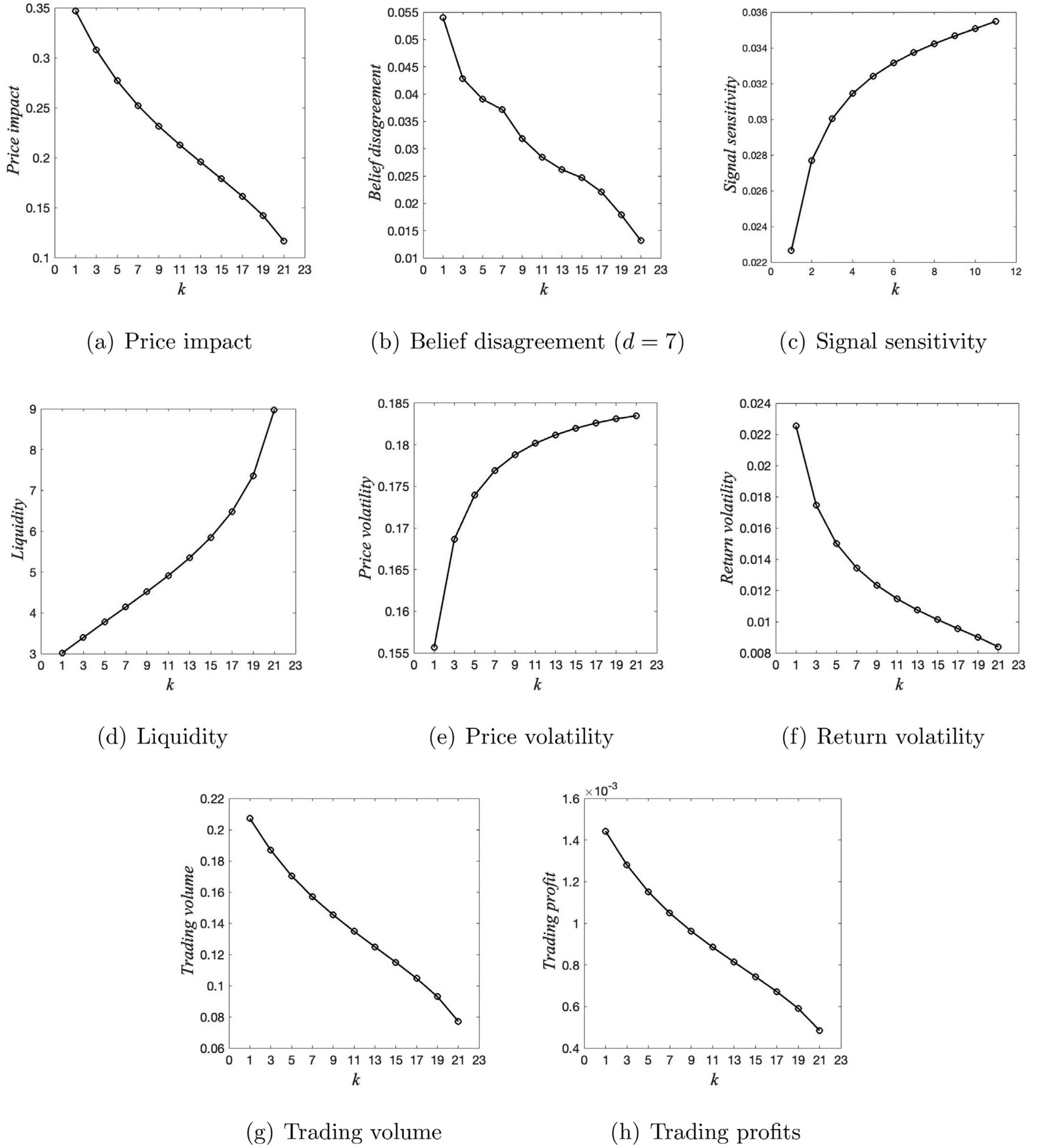
$$p_k = \frac{\sum_{i=1}^n \mathbb{E}[\theta | y_j, j \in \mathcal{N}_{i,k}, p_k]}{n} + \frac{\lambda_k + \xi}{n} u,$$

which is the sum of two terms, the first being an average of speculators' beliefs $\mathbb{E}[\theta | y_j, j \in \mathcal{N}_{i,k}, p_k]$, the second a noise-trading driven term. Intuitively, as the number of information linkages increases, each speculator's belief/valuation is closer to the asset payoff, and consequently, as an average of speculators' beliefs (disturbed by a noise-trading driven term, which is decreasing in k, see Part (i)), the price is also closer to the asset payoff. This is shown in Part (vi) of Proposition 4 and Panel (f) of Fig. 2. Moreover, return volatility and belief disagreement move together, which is consistent with the result

[14] The unique positive root to the cubic equation $a_3 z^3 - a_2 z^2 - a_1 z - a_0 = 0$, $a_i > 0$, $i = 0, 1, 2, 3$ is given by

$$z = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{m^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{m^3}{27}}} + \frac{a_2}{3a_3},$$

where $m = \frac{-3a_3a_1 - a_2^2}{3a_3^2}$, $q = \frac{-2a_2^3 - 9a_3a_2a_1 - 27a_3^2a_0}{27a_3^3}$.



(a) Price impact (b) Belief disagreement ($d = 7$) (c) Signal sensitivity

(d) Liquidity (e) Price volatility (f) Return volatility

(g) Trading volume (h) Trading profits

Fig. 2. Implications of Information Linkages. *Notes.* This figure shows the impacts of the number of information linkages k on price impact, belief disagreement, signal sensitivity, liquidity, price volatility, return volatility, trading volume, and trading profits with the parameter setting of $n = 23$, $\tau_\theta = 25$, $\tau_u = 10$, $\tau_\epsilon = 5$ and $\xi = 2$.

that the investor opinion disagreement and stock return volatility are positively correlated for multiple-period models in Shalen (1993) and Wang (1998).

The trading volume can also be decomposed into an information-driven component, $\sqrt{\text{Var}(\theta - p_k) - \text{Var}[\theta | y_j, j \in \mathcal{N}_{i,k}, p_k]}$, and a price-impact driven component, λ_k; see Eq. (20). As previously discussed, as the number of information linkages k increases, on the one hand, the risk faced by speculators is reduced, that is, the conditional variance $\text{Var}[\theta | y_j, j \in \mathcal{N}_{i,k}, p_k]$ decreases with k, which potentially drives an increase in trading volume. On the other hand, when the price p_k is closer to the asset payoff θ (Part (vi)), there is a lower expected return, referred to as "return effect" in He et al. (2021). The return effect dominates the conditional-variance caused effect and consequently, the information-driven component decreases with k. Moreover, the decreasing price impact (Part (i)) incentivizes agents to increase their trading. However, the price-impact driven component completely offsets the information-driven component so that the trading volume is independent of k in the case of $\xi = 0$. When $\xi > 0$, the information-driven component dominates the price-impact driven component, and then the trading volume strictly decreases with k. The intuition behind the difference between the results in the two cases of $\xi = 0$ and $\xi > 0$ is that when there is an additional holding cost in the case of $\xi > 0$, speculators will reduce their trading aggressiveness to save the resulting cost and the trading volume consequently decreases with k. This is the result in Part (vii) of Proposition 4 and Panel (g) of Fig. 2. Intuitively, this is consistent with the idea that disagreement among traders contributes to trading volume documented, see Hong and Stein (2007), Banerjee and Kremer (2010), Kyle et al. (2018) and so on,[15] and that information symmetry/homogeneity across speculators will lead to similar beliefs of speculators (Part (ii)), which destroys the potential trading opportunities and reduces trading volume.[16]

Analogous to trading volume, the expected trading profits can also be decomposed into an information-driven component, $\mathrm{Var}(\theta - p_k) - \mathrm{Var}[\theta | y_j, j \in \mathcal{N}_{i,k}, p_k]$, and a price-impact driven component, λ_k; see Eq. (21). From the above discussions on trading volume, the information-driven component dominates the price-impact driven component so that the trading profits decrease with k. Moreover, the implication can also be observed intuitively from the definition of trading profits $\mathbb{E}[(\theta - p_k)x^*_{i,k}]$ since the price p_k is closer to θ (Part (vi)) and the trading volume $\mathbb{E}|x^*_{i,k}|$ is lower (Part (vii)) as shown above. This is the result in Part (viii) of Proposition 4 and Panel (h) of Fig. 2. The result is in line with the well-known Hirshleifer effect (Hirshleifer, 1971), which refers to a welfare loss when market participants possess more information. Furthermore, we see that trading profits and trading volume move together, that is, lower trading volume leads to lower trading profits.

At the end of this section, we make some comparisons between our results and that in the large economies of Ozsoylev and Walden (2011) and Han and Yang (2013). Note that our model is distinguished from Ozsoylev and Walden (2011) and Han and Yang (2013) by the following main differences. The first is that while Ozsoylev and Walden (2011) and Han and Yang (2013) consider a large economy (the number of speculators tends to infinity), we consider a finite-agent economy in which agents have price impact. The second is that while the (scaled) average node degree in Ozsoylev and Walden (2011) and the group size in Han and Yang (2013) can take any value of positive integers, the number of information linkages in our finite-agent economy naturally can only take (odd) values in $[1, n-2]$.

In the large economy of Ozsoylev and Walden (2011), liquidity, price volatility, and trading profits are shown to be non-monotonic functions of network connectedness under some parameters, but they are monotonic in our economy with imperfect competition for any model parameter. The main reason for the non-monotonicity of γ_k (note that the liquidity is defined as $1/\gamma_k$) in Ozsoylev and Walden (2011) and Han and Yang (2013) is that speculators' demand first rely more on the price as an information source as speculators have less information and learn more additional information for a low network connectedness, and rely less on the price as speculators have enough information and learn less additional information from the price for a high network connectedness. However, this is not the case in our finite-agent economy since any increase in the number of information linkages exhibits a non-negligible influence, which corresponds to a similar setup of a relatively large network connectedness in Ozsoylev and Walden (2011). Hence, our result in Part (iv) of Proposition 4 is consistent with Proposition 3 (b) in Ozsoylev and Walden (2011) that γ_k decreases with a large network connectedness. Similar explanations also apply to price volatility and trading profits and our results in Parts (v) and (viii) of Proposition 4 are consistent with Proposition 3 (c) in Ozsoylev and Walden (2011) that price volatility increases with, and Proposition 7 in Ozsoylev and Walden (2011) that trading profits decrease with, a large network connectedness, respectively.

In Ozsoylev and Walden (2011), the trading volume is shown to be increasing in network connectedness for a low variance of network connectedness,[17] and Han and Yang (2013), which contrasts with our finding. The main reason for the opposite effect is that in the denominator of trading volume, there is respectively a conditional-variance adjusted term in the two large economies with CARA utility functions (see the expression (A.3) for the trading volume in Lou and Yang (2022)), and a price-impact term in our finite-agent economy with linear utility but with a quadratic holding cost (see Eq. (20)). Although like the conditional variance, the price impact is also decreasing in k (Part (i)), the decay is not strong so that traders trade less in our economy compared to that in the large economies.

7. Empirical implications

Our model suggests that information networks have an important influence on traders' behavior and market equilibrium outcomes. Next, we briefly outline how our predictions can be tested. The predictions below follow from the results in the previous section.

[15] Note that the meanings of *disagreement* in these studies differ from that in our setup. While it refers to the difference on the interpretation of the same information, for example, traders may have heterogeneous subjective priors on the means or precision even for the same information, due to behavioral biases, it means objective difference between available information of traders in our model.

[16] Consider the case of $k = n$ where speculators have a completely identical information set. In this case, speculators have the same beliefs on the asset payoff and have the same trading portfolio. In fact, speculators' equilibrium portfolios are only a function of noise trading, without depending on the signals. That is, speculators only trade with noise traders and there is no trading among speculators. Note that we exclude the uninteresting case of $k = n$. Please see also Footnote 8 for more related discussions.

[17] The network connectedness may differ across traders in Ozsoylev and Walden (2011), but is the same for all traders in our economy. Hence, our setup corresponds to the setting of zero variance of network connectedness in Ozsoylev and Walden (2011).

Predictions

(a) *Traders in a market with more information linkages hold more similar portfolios.*
(b) *Price impact is low in markets with high levels of information linkages.*
(c) *Price volatility is high in markets with high levels of information linkages.*
(d) *Return volatility is low in markets with high levels of information linkages.*
(e) *Trading volume is low in markets with high levels of information linkages.*
(f) *Trading profits is low in markets with high levels of information linkages.*

The above predictions can be tested by comparing the implications across markets. Different markets can be interpreted as different exchanges (especially the exchanges in which the same asset is traded), stock types, asset classes, or even geographical regions (Ozsoylev and Walden, 2011). Before comparisons, a preliminary task is to estimate the parameter k of information linkages. We can employ the methods based on the portfolios of agents indirectly used in Ozsoylev et al. (2014) to identify the numbers of information linkages of agents, or the geographic distance between trader locations to proxy for the density of information linkages (Hau, 2001). We can take the average of the numbers of information linkages of all traders in the empirical samples as the parameter of information linkages.

To test the trading correlation, we can use the absolute value of the difference between two traders' holdings in the risky asset to measure the trading correlation, and the larger the difference, the lower the trading correlation. Prediction (a) has been well understood in the literature and is consistent with the empirical findings in Hong et al. (2005) and Pool et al. (2015) that socially connected traders have more similar asset holdings. Moreover, we can use the ratio of the absolute value of price changes and trading volume over the sample period to measure the price impact. As for price volatility and return volatility, we can use the respective sample variance to proxy for these. Once we obtain the statistical values of these market quality indexes, we can immediately test these predictions across markets.

Hau (2001) examines the trading profits of informationally asymmetric traders where traders' informational advantage is measured according to their geographical proximity to corporate headquarters of equities they trade in and reveals that those located in the financial center (Frankfurt) do not outperform those in other locations in Germany, suggesting that local interaction between traders is not crucial to trading performance. It is reasonable to expect that traders located in the financial center have information advantage relative to those in other locations. Hau's finding is consistent with Prediction (f) to some extent.

8. Concluding remarks

We investigated an imperfectly competitive economy based on Kyle (1989) in which speculators are socially connected via an information network. This economy is analytically tractable and help us consider how the information linkages among speculators affect trading behavior and market quality parameters including market efficiency, trading volume, trading profits, and so on.

One interesting extension would be to consider a multiple-period version of the model in which the extent of information overlap becomes higher over time. Speculators can obtain access to more information from the other channels at distant locations over time. This extension allows us to examine the effects of information networks on the dynamics of market quality parameters, and may potentially shed further light on the relationship between return volatility and trading volume.[18]

Another potential extension would be to analyze the case where the information is endogenous in the sense that speculators acquire information from some close channels by paying a precision-dependent cost (Lou et al. 2022; Verrecchia 1982). With such an extension, we can analyze the effects of the number of information linkages on the precision of endogenous information as well as the resulting market equilibrium parameters analogous to the setting of exogenous information. As in Han and Yang (2013), we would make comparisons to determine whether the implications of information networks in both cases of exogenous and endogenous information display the similar pattern. Furthermore, we would also make comparisons to examine whether the implications in the two setups of the imperfect and perfect competition considered in Lou and Yang (2022) are the same.

Acknowledgments

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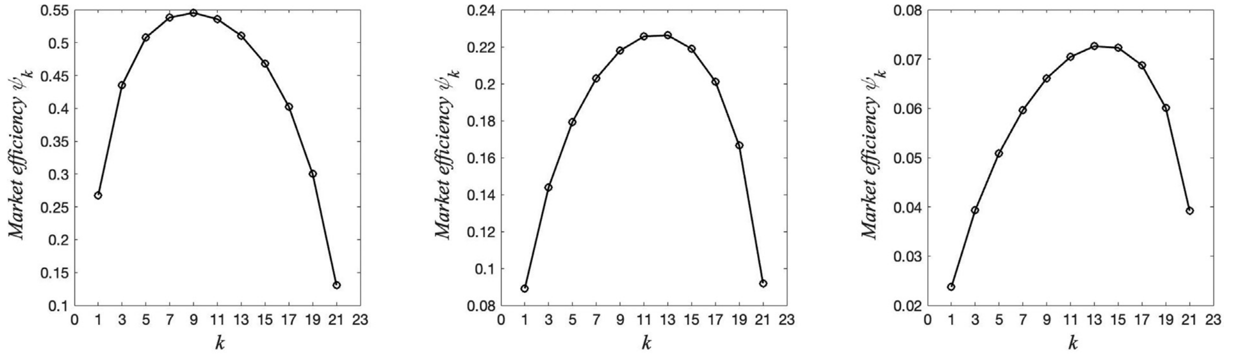
Appendix A

In Section 5, we demonstrated that the measure of market efficiency ψ_k is monotonically increasing in k under the limit of $\tau_u \to \infty$. Here, we numerically show that the measure of market efficiency ψ_k may be not monotonic for general $\tau_u > 0$.

[18] Please refer to an earlier survey by Karpoff (1987) and a recent survey by Yamani (2022) for more related work.



(a) (b) (c)

Fig. A.1. Market Efficiency. *Notes.* This figure shows the impact of the number of information linkages k on the market efficiency ψ_k. The parameter values are given as follows: $n = 23$, $\tau_\theta = 25$, $\tau_\epsilon = 5$ and $\xi = 2$. Moreover, τ_u is taken as 0.1 in Panel (a), as 10 in Panel (b), and as 100 in Panel (c).

Figure A.1 indicates that the measure of market efficiency used in Kyle (1989) may be initially increasing and eventually decreasing in the number of information linkages.

Appendix B

Here we test the robustness of the implications of information linkages on market equilibrium outcomes for more general network structures. To keep tractability, in the baseline model we use a special cyclical graph to describe the information network of speculators. Now we analyze whether the implications in the setting of cyclical networks still hold when the information network is general and becomes more dense. To this end, we compare the implications under two network structures: the first one is a general sparse (undirected) network, and the second one is a dense network, which is defined as the union graph of the sparse network and a cyclical graph (where each node has two neighbors).

As the first step, we first derive the system of equations that the market equilibrium under a general network satisfies. We still let $\mathcal{N}_i$ denote the neighbor set of agent i including himself/herself and suppose $p = \sum_{i=1}^n \bar{\pi}_i y_i + \gamma u$. Then from the projection theorem for normal random variables, we have

$$\begin{aligned}
\mathbb{E}[\theta | y_j, j \in \mathcal{N}_i, p] &= \mathbb{E}\left[\theta \middle| y_j, j \in \mathcal{N}_i, \sum_{r \notin \mathcal{N}_i} \bar{\pi}_r y_r + \gamma u\right] \\
&= \frac{\tau_\epsilon \sum_{j \in \mathcal{N}_i} y_j + \frac{1}{\frac{\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r^2}{\left(\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r\right)^2 \tau_\epsilon} + \frac{\gamma^2}{\left(\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r\right)^2 \tau_u}} \left(\theta + \frac{\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r \epsilon_r}{\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r} + \frac{\gamma}{\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r} u\right)}{\tau_\theta + |\mathcal{N}_i| \tau_\epsilon + \frac{1}{\frac{\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r^2}{\left(\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r\right)^2 \tau_\epsilon} + \frac{\gamma^2}{\left(\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r\right)^2 \tau_u}}} \\
&= \frac{\tau_\epsilon \sum_{j \in \mathcal{N}_i} y_j + \frac{1}{\frac{\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r^2}{\left(\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r\right)^2 \tau_\epsilon} + \frac{\gamma^2}{\left(\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r\right)^2 \tau_u}} \frac{1}{\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r} \left(p - \sum_{j \in \mathcal{N}_i} \bar{\pi}_j y_j\right)}{\tau_\theta + |\mathcal{N}_i| \tau_\epsilon + \frac{1}{\frac{\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r^2}{\left(\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r\right)^2 \tau_\epsilon} + \frac{\gamma^2}{\left(\sum_{r \notin \mathcal{N}_i} \bar{\pi}_r\right)^2 \tau_u}}} \\
&=: \sum_{j \in \mathcal{N}_i} \alpha_{ij} y_j + \beta_i p,
\end{aligned}$$

where α_{ij}, β_i are functions of $\{\bar{\pi}_1, \dots, \bar{\pi}_n, \gamma\}$ defined by the second last equality. Let λ_i denote the price impact parameter of speculator i. Then the market-clearing condition becomes

$$\sum_{i=1}^n x_i^* + u = \sum_{i=1}^n \frac{\sum_{j \in \mathcal{N}_i} \alpha_{ij} y_j + (\beta_i - 1) p}{\lambda_i + \xi} + u = 0,$$

from which we have

$$p = \left[\sum_{i=1}^n \frac{1 - \beta_i}{\lambda_i + \xi}\right]^{-1} \left[\sum_{i=1}^n \frac{\sum_{j \in \mathcal{N}_i} \alpha_{ij} y_j}{\lambda_i + \xi} + u\right].$$

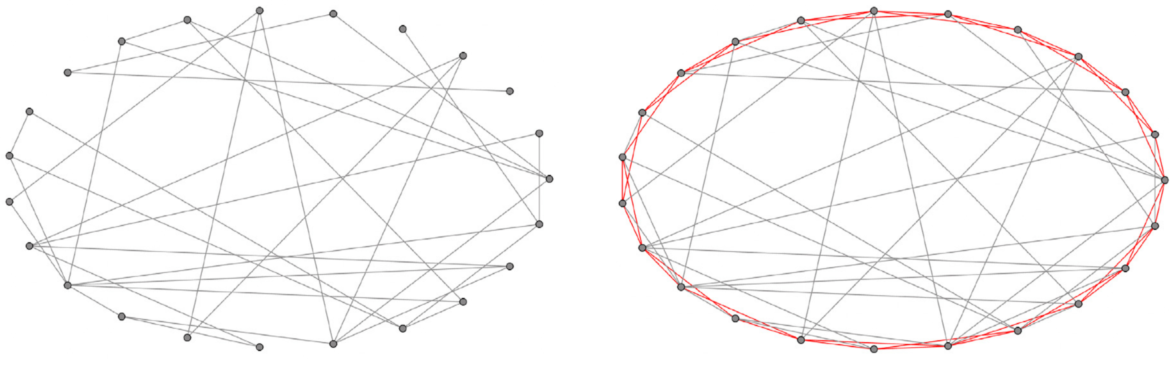


Fig. B.1. Two Information Networks. *Notes.* This figure shows the two networks used in the numerical example. Panel (a) shows an Erdős-Rényi random graph with $n = 23$ nodes, where each edge is included in the graph with probability $p = 0.15$ independently. Panel (b) shows the union graph of the graph in Panel (a) and the 2-cyclical graph.

Table B.1
Implications of Information Linkages for General Network Structures.

	price impact	belief disagreement	signal sensitivity	liquidity
the sparse network	0.1376	0.0707	0.0109	7.5996
the dense network	0.1341	0.0667	0.0153	7.7949

	price volatility	return volatility	trading volume	trading profits
the sparse network	0.0049	0.0248	0.8108	0.0042
the dense network	0.0078	0.0196	0.7575	0.0036

Notes. This table shows the market outcomes under the sparse network and the dense network in Fig. B.1, where the parameter setting is the same as the ones in the main model: $n = 23$, $\tau_\theta = 25$, $\tau_u = 10$, $\tau_\epsilon = 5$ and $\xi = 2$.

Matching coefficients leads to

$$\gamma = \left[\sum_{i=1}^{n} \frac{1 - \beta_i}{\lambda_i + \xi} \right]^{-1}, \tag{B.1}$$

$$\bar{\pi}_i = \gamma \sum_{j \in \mathcal{N}_i} \frac{\alpha_{ji}}{\lambda_j + \xi}, \quad i = 1, \ldots, n. \tag{B.2}$$

Moreover, the price impact parameters endogenously satisfy

$$\lambda_i = \left[\sum_{j \neq i} \frac{1 - \beta_j}{\lambda_j + \xi} \right]^{-1}, \quad i = 1, \ldots, n. \tag{B.3}$$

From (B.1)–(B.3), we see that there are $(2n + 1)$ equations with $(2n + 1)$ variables $\{\bar{\pi}_i, \lambda_i, i = 1, \ldots, n, \gamma\}$. It is worth noting that the economy under the general network structure is no longer symmetric, i.e., $\lambda_i \neq \lambda_j$, $\bar{\pi}_i \neq \bar{\pi}_j$ for $i \neq j$. The proof of equilibrium existence is challenging due to the generality of network structures. Instead, below we use one numerical example to illustrate the robustness.

We consider the sparse network in Fig. B.1(a), and the dense network in Fig. B.1(b). Here we use the average $\sum_{i=1}^{n} \lambda_i / n$, the average $\sum_{i=1}^{n} \bar{\pi}_i / n$, the maximum $\max_{i,i+d} \mathbb{E}|\mathbb{E}[\theta | y_r, r \in \mathcal{N}_i, p] - \mathbb{E}[\theta | y_r, r \in \mathcal{N}_{i+d}, p]|$ (here we set $d = 7$ to be consistent with the setting in the baseline model), the average $\sum_{i=1}^{n} \mathbb{E}[(\theta - p)x_i^*]/n$ to respectively measure price impact, signal sensitivity, belief disagreement, and trading profits. The numerical results are summarized in Table B.1 and it shows that the implications of information linkages on market equilibrium outcomes in the main model are robust to the network structure.

Appendix C

In the baseline model, for analytical tractability we assume that speculators share signals with their neighbors perfectly. But it may not be the case in reality and speculators may have an incentive to share only noisy signals with their neighbors due to various reasons, for example, privacy concerns, market competition, etc. An alternative explanation on the imperfectness of signal sharing is that speculators do not directly share signals with each other, but instead get access to other speculators' (noisy) signals through information leakage. Here we consider a more general setting that speculators cannot perfectly observe neighbors' information but corrupted by noise. Specifically, each speculator i can only observe a signal $z_j = y_j + \eta_j$ of her neighbor $j \in \mathcal{N}_i$, where $\eta_j \sim N(0, 1/\tau_\eta)$, $\tau_\eta > 0$, are the communication noise independent of other random variables in the model. The other setup is in line with the baseline model.

We now derive the system of equations that the market equilibrium satisfies. Suppose that the conjectured linear price function is $p = \pi_1 \sum_{i=1}^n y_i + \pi_2 \sum_{i=1}^n z_i + \gamma u$, where π_1, π_2 and γ are three endogenous positive constants to be determined later. To simplify the expression, we denote $\boldsymbol{s}_i := (y_i, z_j, j \in \mathcal{N}_i/\{i\}, p)$ as the signal vector of speculator i. Following the projection theorem for normal random variables, we can see that

$$\begin{aligned}\mathbb{E}\left[\theta \middle| y_i, z_j, j \in \mathcal{N}_i/\{i\}, \pi_1 \sum_{r=1}^n y_r + \pi_2 \sum_{r=1}^n z_r + \gamma u\right] &= \Sigma_{\theta,\boldsymbol{s}_i} \Sigma_{\boldsymbol{s}_i,\boldsymbol{s}_i}^{-1} \boldsymbol{s}_i \\ &=: \alpha_1 y_i + \alpha_2 \sum_{j\in\mathcal{N}_i/\{i\}} z_j + \beta p, \end{aligned} \tag{C.1}$$

where

$$\Sigma_{\theta,\boldsymbol{s}_i} = \mathrm{Cov}(\theta, \boldsymbol{s}_i) = \Big(\underbrace{1/\tau_\theta, 1/\tau_\theta, \ldots, 1/\tau_\theta}_{k}, n(\pi_1+\pi_2)/\tau_\theta\Big),$$

$$\Sigma_{\boldsymbol{s}_i,\boldsymbol{s}_i} := \begin{bmatrix} \frac{1}{\tau_\theta}+\frac{1}{\tau_\epsilon} & \frac{1}{\tau_\theta} & \cdots & \frac{1}{\tau_\theta} & \frac{n(\pi_1+\pi_2)}{\tau_\theta}+\frac{(\pi_1+\pi_2)}{\tau_\epsilon} \\ \frac{1}{\tau_\theta} & \frac{1}{\tau_\theta}+\frac{1}{\tau_\epsilon}+\frac{1}{\tau_\eta} & \cdots & \frac{1}{\tau_\theta} & \frac{n(\pi_1+\pi_2)}{\tau_\theta}+\frac{(\pi_1+\pi_2)}{\tau_\epsilon}+\frac{\pi_2}{\tau_\eta} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{1}{\tau_\theta} & \frac{1}{\tau_\theta} & \cdots & \frac{1}{\tau_\theta}+\frac{1}{\tau_\epsilon}+\frac{1}{\tau_\eta} & \frac{n(\pi_1+\pi_2)}{\tau_\theta}+\frac{(\pi_1+\pi_2)}{\tau_\epsilon}+\frac{\pi_2}{\tau_\eta} \\ \frac{n(\pi_1+\pi_2)}{\tau_\theta}+\frac{(\pi_1+\pi_2)}{\tau_\epsilon} & \frac{n(\pi_1+\pi_2)}{\tau_\theta}+\frac{(\pi_1+\pi_2)}{\tau_\epsilon}+\frac{\pi_2}{\tau_\eta} & \cdots & \frac{n(\pi_1+\pi_2)}{\tau_\theta}+\frac{(\pi_1+\pi_2)}{\tau_\epsilon}+\frac{\pi_2}{\tau_\eta} & \frac{n^2(\pi_1+\pi_2)^2}{\tau_\theta}+\frac{n(\pi_1+\pi_2)^2}{\tau_\epsilon}+\frac{n\pi_2^2}{\tau_\eta}+\frac{\gamma^2}{\tau_u} \end{bmatrix},$$

and $\alpha_1, \alpha_2, \beta$ are the functions of π_1, π_2, γ and satisfy the following equation

$$\Big(\alpha_1, \underbrace{\alpha_2, \cdots, \alpha_2}_{k-1}, \beta\Big) = \Sigma_{\theta,\boldsymbol{s}_i} \Sigma_{\boldsymbol{s}_i,\boldsymbol{s}_i}^{-1},$$

which is independent of i due to the symmetry of the signal structure.

Following from (C.1) and the market-clearing condition, we have

$$\begin{aligned}\sum_{i=1}^n x_i^* + u &= \sum_{i=1}^n \frac{\mathbb{E}[\theta | y_i, z_j, j \in \mathcal{N}_i/\{i\}, p] - p}{\lambda+\xi} + u \\ &= \sum_{i=1}^n \frac{\alpha_1 y_i + \alpha_2 \sum_{j\in\mathcal{N}_i/\{i\}} z_j + \beta p - p}{\lambda+\xi} + u = 0.\end{aligned}$$

This implies

$$p = \frac{1}{n(1-\beta)}\left[\alpha_1 \sum_{i=1}^n y_i + (k-1)\alpha_2 \sum_{i=1}^n z_i + (\lambda+\xi) u\right].$$

Furthermore, similar to the Eq. (12) we also have $\lambda = \frac{n}{n-1}\gamma$. Then, identifying coefficients of the previous price function and the conjectured one and using the preceding relation yields

$$\begin{aligned}\pi_1 &= \frac{1}{n(1-\beta)}\alpha_1, \\ \pi_2 &= \frac{1}{n(1-\beta)}(k-1)\alpha_2, \\ \gamma &= \frac{1}{n(1-\beta)}(\lambda+\xi) = \frac{\xi}{n(1-\beta-\frac{1}{n-1})}.\end{aligned}$$

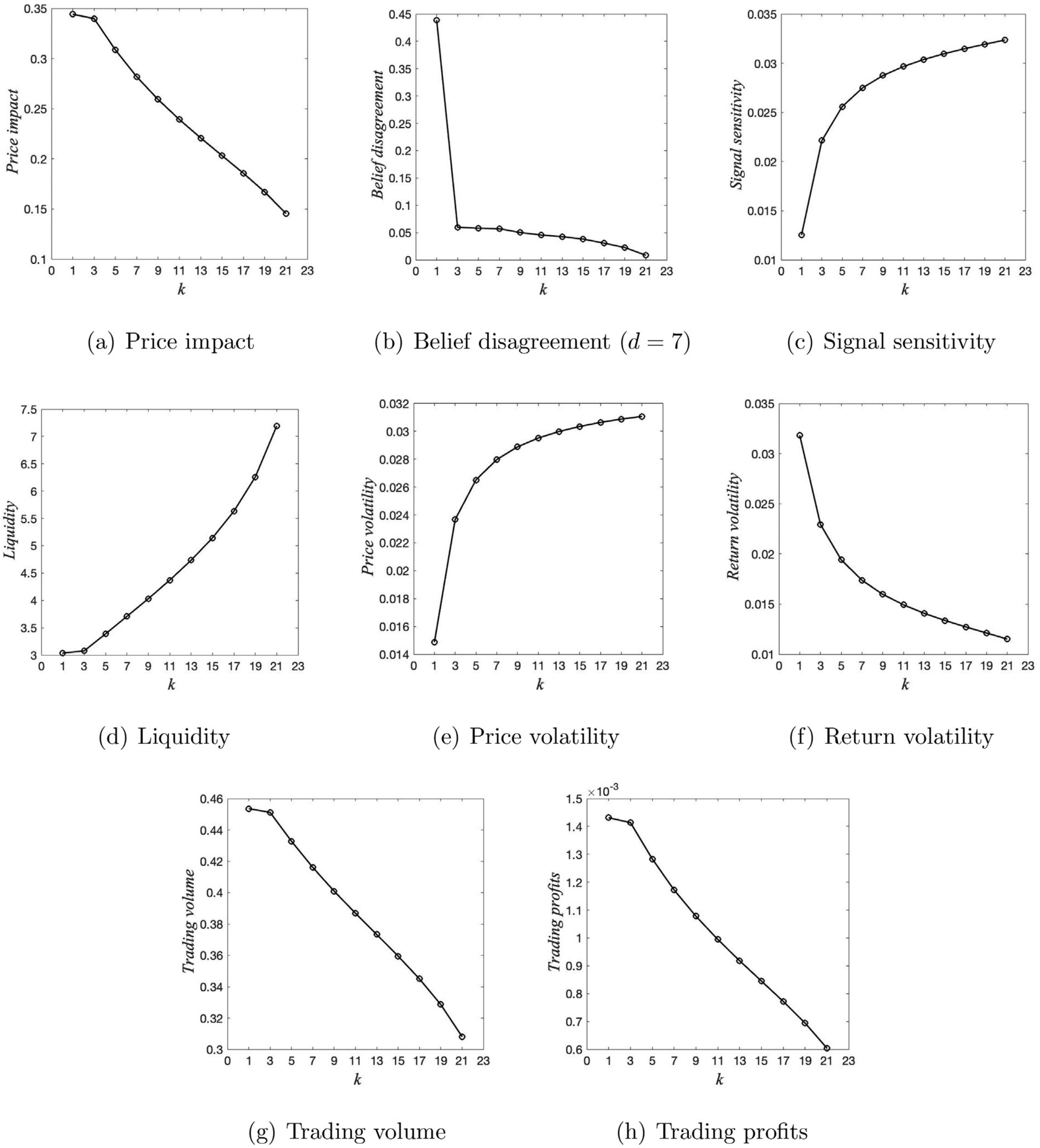


Fig. C.1. Implications of Information Linkages in Presence of Imperfectly Shared Signals. *Notes.* This figure shows the impacts of the number of information linkages k on price impact, belief disagreement, signal sensitivity, liquidity, price volatility, return volatility, trading volume, and trading profits, where the parameters are set as $n = 23$, $\tau_\theta = 25$, $\tau_u = 10$, $\tau_\epsilon = 5$, $\tau_\eta = 10$ and $\xi = 2$. Here we use $\pi_1 + \pi_2$ to measure the signal sensitivity. Our simulations show that the implications in this figure continue to hold for larger values of $\tau_\eta = 20, 50, 100$.

This is a system of three equations with three variables $\{\pi_1, \pi_2, \gamma\}$. Since it is quite difficult to get the explicit solution, here we illustrate the robustness with the help of numerical examples. The implications of information linkages on market equilibrium outcomes are summarized in Fig. C.1 and it shows that the implication results in the baseline model also hold for the more general setting where the shared signals between speculators are imperfect.

Appendix D

In this appendix, we provide the proofs for all the propositions.

Proof of Proposition 1.

From the discussions before Proposition 1, it is sufficient to demonstrate that the system of Eqs. (10), (11) and (12) has a unique positive solution (π, γ, λ). From (10), (11) and (12), we have

$$\frac{\pi}{\gamma} = \frac{k\alpha}{\lambda + \xi} = \frac{k\alpha}{\frac{n}{n-1}\gamma + \xi}, \tag{D.1}$$

and hence

$$\gamma = \frac{n-1}{n}\left(\frac{k\alpha}{\frac{\pi}{\gamma}} - \xi\right). \tag{D.2}$$

Moreover, combining (12) with (11), we obtain

$$n\left(1 - \beta - \frac{1}{n-1}\right)\gamma = \xi. \tag{D.3}$$

It then follows from (D.3), (D.2), (7), and (8) that

$$\begin{aligned}
\frac{\xi}{n} &= \left(1 - \frac{1}{n-1}\right)\gamma - \beta\gamma \\
&= \frac{n-2}{n-1}\frac{n-1}{n}\left(\frac{k\alpha}{\frac{\pi}{\gamma}} - \xi\right) - \beta\gamma \\
&= \frac{n-2}{n}\left[\frac{1}{\frac{\pi}{\gamma}}\frac{k\left(\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon} + \frac{\gamma^2}{(n-k)\pi^2\tau_u}}\right)}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}}} - \xi\right] - \frac{\frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}}\frac{\gamma}{(n-k)\pi}}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{\gamma^2}{(n-k)^2\pi^2\tau_u}}}.
\end{aligned} \tag{D.4}$$

Denote by the ratio of the weight on noise trade to that on signals $z := \gamma/\pi$. With the replacement of γ/π with z in (D.4), we have the equation

$$\frac{n-2}{n}\left[z\frac{k\left(\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon} + \frac{z^2}{(n-k)\tau_u}}\right)}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{z^2}{(n-k)^2\tau_u}}} - \xi\right] - \frac{\frac{z}{\frac{1}{\tau_\epsilon} + \frac{z^2}{(n-k)\tau_u}}}{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon} + \frac{z^2}{(n-k)^2\tau_u}}} - \frac{\xi}{n} = 0.$$

The previous equation can be alternatively written as

$$\frac{n-2}{n}z\frac{k\frac{z^2\tau_\epsilon}{(n-k)\tau_u}}{(\tau_\theta + k\tau_\epsilon)\left(\frac{1}{\tau_\epsilon} + \frac{z^2}{(n-k)\tau_u}\right) + n - k} - \frac{z}{(\tau_\theta + k\tau_\epsilon)\left(\frac{1}{\tau_\epsilon} + \frac{z^2}{(n-k)\tau_u}\right) + n - k} = \frac{n-1}{n}\xi,$$

which is equivalent to

$$\left[k\frac{z^2\tau_\epsilon}{(n-k)\tau_u} - \frac{n}{n-2}\right]z - \frac{n-1}{n-2}\xi\left[(\tau_\theta + k\tau_\epsilon)\left(\frac{1}{\tau_\epsilon} + \frac{z^2}{(n-k)\tau_u}\right) + n - k\right] = 0.$$

We denote the previous equation as $f(z) := a_3z^3 - a_2z^2 - a_1z - a_0 = 0$, where a_3, a_2, a_1, a_0 are as in the proposition.

When $n \geq 3$, by applying Descartes' Rule, a unique positive solution, still denoted as z, to the equation $f(z) = 0$ exists. From the solution z to the cubic equation we can obtain the rest of the equilibrium parameters. In fact, replacing γ/π with z in (7), we obtain the parameter α. Substituting z and α into (D.2), we obtain the parameter γ, and also the parameter π by the relation $\pi = \gamma/z$. Plugging γ and π into (8), we obtain the parameter β. Finally, we claim that the equilibrium parameter $(\alpha, \beta, \pi, \gamma, \lambda)$ is positive. If $\pi < 0$, it then follows from (8) that $\beta < 0$. Consequently, from (10) we have $\alpha < 0$. However, it follows from (7) that $\alpha > 0$, a contradiction. Furthermore, it follows from (10) that $\pi - \beta\pi = k\alpha/n$, from which and (8) we can conclude that $\pi \neq 0$ since otherwise, $\beta\pi$ is negative, which contradicts (8). Hence, it follows that $\pi > 0$. Furthermore, it is easy to see that $(\alpha, \beta, \gamma, \lambda)$ is positive (in fact, we further have $\beta < 1$ by (8)). This completes the proof.

Proof of Proposition 2.

From (9) and (6), we have

$$\begin{aligned}
\mathrm{Corr}(x^*_{i,k}, x^*_{i+d,k}) &= \frac{\mathrm{Cov}(x^*_{i,k}, x^*_{i+d,k})}{\sqrt{\mathrm{Var}(x^*_{i,k})\mathrm{Var}(x^*_{i+d,k})}} \\
&= \frac{\alpha_k^2k^2\mathrm{Cov}(\bar{y}_{i,k}, \bar{y}_{i+d,k}) + (\beta_k - 1)^2\mathrm{Var}(p_k) + 2\alpha_k k(\beta_k - 1)\mathrm{Cov}(\bar{y}_{i,k}, p_k)}{\alpha_k^2k^2\mathrm{Var}(\bar{y}_{i,k}) + (\beta_k - 1)^2\mathrm{Var}(p_k) + 2\alpha_k k(\beta_k - 1)\mathrm{Cov}(\bar{y}_{i,k}, p_k)},
\end{aligned} \tag{D.5}$$

where $\bar{y}_{i,k} = \frac{1}{k}\sum_{r\in\mathcal{N}_{i,k}} y_r$ denotes the average signal of speculator i, (D.5) follows from the facts that $\mathrm{Var}(x^*_{i,k}) = \mathrm{Var}(x^*_{i+d,k})$ and $\mathrm{Cov}(\bar{y}_{i,k}, p_k) = \mathrm{Cov}(\bar{y}_{i+d,k}, p_k)$. It is easy to see that $\mathrm{Cov}(x^*_{i,k}, x^*_{i+d,k})$ is decreasing in d and consequently, $\mathrm{Corr}(x^*_{i,k}, x^*_{i+d,k})$ is also decreasing in d.

Proof of Proposition 3.

See the arguments in the main body of the paper.

Proof of Proposition 4.

(i) Substituting (22) into (16) (setting $\xi = 0$), we have

$$\begin{aligned}
\lambda_k &\propto \frac{n-k}{k}\left[\frac{k - \frac{k}{1+\frac{n}{n-2}\frac{1}{k}}}{\tau_\theta + k\tau_\epsilon + \frac{(n-k)\tau_\epsilon}{1+\frac{n}{n-2}\frac{1}{k}}}\right]^2 \\
&= \frac{n-k}{k}\left[\frac{k - \frac{k^2}{k+\frac{n}{n-2}}}{\tau_\theta + k\tau_\epsilon + \frac{(n-k)k\tau_\epsilon}{k+\frac{n}{n-2}}}\right]^2 \\
&= \frac{n-k}{k}\left[\frac{k(k+\frac{n}{n-2}) - k^2}{(\tau_\theta + k\tau_\epsilon)(k+\frac{n}{n-2}) + (n-k)k\tau_\epsilon}\right]^2 \\
&= \frac{n-k}{k}\frac{k^2(\frac{n}{n-2})^2}{\left(\tau_\theta(k+\frac{n}{n-2}) + (n+\frac{n}{n-2})k\tau_\epsilon\right)^2} \\
&\propto \frac{(n-k)k}{\left[\tau_\theta\frac{n}{n-2} + k((n+\frac{n}{n-2})\tau_\epsilon + \tau_\theta)\right]^2}.
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial\lambda_k}{\partial k} &\propto (n-2k)\left[\tau_\theta\frac{n}{n-2} + k\left(\left(n+\frac{n}{n-2}\right)\tau_\epsilon + \tau_\theta\right)\right] - 2(n-k)k\left(\left(n+\frac{n}{n-2}\right)\tau_\epsilon + \tau_\theta\right) \\
&\propto -n\left(\left(n+\frac{n}{n-2}\right)\tau_\epsilon + \tau_\theta\right)k + (n-2k)\tau_\theta\frac{n}{n-2} \\
&< -nk\tau_\theta + (n-2k)\tau_\theta\frac{n}{n-2} \\
&< 0.
\end{aligned}$$

This implies that price impact strictly decreases with k.

Here we first show Part (iii) before showing Part (ii). Substituting (22) into (17) (setting $\xi = 0$), we have

$$\begin{aligned}
\pi_k &= \frac{\tau_\epsilon}{\tau_\theta + k\tau_\epsilon + \frac{(n-k)\tau_\epsilon}{1+\frac{n}{k(n-2)}}}\left[\frac{1}{1+\frac{n}{k(n-2)}} + \frac{k\frac{\frac{n}{k(n-2)}}{1+\frac{n}{k(n-2)}}}{n}\right] \\
&= \frac{\tau_\epsilon}{(\tau_\theta + k\tau_\epsilon)\left(1+\frac{n}{k(n-2)}\right) + (n-k)\tau_\epsilon}\frac{n-1}{n-2} \\
&= \frac{n-1}{n-2}\frac{\tau_\epsilon k(n-2)}{(\tau_\theta + k\tau_\epsilon)(k(n-2)+n) + (n-k)k(n-2)\tau_\epsilon} \\
&= \frac{n-1}{n-2}\frac{k(n-2)}{(\tau_\theta/\tau_\epsilon + k)(k(n-2)+n) + (n-k)k(n-2)} \\
&= \frac{n-1}{n-2}\frac{k(n-2)}{k\left(n(n-2) + n + \frac{(n-2)\tau_\theta}{\tau_\epsilon}\right) + \frac{n\tau_\theta}{\tau_\epsilon}} \\
&= \frac{n-1}{n(n-2) + n + \frac{(n-2)\tau_\theta}{\tau_\epsilon} + \frac{n\tau_\theta}{\tau_\epsilon k}}.
\end{aligned} \tag{D.6}$$

Hence,

$$\frac{\partial\pi_k}{\partial k} = \frac{n\tau_\theta/\tau_\epsilon}{(n-1)k^2}\pi_k^2 > 0, \tag{D.7}$$

which implies that π_k strictly increases with k.

(ii) We have

$$\begin{aligned}\alpha_k \mathbb{E}\left|\sum_{r\in\mathcal{N}_{i,k}} y_r - \sum_{r\in\mathcal{N}_{j,k}} y_r\right| &= \alpha_k \sqrt{\frac{2}{\pi}\mathrm{Var}\left(\sum_{r\in\mathcal{N}_{i,k}} y_r - \sum_{r\in\mathcal{N}_{j,k}} y_r\right)} \\ &= \sqrt{\frac{2}{\pi}}\alpha_k k\sqrt{\mathrm{Var}\big(\bar{y}_{i,k} - \bar{y}_{j,k}\big)} \\ &\propto \alpha_k k\sqrt{\mathrm{Var}(\bar{y}_{i,k}) + \mathrm{Var}(\bar{y}_{j,k}) - 2\mathrm{Cov}(\bar{y}_{i,k}, \bar{y}_{j,k})} \\ &\propto \alpha_k k\sqrt{\mathrm{Var}(\bar{y}_{i,k}) - \mathrm{Cov}(\bar{y}_{i,k}, \bar{y}_{j,k})}. \end{aligned} \tag{D.8}$$

For notational convenience, we denote $j = i + d$ for some $d \in [1, \frac{n-1}{2}]$. It is easy to see

$$\mathrm{Cov}(\bar{y}_{i,k}, \bar{y}_{i+d,k}) = \begin{cases} \tau_\theta^{-1}, & k \in [1, d], \\ \tau_\theta^{-1} + \frac{1}{k}\tau_\epsilon^{-1} - \frac{d}{k^2}\tau_\epsilon^{-1}, & k \in [d, n-d], \\ \tau_\theta^{-1} + \frac{2}{k}\tau_\epsilon^{-1} - \frac{n}{k^2}\tau_\epsilon^{-1}, & k \in [n-d, n-2], \end{cases} \tag{D.9}$$

First, from (D.9) and (D.8), when $k \in [1, d]$,

$$\alpha_k \mathbb{E}\left|\sum_{r\in\mathcal{N}_{i,k}} y_r - \sum_{r\in\mathcal{N}_{i+d,k}} y_r\right| \propto \alpha_k\sqrt{\tau_\epsilon^{-1} k}.$$

From (D.1) (setting $\xi = 0$), we have $\alpha_k\sqrt{k} = \frac{n}{n-1}\frac{\pi_k}{\sqrt{k}}$. By (D.6) and (D.7), taking derivative of $\alpha_k\sqrt{k}$ with respect to k leads to

$$\begin{aligned}\frac{\partial(\alpha_k\sqrt{k})}{\partial k} &\propto \frac{\partial\pi_k}{\partial k}\frac{1}{\sqrt{k}} - \frac{\pi_k}{2k\sqrt{k}} \\ &\propto \frac{\partial\pi_k}{\partial k} - \frac{\pi_k}{2k} \\ &= \frac{n\tau_\theta/\tau_\epsilon}{(n-1)k^2}\pi_k^2 - \frac{\pi_k}{2k} \\ &\propto \frac{n\tau_\theta/\tau_\epsilon}{(n-1)k}\pi_k - \frac{1}{2} \\ &= \frac{\tau_\theta/\tau_\epsilon}{\left((n-1) + \frac{n-2}{n}\tau_\theta/\tau_\epsilon\right)k + \tau_\theta/\tau_\epsilon} - \frac{1}{2},\end{aligned}$$

which is negative if $k \geq 3$. We next demonstrate that $\alpha_k\sqrt{k}|_{k=1} > \alpha_k\sqrt{k}|_{k=3}$. From (D.6), we have

$$\begin{aligned}\alpha_k\sqrt{k} &= \frac{n}{n-1}\frac{\pi_k}{\sqrt{k}} \\ &= \frac{1}{\left(n - 1 + \frac{n-2}{n}\frac{\tau_\theta}{\tau_\epsilon}\right)\sqrt{k} + \frac{\tau_\theta}{\tau_\epsilon}\frac{1}{\sqrt{k}}} \\ &=: \frac{1}{A(k)}.\end{aligned}$$

With some simple calculations, $A(1) < A(3)$ and consequently, $\alpha_k\sqrt{k}|_{k=1} > \alpha_k\sqrt{k}|_{k=3}$. Thus, $\partial(\alpha_k\sqrt{k})/\partial k < 0$ for any $k \in [1, d]$.

Next, when $k \in [d, n-d]$, replacing $\mathrm{Cov}(\bar{y}_{i,k}, \bar{y}_{j,k})$ in (D.8) with (D.9),

$$\alpha_k \mathbb{E}\left|\sum_{r\in\mathcal{N}_{i,k}} y_r - \sum_{r\in\mathcal{N}_{i+d,k}} y_r\right| \propto \alpha_k\sqrt{\tau_\epsilon^{-1} d}.$$

Recall that $\alpha_k = \frac{n}{n-1}\frac{\pi_k}{k}$. Then, according to (D.6) and (D.7),

$$\begin{aligned}\frac{\partial\alpha_k}{\partial k} &\propto \frac{\partial\pi_k}{\partial k}\frac{1}{k} - \frac{\pi_k}{k^2} \\ &= \frac{n\tau_\theta/\tau_\epsilon}{(n-1)k^2}\pi_k^2 - \frac{\pi_k}{k} \\ &\propto \frac{n\tau_\theta/\tau_\epsilon}{(n-1)k}\pi_k - 1\end{aligned}$$

$$= \frac{\tau_\theta/\tau_\epsilon}{\left((n-1)+\frac{n-2}{n}\tau_\theta/\tau_\epsilon\right)k+\tau_\theta/\tau_\epsilon} - 1 \\ < 0.$$

Finally, when $k \in [n-d, n-2]$, we substitute $\mathrm{Cov}(\bar{y}_{i,k}, \bar{y}_{j,k})$ in (D.8) with (D.9) to obtain that

$$\alpha_k \mathbb{E}\left| \sum_{r\in\mathcal{N}_{i,k}} y_r - \sum_{r\in\mathcal{N}_{i+d,k}} y_r \right| \propto \alpha_k \sqrt{\tau_\epsilon^{-1}(n-k)}.$$

The proof of Part (ii) is completed by noting that $\alpha_k > 0$, $\sqrt{\tau_\epsilon^{-1}(n-k)} > 0$ and are both decreasing in $k \in [n-d, n-2]$.

(iv) It follows immediately from the relation $1/\gamma_k = n/((n-1)\lambda_k)$ (see Eq. (12)) and the result in Part (i).

(v) It is sufficient to show that $\partial \mathrm{Var}(p_k)/\partial k > 0$. Substituting (22) and (D.6) into (18) (setting $\xi = 0$), we have

$$\mathrm{Var}(p_k) \propto \left[\frac{n-1}{n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}}\right]^2 \left[\frac{n^2}{\tau_\theta}+\frac{n}{\tau_\epsilon}+\frac{n(n-k)}{(n-2)k\tau_\epsilon}\right].$$

Hence,

$$\begin{aligned}
\frac{\partial \mathrm{Var}(p_k)}{\partial k} &= 2\left[\frac{n-1}{n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}}\right]\frac{n-1}{\left[n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}\right]^2}\frac{n\tau_\theta}{\tau_\epsilon k^2} \\
&\quad \times \left[\frac{n^2}{\tau_\theta}+\frac{n}{\tau_\epsilon}+\frac{n(n-k)}{(n-2)k\tau_\epsilon}\right] - \left[\frac{n-1}{n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}}\right]^2 \frac{n^2}{(n-2)\tau_\epsilon k^2} \\
&\propto \frac{2\tau_\theta}{n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}}\left[\frac{n^2}{\tau_\theta}+\frac{n}{\tau_\epsilon}+\frac{n}{(n-2)\tau_\epsilon}\left(\frac{n}{k}-1\right)\right] - \frac{n}{n-2} \\
&\propto 2(n-2)\left[n^2+\left(n+\frac{n}{n-2}\left(\frac{n}{k}-1\right)\right)\frac{\tau_\theta}{\tau_\epsilon}\right] - n\left[n(n-2)+n+\left(n-2+\frac{n}{k}\right)\frac{\tau_\theta}{\tau_\epsilon}\right] \\
&> 0.
\end{aligned}$$

(vi) It is sufficient to show that $\partial \mathrm{Var}(\theta - p_k)/\partial k < 0$. Substituting (22) into (19) (setting $\xi = 0$), we have

$$\mathrm{Var}(\theta - p_k) = \frac{(1-n\pi_k)^2}{\tau_\theta} + \frac{n\pi_k^2}{\tau_\epsilon} + \frac{n(n-k)}{(n-2)k\tau_\epsilon}\pi_k^2. \tag{D.10}$$

Hence, it follows from (D.6) that

$$\begin{aligned}
&\frac{\partial \mathrm{Var}(\theta - p_k)}{\partial k} \\
&= \left[\frac{-2n(1-n\pi_k)}{\tau_\theta}+\frac{2n\pi_k}{\tau_\epsilon}+\frac{2n(n-k)\pi_k}{(n-2)k\tau_\epsilon}\right]\frac{\partial \pi_k}{\partial k} + n\frac{\partial \frac{(n-k)}{(n-2)k\tau_\epsilon}}{\partial k}\pi_k^2 \\
&= \left[\frac{2n^2}{\tau_\theta}+\frac{2n}{\tau_\epsilon}+\frac{2n(n-k)}{(n-2)k\tau_\epsilon}\right]\pi_k\frac{\partial \pi_k}{\partial k} - \frac{2n}{\tau_\theta}\frac{\partial \pi_k}{\partial k} - \frac{n^2}{(n-2)\tau_\epsilon k^2}\pi_k^2 \\
&\propto \left[\frac{2n^2}{\tau_\theta}+\frac{2n}{\tau_\epsilon}+\frac{2n(n-k)}{(n-2)k\tau_\epsilon}\right]\frac{n\frac{\tau_\theta}{\tau_\epsilon}}{n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}} - \frac{2n}{\tau_\theta}\frac{n\frac{\tau_\theta}{\tau_\epsilon}}{n-1} - \frac{n^2}{(n-2)\tau_\epsilon} \\
&\propto \left[\frac{2n^2}{\tau_\theta}+\frac{2n}{\tau_\epsilon}+\frac{2n(n-k)}{(n-2)k\tau_\epsilon}\right]\frac{\tau_\theta}{n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}} - \frac{2n}{n-1} - \frac{n}{n-2} \\
&= \frac{2n^2+\left(2n+\frac{2n}{n-2}\left(\frac{n}{k}-1\right)\right)\frac{\tau_\theta}{\tau_\epsilon}}{n(n-2)+n+\left(n-2+\frac{n}{k}\right)\frac{\tau_\theta}{\tau_\epsilon}} - \frac{(3n-5)n}{(n-1)(n-2)} \\
&=: B(k).
\end{aligned}$$

By some simple computations, we can see that

$$\frac{\partial B(k)}{\partial k} < 0, \quad B(0) = \frac{2n^2+\left(2n+\frac{2n}{n-2}(n-1)\right)\frac{\tau_\theta}{\tau_\epsilon}}{n(n-2)+n+2(n-1)\frac{\tau_\theta}{\tau_\epsilon}} - \frac{(3n-5)n}{(n-1)(n-2)} < 0,$$

which implies that $\mathrm{Var}(\theta - p_k)$ strictly decreases with k.

(vii) First, we have

$$
\begin{aligned}
\frac{1}{\lambda_k} &\overset{(D.1)}{=} \frac{\pi_k/\gamma_k}{k\alpha_k} \\
&\overset{(7)}{=} z_k^{-1}\frac{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}}{k\left(\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{z_k^2}{(n-k)\tau_u}}\right)} \\
&\overset{(22)}{=} \sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{z_k^2}{(n-k)^2\tau_u}}}{k\left(\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{z_k^2}{(n-k)\tau_u}}\right)} \\
&= \sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{\tau_\theta + k\tau_\epsilon + \frac{(n-k)\tau_\epsilon}{1+\frac{n}{(n-2)k}}}{k\left(\tau_\epsilon - \frac{\tau_\epsilon}{1+\frac{n}{(n-2)k}}\right)} \\
&= \sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{\tau_\theta + k\tau_\epsilon + \frac{(n-k)(n-2)k\tau_\epsilon}{(n-2)k+n}}{\frac{kn\tau_\epsilon}{(n-2)k+n}}
\end{aligned} \tag{D.11}
$$

$$
\begin{aligned}
&= \sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{(\frac{\tau_\theta}{\tau_\epsilon}+k)((n-2)k+n)+(n-k)(n-2)k}{kn} \\
&= \sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)+kn(n-1)}{nk}.
\end{aligned} \tag{D.12}
$$

Then, we obtain

$$
\begin{aligned}
&\frac{1}{\lambda_k^2}\left(\mathrm{Var}(\theta - p_k) - \mathrm{Var}[\theta|y_j, j\in\mathcal{N}_{i,k}, p_k]\right) \\
&\overset{(D.10),(15)}{=} \frac{1}{\lambda_k^2}\left[\frac{(1-n\pi_k)^2}{\tau_\theta}+\frac{n\pi_k^2}{\tau_\epsilon}+\frac{n(n-k)}{(n-2)k\tau_\epsilon}\pi_k^2 - \left(\tau_\theta + k\tau_\epsilon + \frac{1}{\frac{1}{(n-k)\tau_\epsilon}+\frac{\gamma_k^2}{(n-k)^2\pi_k^2\tau_u}}\right)^{-1}\right] \\
&\overset{(D.6)}{=} \frac{1}{\lambda_k^2}\left[\tau_\theta\frac{\left(\frac{n-2}{n\tau_\epsilon}+\frac{1}{k\tau_\epsilon}\right)^2}{\left((n-1)+\frac{n-2}{n}\frac{\tau_\theta}{\tau_\epsilon}+\frac{1}{k}\frac{\tau_\theta}{\tau_\epsilon}\right)^2}+\frac{n}{\tau_\epsilon}\left(1+\frac{n-k}{(n-2)k}\right)\frac{(n-1)^2}{\left(n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}\right)^2}\right. \\
&\qquad\left. - \left(\frac{1}{\tau_\theta + k\tau_\epsilon + \frac{(n-k)\tau_\epsilon}{1+\frac{n}{(n-2)k}}}\right)\right] \\
&= \frac{1}{\lambda_k}\left[\frac{1}{\lambda_k}\left(\tau_\theta\frac{\left(\frac{n-2}{n\tau_\epsilon}+\frac{1}{k\tau_\epsilon}\right)^2}{\left((n-1)+\frac{n-2}{n}\frac{\tau_\theta}{\tau_\epsilon}+\frac{1}{k}\frac{\tau_\theta}{\tau_\epsilon}\right)^2}+\frac{n}{\tau_\epsilon}\left(1+\frac{n-k}{(n-2)k}\right)\frac{(n-1)^2}{\left(n(n-2)+n+\frac{(n-2)\tau_\theta}{\tau_\epsilon}+\frac{n\tau_\theta}{k\tau_\epsilon}\right)^2}\right)\right. \\
&\qquad\left. - \frac{1}{\lambda_k}\left(\frac{1}{\tau_\theta + k\tau_\epsilon + \frac{(n-k)\tau_\epsilon}{1+\frac{n}{(n-2)k}}}\right)\right] \\
&\overset{(D.12),(D.11)}{=} \frac{1}{\lambda_k}\left[\sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)+kn(n-1)}{nk}\left(\frac{\tau_\theta}{\tau_\epsilon^2}\left(\frac{(n-2)k+n}{n(n-1)k+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)}\right)^2\right.\right. \\
&\qquad\left. + \frac{1}{n\tau_\epsilon}\frac{(n-2)k+(n-k)}{(n-2)k}\left(\frac{n-1}{n-1+\frac{\tau_\theta}{\tau_\epsilon}\left(\frac{n-2}{n}+\frac{1}{k}\right)}\right)^2\right) \\
&\qquad\left. - \sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{\tau_\theta + k\tau_\epsilon + \frac{(n-k)(n-2)k\tau_\epsilon}{(n-2)k+n}}{\frac{\tau_\epsilon kn}{(n-2)k+n}}\frac{1}{\tau_\theta + k\tau_\epsilon + \frac{(n-k)(n-2)k\tau_\epsilon}{(n-2)k+n}}\right] \\
&= \frac{1}{\lambda_k}\left[\sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\left(\frac{\tau_\theta}{\tau_\epsilon^2}\frac{((n-2)k+n)^2}{nk\left(n(n-1)k+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)\right)}\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{\tau_\epsilon}\frac{(n-1)^2}{(n-1)nk+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)}\frac{(n-2)k+(n-k)}{n-2}\Bigg)-\frac{1}{\tau_\epsilon}\sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{1}{\frac{kn}{(n-2)k+n}}\Bigg]\\
&=\frac{1}{\lambda_k}\Bigg[\frac{1}{\tau_\epsilon}\sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{1}{(n-1)nk+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)}\Bigg(\frac{\tau_\theta}{\tau_\epsilon}\frac{((n-2)k+n)^2}{nk}\\
&\quad+(n-1)^2\Big(k+\frac{n-k}{n-2}\Big)\Bigg)-\frac{1}{\tau_\epsilon}\sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\Big(\frac{n-2}{n}+\frac{1}{k}\Big)\Bigg]\\
&=\frac{1}{\lambda_k}\Bigg[\frac{1}{\tau_\epsilon}\sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{1}{(n-1)nk+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)}\Bigg(\frac{\tau_\theta}{\tau_\epsilon}\frac{((n-2)k+n)^2}{nk}\\
&\quad+(n-1)^2\Big(k+\frac{n-k}{n-2}\Big)-\Big(\frac{n-2}{n}+\frac{1}{k}\Big)\Big((n-1)nk+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)\Big)\Bigg)\Bigg]\\
&\overset{(D.12)}{=}\sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)+kn(n-1)}{nk}\frac{1}{\tau_\epsilon}\sqrt{\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}}\frac{1}{(n-1)nk+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)}\\
&\times\Bigg[\frac{\tau_\theta}{\tau_\epsilon}\frac{((n-2)k+n)^2}{nk}+(n-1)^2\Big(k+\frac{n-k}{n-2}\Big)-\Big(\frac{n-2}{n}+\frac{1}{k}\Big)\Big((n-1)nk+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)\Big)\Bigg]\\
&=\frac{1}{\tau_\epsilon}\frac{(n-2)k\tau_\epsilon}{n\tau_u(n-k)}\frac{1}{n^2k^2(n-2)}\Bigg[(n-1)^2nk((n-2)k+n-k)+\frac{\tau_\theta}{\tau_\epsilon}((n-2)k+n)^2(n-2)\\
&\quad-(n-2)\Big(((n-2)k+n)\frac{\tau_\theta}{\tau_\epsilon}+kn(n-1)\Big)((n-2)k+n)\Bigg]\\
&=\frac{1}{n^3\tau_u(n-k)k}\big[(n-1)^2kn(k(n-2)+n-k)-(n-2)(n-1)kn((n-2)k+n)\big]\\
&=\frac{n-1}{n^2\tau_u(n-k)}[k(n-2)+n-(n-1)k]\\
&=\frac{n-1}{n^2\tau_u}.
\end{aligned}
$$

Hence, the result in Part (vii) follows.

(viii) From Parts (i), (vii), and (20), $\sqrt{\mathrm{Var}(\theta-p_k)-\mathrm{Var}[\theta|y_j,\, j\in\mathcal{N}_{i,k},\, p_k]}$ decreases with k. Hence, the conclusion follows from (20), Part (vii), and the relation

$$
\begin{aligned}
&\frac{1}{\lambda_k}\big(\mathrm{Var}(\theta-p_k)-\mathrm{Var}[\theta|y_j,\, j\in\mathcal{N}_{i,k},\, p_k]\big)\\
&=\Big(\frac{1}{\lambda_k}\sqrt{\mathrm{Var}(\theta-p_k)-\mathrm{Var}[\theta|y_j,\, j\in\mathcal{N}_{i,k},\, p_k]}\Big)\sqrt{\mathrm{Var}(\theta-p_k)-\mathrm{Var}[\theta|y_j,\, j\in\mathcal{N}_{i,k},\, p_k]}.
\end{aligned}
$$

This completes the proof.

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