Online Appendix

Supplemental Material for "Information, Market Power and Welfare"

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A1 Introduction

In this Online Appendix we provide some additional results. The numbers for equations, lemmas and propositions correspond to those in the main paper, unless they are specific to the Online Appendix, in which case the numbers are prefixed by an "A".

A2 Equilibrium

Here we restate Lemma 3.3 and provide a proof.

Lemma 3.3 *Suppose one of the following conditions is satisfied: (i) $N_i^I \geq 2$; (ii) $N_i^I \geq 1$ and $R \geq 0$; or (iii) $\rho_{ij} = \rho$ for all $i \neq j$. Then $R_i^\top \eta_I / \eta_I^\top R \eta_I \leq 1/2$.*

Proof We have

$$\frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} = \frac{R_i^\top \eta_I}{N_i^I R_i^\top \eta_I + \sum_{j \neq i} N_j^I R_j^\top \eta_I}. \tag{A1}$$

We will invoke our standing assumptions that $L_I \geq 2$, and $R_j^\top \eta_I \geq 0$ for all j.

Condition (i): The sufficiency of this condition is immediate from (A1).

Condition (ii): If $N_i^I \geq 2$, condition (i) applies. If $N_i^I = 1$, and all correlations are nonnegative, we have

$$\begin{aligned}
\frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} &= \frac{1 + \sum_{j \neq i} \rho_{ij} N_j^I}{\left[1 + \sum_{j \neq i} \rho_{ij} N_j^I\right] + \sum_{j \neq i} N_j^I \left[N_j^I + \sum_{\ell \neq j} \rho_{j\ell} N_\ell^I\right]} \\
&\leq \frac{1 + \sum_{j \neq i} \rho_{ij} N_j^I}{\left[1 + \sum_{j \neq i} \rho_{ij} N_j^I\right] + \sum_{j \neq i} N_j^I [N_j^I + \rho_{ij}]} \\
&= \frac{1 + \sum_{j \neq i} \rho_{ij} N_j^I}{2\left[1 + \sum_{j \neq i} \rho_{ij} N_j^I\right] + \sum_{j \neq i} (N_j^I)^2 - 1} \\
&\leq \frac{1}{2}.
\end{aligned}$$

Condition (iii): If all pairwise correlations are equal to ρ, we have

$$\frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} = \frac{(1-\rho) N_i^I + \rho N^I}{(1-\rho) \sum_j (N_j^I)^2 + \rho (N^I)^2}.$$

If $N_i^I \geq 2$, condition (i) applies. If N_i^I is equal to 0 or 1, we have

$$\frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} \leq \frac{1}{N^I} \frac{(1-\rho) N_i^I + \rho N^I}{(1-\rho) + \rho N^I} \leq \frac{1}{N^I} \leq \frac{1}{2}.$$

This completes the proof. $\square$

A3 Convergence to Competitive Equilibrium

The following result is used in the proof of Proposition 5.2:

Lemma A3.1 *In the economy parametrized by $(\lambda\eta_I, \lambda\eta_U)$, $\lambda \geq 1$, we have*

$$\lim_{\lambda\to\infty} \frac{k\phi^I(\lambda)}{\lambda} = \gamma,$$

where γ is defined by (27).

Proof From (55), we have

$$\begin{aligned} 0 &= \frac{k\phi^I + 2}{k\phi^I + 1} - \frac{N^I + N^U}{k\phi^I + 1}\lambda + \sum_{i\in L_U} N_i^U \left[\frac{g_i(\phi^I;\lambda)}{\phi^I} - 1\right]\lambda \\ &= \frac{k\phi^I + 2}{k\phi^I + 1} - \frac{N^I + N^U}{k\phi^I + 1}\lambda + \sum_{i\in L_U} N_i^U \left[h_i(\phi^I;\lambda) - \lambda\right], \end{aligned} \tag{A2}$$

where

$$h_i(\phi^I;\lambda) := \frac{g_i(\phi^I;\lambda)\lambda}{\phi^I} = -\frac{b_i(\phi^I;\lambda)\lambda}{2k\phi^I} + \sqrt{\left[\frac{b_i(\phi^I;\lambda)\lambda}{2k\phi^I}\right]^2 + \frac{k\phi^I + 2}{k\phi^I(k\phi^I + 1)}\lambda^2}. \tag{A3}$$

Note that $h_i(\phi^I;\lambda)$ is strictly positive and satisfies

$$\begin{aligned} 0 &= [h_i(\phi^I;\lambda)]^2 + \frac{b_i(\phi^I;\lambda)\lambda}{k\phi^I} h_i(\phi^I;\lambda) - \frac{k\phi^I + 2}{k\phi^I(k\phi^I + 1)}\lambda^2 \\ &= h_i(\phi^I;\lambda)\left[h_i(\phi^I;\lambda) - \lambda - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}\left(1 + \frac{2}{k\phi^I}\right) + \frac{2\lambda}{k\phi^I} - \frac{\lambda}{k\phi^I + 1}\right] - \frac{k\phi^I + 2}{k\phi^I(k\phi^I + 1)}\lambda^2 \\ &= h_i(\phi^I;\lambda)\left[h_i(\phi^I;\lambda) - \lambda - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}\left(1 + \frac{2}{k\phi^I}\right)\right] + \frac{k\phi^I + 2}{k\phi^I(k\phi^I + 1)}\left[h_i(\phi^I;\lambda) - \lambda\right]\lambda. \end{aligned}$$

Dividing both sides of this equation by $h_i(\phi^I;\lambda)$, and noting that $\lambda/h_i(\phi^I;\lambda) = \phi^I/g_i(\phi^I;\lambda)$, we obtain

$$h_i(\phi^I;\lambda) - \lambda = \left[1 + \frac{k\phi^I + 2}{k\phi^I(k\phi^I + 1)} \frac{\phi^I}{g_i(\phi^I;\lambda)}\right]^{-1} \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}\left(1 + \frac{2}{k\phi^I}\right). \tag{A4}$$

In the proof of Proposition 5.2, we established that $\phi^I \to \infty$ as $\lambda \to \infty$. From (A3),

$$\frac{g_i(\phi^I;\lambda)}{\phi^I} = -\frac{b_i(\phi^I;\lambda)}{2k\phi^I} + \sqrt{\left[\frac{b_i(\phi^I;\lambda)}{2k\phi^I}\right]^2 + \frac{k\phi^I + 2}{k\phi^I(k\phi^I + 1)}}.$$

Since $b_i(\phi^I;\lambda)/\phi^I \to -k$, we see that $g_i(\phi^I;\lambda)/\phi^I \to 1$, so that $h_i(\phi^I;\lambda) - \lambda \to R_i^\top \eta_I/\eta_I^\top R \eta_I$ from (A4). Now the result follows from (A2). □

The proof of Proposition 5.3 is analogous to that of Proposition 5.2. Here we restate Proposition 5.3 and provide a full proof.

Proposition 5.3 (Convergence II) *We have the following convergence results:*

i. $\lim_{\lambda\to\infty} \mathcal{E}(\lambda\eta_I, \eta_U) = \lim_{\lambda\to\infty} \hat{\mathcal{E}}(\lambda\eta_I, \eta_U)$. *Price informativeness does not depend on λ, and ϕ^I and ϕ^U are strictly increasing in λ.*

ii. Suppose $R_\ell \geq 0$. Then, $\lim_{N_\ell^I\to\infty} \mathcal{E}(\eta_I, \eta_U) = \lim_{N_\ell^I\to\infty} \hat{\mathcal{E}}(\eta_I, \eta_U)$.

Proof *Proof of (i):* The depth parameter ϕ^I satisfies the following equation:

$$\frac{k\phi^I + 2}{k\phi^I + 1} - \frac{\lambda N^I}{k\phi^I + 1} + \sum_{i \in L_U} N_i^U \left[\frac{g_i(\phi^I; \lambda)}{\phi^I} - \frac{k\phi^I + 2}{k\phi^I + 1} \right] = 0, \tag{A5}$$

which is the same as (55), except that λ does not multiply N_i^U. The proof that $\phi^I \to \infty$, $\phi_i^U \to \infty$, $\alpha^I \to \hat{\alpha}^I$, and $g_i(\phi^I; \lambda)/\phi^I \to 1$, is identical to that in Proposition 5.2. Using the last of these results, it follows from (A5) that $k\phi^I/\lambda \to N^I$. Therefore, from (58) and (59),

$$\lim_{\lambda \to \infty} \alpha_i^U = k^{-1} \left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} N^I \right], \qquad \text{and} \qquad \lim_{\lambda \to \infty} p = (N^I)^{-1} \eta_I^\top \theta.$$

From (27), for the competitive economy parametrized by $(\lambda \eta_I, \eta_U)$,

$$\gamma = \frac{\lambda N^I + N^U}{1 + \sum_{i \in L_U} N_i^U \frac{R_i^\top \eta_I}{\lambda \eta_I^\top R \eta_I}},$$

so that $\gamma/\lambda \to N^I$. Using (26) and (30), we conclude that $\lim_{\lambda \to \infty} \alpha_i^U = \lim_{\lambda \to \infty} \hat{\alpha}_i^U$ and $\lim_{\lambda \to \infty} p = \lim_{\lambda \to \infty} \hat{p}$.

We establish the monotonicity properties in the same way as in the proof of Proposition 5.2. Here we have

$$f(\phi^I(\lambda); \lambda) := \frac{\phi^I}{k\phi^I + 1} \left[(k\phi^I + 2) - \lambda N^I \right] + \sum_{i \in L_U} N_i^U \left[g_i(\phi^I; \lambda) - \phi^I \frac{k\phi^I + 2}{k\phi^I + 1} \right] = 0,$$

so that

$$\frac{\partial f(\phi^I(\lambda); \lambda)}{\partial \lambda} = -\frac{\phi^I}{k\phi^I + 1} N^I + \sum_{i \in L_U} N_i^U \frac{\partial g_i(\phi^I, \lambda)}{\partial \lambda} < 0,$$

implying that ϕ^I is strictly increasing in λ. Also,

$$\phi^U(\phi^I(\lambda); \lambda) = \frac{N^U - 1}{N^U} \phi^I \frac{k\phi^I + 2}{k\phi^I + 1} + \frac{\lambda N^I}{N^U} \frac{\phi^I}{k\phi^I + 1},$$

from which we can conclude that ϕ^U is strictly increasing in λ. The result that price informativeness does not depend on λ follows from (22).

Proof of (ii): We can write

$$\frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} = \frac{\rho_{i\ell} N_\ell^I + \sum_{j \neq \ell} \rho_{ij} N_j^I}{N_\ell^I \left[N_\ell^I + \sum_{j \neq \ell} \rho_{\ell j} N_j^I \right] + \sum_{j \neq \ell} N_j^I \left[\rho_{j\ell} N_\ell^I + \sum_{m \neq \ell} \rho_{jm} N_m^I \right]}. \tag{A6}$$

Hence, if $N_\ell^I \to \infty$,

$$\frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} \to 0. \tag{A7}$$

By exactly the same arguments as in the proof of part (i), we can show that $\phi^I \to \infty$, $\phi_i^U \to \infty$ for all $i \in L_U$, $\alpha^I \to \hat{\alpha}^I$, and

$$\frac{k\phi^I}{N_\ell^I} \to 1. \tag{A8}$$

Using (A6) and (A8),

$$\frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} k\phi^I = \frac{N_\ell^I \left[\rho_{i\ell} N_\ell^I + \sum_{j\neq \ell} \rho_{ij} N_j^I\right]}{N_\ell^I \left[N_\ell^I + \sum_{j\neq\ell} \rho_{\ell j} N_j^I\right] + \sum_{j\neq \ell} N_j^I \left[\rho_{j\ell} N_\ell^I + \sum_{m\neq\ell} \rho_{jm} N_m^I\right]} \left(\frac{k\phi^I}{N_\ell^I}\right),$$

which converges to $\rho_{i\ell}$. From (15) and (A7), we conclude that $\alpha_i^U \to k^{-1}(1-\rho_{i\ell})$ which, from (30), is equal to $\lim_{N_\ell^I\to\infty} \hat{\alpha}_i^U$. Finally, from (16) and (A8),

$$p = (k\phi^I + 2)^{-1} \sum_i N_i^I \theta_i = \left(\frac{k\phi^I}{N_\ell^I} + \frac{2}{N_\ell^I}\right)^{-1} \left(\theta_\ell + \sum_{i\neq\ell} \frac{N_i^I}{N_\ell^I}\theta_i\right),$$

which converges to θ_ℓ as $\lambda \to \infty$. From (26) and (27), $\hat{p}$ converges to θ_ℓ as well. □

A4 Market Size and Price Informativeness

In the paper we show that market size has no impact on price informativeness (Proposition 5.2), where we increase market size by scaling up $(N_i^I, N_i^U)_{i\in L}$. The purpose of this section is to investigate the effect of market size on price informativeness when informed agents in any given group have diverse information about their value for the asset. This allows us to place the price informativeness result in our paper, and those in Vives (2011) and Rostek and Weretka (2012, 2015), in a more general context.

We consider a generalization of the Rostek and Weretka (2012, 2015) model to allow for multiple agents who have the same value for the asset. We call this the "differential information model". There are L groups, with the set of agents who belong to group i denoted by N_i. We assume that $\#N_i = N$ for all $i \in L$. Agent $n \in N_i$ observes a private signal $s_{in} = \theta_i + \epsilon_{in}$. There are no uninformed traders. All random variables are joint normally distributed with zero mean. The values $(\theta_i)_{i\in L}$ have a common variance σ_θ^2, and a correlation matrix R satisfying the"equicommonality" assumption, which means that the average correlation of θ_i with $\{\theta_j\}_{j\neq i}$ is the same for all i, i.e. $(L-1)^{-1}\sum_{j\neq i} \rho_{ij} = \bar{\rho}$, for all i. The random variables $(\epsilon_{in})_{i\in L, n\in N_i}$ are mutually independent, independent of $(\theta_i)_{i\in L}$, and have a common variance σ_ϵ^2.

If we set $N = 1$ in the differential information model, we obtain the Rostek and Weretka (2012, 2015) model. If, in addition, $\rho_{ij} = \rho$ for all $i \neq j$, we get the Vives (2011) model. Taking the limit as σ_ϵ^2 goes to zero in the differential information model yields a special case of the model in our paper, with $N_i^I = N$ and $N_i^U = 0$ for all $i \in L$.

Rostek and Weretka (2012, 2015) postulate a function $\bar{\rho}(L)$, which describes how the "commonality" parameter $\bar{\rho}$ varies with L. The shape of this commonality function depends on how heterogeneity in values arises (e.g. through differences in geographical location or from group affiliations). In our discussion here we will focus on the baseline case in which $\bar{\rho}$ does not depend on L, which is also the case that corresponds to the assumption of an unvarying correlation parameter in Vives (2011).

In our model, market size is measured by N, and price informativeness does not depend on it. In Vives (2011) and Rostek and Weretka (2012, 2015) market size is measured by L, and price informativeness is increasing in it (given that $\bar{\rho}$ is constant). With this in mind, we ask what happens to price informativeness in the differential information model when N or L goes up. An increase in N means that there are more agents in each group, each armed with their own signal about their own value. An increase in L, on the other hand, means that there are more groups, with their own values for the asset, and signals about these values. We show that regardless of the notion of market size, N or L, price informativeness can go up or down with market size. The reason is that while there are more signals in a larger economy, from the perspective of any agent in the economy, some of these signals contribute to price discovery, while other signals add to the "noise" in prices. In this sense, the intuition is the same as in our main model, where an increase in the number of informed agents in group i, which by itself increases price informativeness for group i, is completely offset by the increase in the number of informed agents in other groups, which adds noise to the price from the perspective of group i. In the differential information model, these opposing effects are not exactly offsetting in general, so price informativeness can increase or decrease.

We now proceed with the analysis of the differential information model. Due to the symmetry assumptions of this model, including the equicommonality assumption, all agents face the same depth parameter ϕ, and have the same strategies, given by

$$q_{in}(s_{in}, p) = \frac{\mathbb{E}(\theta_i|s_{in}, p) - p}{k + \phi^{-1}} = \mu s_{in} - \alpha p,$$

for some coefficients μ and α. Using the market-clearing condition, $\sum_{i\in L}\sum_{n\in N_i} q_{in} = 0$, we can solve for the equilibrium price:

$$p = A \sum_{i\in L}\sum_{n\in N_i} s_{in} = A\left[N\sum_{i\in L}\theta_i + \sum_{i\in L}\sum_{n\in N_i}\epsilon_{in}\right],$$

where $A = \mu/(\alpha NL)$. Price informativeness is defined as follows:

$$\psi^+ := \frac{\text{Var}(\theta_i|s_{in}) - \text{Var}(\theta_i|s_{in}, p)}{\text{Var}(\theta_i|s_{in})}. \tag{A9}$$

This is the same definition and notation as in Rostek and Weretka (2012), and we also follow their notation in defining $\sigma^2 := \sigma_\epsilon^2/\sigma_\theta^2$, which measures how noisy signals are. Due to symmetry, ψ^+ does not depend on i or n. For the limiting case of completely uninformative signals ($\sigma_\epsilon^2 \to \infty$), ψ^+ is the same as $\mathcal{V}_i$, the measure of price informativeness in our paper (see (21)).

We first consider the same measure of market size as in the main paper, here applied to the differential information model.

Proposition A4.1 (Price informativeness and N) *Price informativeness ψ^+ may be increasing or decreasing in N. We have*

i. $\partial\psi^+/\partial N > 0$ *if* $\bar{\rho} \geq 0$.

ii. $\partial\psi^+/\partial N < 0$ *if* $\bar{\rho} < 0$ *and sufficiently close to* $-(L-1)^{-1}$.

Proof Using the normal projection theorem, we have

$$\begin{aligned}
&\text{Var}(\theta_i|s_{in}, p)\\
&= \sigma_\theta^2 - \begin{bmatrix}\sigma_\theta^2 & \text{Cov}(\theta_i, p)\end{bmatrix}\begin{bmatrix}\text{Var}(s_{in}) & \text{Cov}(s_{in}, p)\\ \text{Cov}(s_{in}, p) & \text{Var}(p)\end{bmatrix}^{-1}\begin{bmatrix}\sigma_\theta^2\\ \text{Cov}(\theta_i, p)\end{bmatrix}\\
&= \sigma_\theta^2 - \frac{1}{\text{Var}(s_{in})\text{Var}(p) - [\text{Cov}(s_{in}, p)]^2}\begin{bmatrix}\sigma_\theta^2 & \text{Cov}(\theta_i, p)\end{bmatrix}\begin{bmatrix}\text{Var}(p) & -\text{Cov}(s_{in}, p)\\ -\text{Cov}(s_{in}, p) & \text{Var}(s_{in})\end{bmatrix}\begin{bmatrix}\sigma_\theta^2\\ \text{Cov}(\theta_i, p)\end{bmatrix}\\
&= \sigma_\theta^2 - \frac{\sigma_\theta^4\text{Var}(p) - 2\sigma_\theta^2\text{Cov}(s_{in}, p)\text{Cov}(\theta_i, p) + \text{Var}(s_{in})[\text{Cov}(\theta_i, p)]^2}{\text{Var}(s_{in})\text{Var}(p) - [\text{Cov}(s_{in}, p)]^2}.
\end{aligned}$$

Let $C := 1 + (L-1)\bar{\rho}$, and let $\mathbf{1}$ denote the L-vector each element of which is 1. Since $\mathrm{Var}(\mathbf{1}^\top\theta) = \sigma_\theta^2 \mathbf{1}^\top R \mathbf{1} = \sigma_\theta^2 L[1+(L-1)\bar{\rho}] > 0$, we must have $C > 0$ or equivalently, $\bar{\rho} > -(L-1)^{-1}$. We have

$$\begin{aligned}\mathrm{Cov}(\theta_i, p) &= \sigma_\theta^2 ANC,\\ \mathrm{Cov}(s_{in}, p) &= \sigma_\theta^2 A(NC+\sigma^2),\\ \mathrm{Var}(p) &= \sigma_\theta^2 A^2 NL(NC+\sigma^2).\end{aligned}$$

Note that $L - C = (1-\bar{\rho})(L-1)$. Therefore, we have

$$\begin{aligned}\mathrm{Var}(\theta_i|s_{in}, p) &= \sigma_\theta^2 - \sigma_\theta^2 \frac{NL(NC+\sigma^2) - 2N(NC+\sigma^2)C + (1+\sigma^2)N^2C^2}{(1+\sigma^2)NL(NC+\sigma^2) - (NC+\sigma^2)^2}\\ &= \sigma_\theta^2 - \sigma_\theta^2 \frac{N(L-C)(NC+\sigma^2) + \sigma^2 NC(NC-1)}{(NC+\sigma^2)\big[N(L-C)+\sigma^2(NL-1)\big]}\\ &= \sigma_\epsilon^2 \frac{N^2(L-C)C + \sigma^2(NL-1)}{(NC+\sigma^2)\big[N(L-C)+\sigma^2(NL-1)\big]}\\ &= \sigma_\epsilon^2 \frac{(1-\bar{\rho})N^2(L-1)C + \sigma^2(NL-1)}{(NC+\sigma^2)\big[(1-\bar{\rho})N(L-1)+\sigma^2(NL-1)\big]}.\end{aligned}$$

Also,

$$\mathrm{Var}(\theta_i|s_{in}) = \sigma_\theta^2 - \frac{\sigma_\theta^4}{\sigma_\theta^2+\sigma_\epsilon^2} = \frac{\sigma_\epsilon^2}{1+\sigma^2}.$$

Hence, using the definition (A9), we have

$$\psi^+ = 1 - (1+\sigma^2)\frac{(1-\bar{\rho})N^2(L-1)C + \sigma^2(NL-1)}{(NC+\sigma^2)\big[(1-\bar{\rho})N(L-1)+\sigma^2(NL-1)\big]}. \tag{A10}$$

Differentiating (A10), we obtain

$$\begin{aligned}\frac{\partial\psi^+}{\partial N} &\propto -(NC+\sigma^2)\Big[(1-\bar{\rho})N(L-1)+\sigma^2(NL-1)\Big]\Big[2(1-\bar{\rho})N(L-1)C+\sigma^2 L\Big]\\ &\quad + \Big[(1-\bar{\rho})N^2(L-1)C+\sigma^2(NL-1)\Big]\\ &\qquad \cdot \Big[(NC+\sigma^2)\big[(1-\bar{\rho})(L-1)+\sigma^2 L\big] + C\big[(1-\bar{\rho})N(L-1)+\sigma^2(NL-1)\big]\Big]\\ &= (NC+\sigma^2)(1-\bar{\rho})(L-1)\Big[-(1-\bar{\rho})N^2(L-1)C - \sigma^2 N(NL-2)C - \sigma^2\Big]\\ &\quad + (1-\bar{\rho})N^2(L-1)C^2\big[(1-\bar{\rho})N(L-1)+\sigma^2(NL-1)\big]\\ &\quad + \sigma^2(1-\bar{\rho})N(NL-1)(L-1)C + \sigma^4(NL-1)^2C\\ &= \sigma^2(1-\bar{\rho})N(L-1)C\big[(NC-1)+(NL-1)-(1-\bar{\rho})N(L-1)\big]\\ &\quad + \sigma^4\big[(1-\bar{\rho})(L-1)(NC-1) - (1-\bar{\rho})N(NL-1)(L-1)C + (NL-1)^2C\big]\\ &= \sigma^2(1-\bar{\rho})N(L-1)C\big[(NC-1)+(N-1)+\bar{\rho}N(L-1)\big]\\ &\quad + \sigma^4\big[(1-\bar{\rho})(L-1)(NC-1) + \bar{\rho}N(NL-1)(L-1)C + (N-1)(NL-1)C\big].\end{aligned}$$

Suppose $\bar{\rho} \geq 0$. Then $NC - 1 > (N-1)C + (C-1) > 0$, and hence $\partial\psi^+/\partial N > 0$. On the other hand, $\lim_{C\downarrow 0}(\partial\psi^+/\partial N) < 0$. Therefore, $\partial\psi^+/\partial N < 0$ when $\bar{\rho}$ is sufficiently close to its infimum, $-(L-1)^{-1}$. □

The proposition says that price informativeness is increasing in market size, as measured by N, when the correlation parameter $\bar{\rho}$ is nonnegative. But when $\bar{\rho}$ is sufficiently negative, a higher N adds more to the "noise" than to the "signal" in the price from the perspective of any agent. This is because price is an ambiguous signal of value when $\bar{\rho} < 0$. For an agent in group i, a higher price may be due to a higher θ_i (which is good news) but may also be due to higher values of θ_j, $j \neq i$ (which is bad news since $\bar{\rho} < 0$). Negative correlations can arise due to hedging motives (see Rahi and Zigrand (2018) for an example).

Next, we look at the case where market size is measured by the number of groups, L.

Proposition A4.2 (Price informativeness and L) *Price informativeness ψ^+ may be increasing or decreasing in L. We have the following results:*

i. Suppose $N = 1$. Then $\partial\psi^+/\partial L \geq 0$. If, in addition, $\bar{\rho} \neq 0$, then $\partial\psi^+/\partial L > 0$.

ii. Suppose $N \geq 2$. Then $\partial\psi^+/\partial L > 0$ if $\bar{\rho} \geq 1/2$ and $\sigma^2\bar{\rho} \geq N-1$.

iii. Suppose $N \geq 2$. Then $\partial\psi^+/\partial L < 0$ if $\bar{\rho} = 0$.

iv. Suppose $N \geq 3$. Then $\partial\psi^+/\partial L < 0$ if $\bar{\rho} \geq -[2(L-1)]^{-1}$, and $\sigma^2 \leq (1-\bar{\rho})(N-2)$.

Proof Differentiating (A10) with respect to L, we have

$$\begin{aligned}
\frac{\partial\psi^+}{\partial L} &\propto -(NC+\sigma^2)\Big[(1-\bar{\rho})N(L-1)+\sigma^2(NL-1)\Big]\Big[(1-\bar{\rho})N^2[C+(L-1)\bar{\rho}]+\sigma^2 N\Big] \\
&\quad + \Big[(1-\bar{\rho})N^2(L-1)C+\sigma^2(NL-1)\Big] \\
&\qquad \cdot \Big[(NC+\sigma^2)(1-\bar{\rho}+\sigma^2)N+\bar{\rho}N\big[(1-\bar{\rho})N(L-1)+\sigma^2(NL-1)\big]\Big] \\
&\propto -(1-\bar{\rho})(NC+\sigma^2)\Big[\bar{\rho}(1-\bar{\rho})N^2(L-1)^2+\sigma^2(N-1)(NC-1)+\sigma^2\bar{\rho}N(L-1)(NL-1)\Big] \\
&\quad + \bar{\rho}\Big[(1-\bar{\rho})^2N^3(L-1)^2C+\sigma^2(1-\bar{\rho})N(L-1)(NL-1)(NC+1)+\sigma^4(NL-1)^2\Big] \\
&\propto \bar{\rho}(1-\bar{\rho})N(L-1)(NL-1)-(1-\bar{\rho})N(N-1)(NC-1)C-\bar{\rho}(1-\bar{\rho})^2N^2(L-1)^2 \\
&\qquad + \sigma^2\bar{\rho}(NL-1)^2-\sigma^2(1-\bar{\rho})(N-1)(NC-1)-\sigma^2\bar{\rho}(1-\bar{\rho})N(L-1)(NL-1) \\
&= -(1-\bar{\rho})N(N-1)^2-2\bar{\rho}(1-\bar{\rho})N(N-1)^2(L-1)-\bar{\rho}^2(1-\bar{\rho})N^2(N-2)(L-1)^2 \\
&\quad + \sigma^2\big[(2\bar{\rho}-1)(N-1)^2+\bar{\rho}^2N^2(L-1)^2+2\bar{\rho}^2N(N-1)(L-1)\big] \\
&= (N-1)^2\big[\sigma^2(2\bar{\rho}-1)-(1-\bar{\rho})N\big]+2\bar{\rho}N(N-1)(L-1)\big[\sigma^2\bar{\rho}-(1-\bar{\rho})(N-1)\big] \\
&\quad + \bar{\rho}^2N^2(L-1)^2\big[\sigma^2-(1-\bar{\rho})(N-2)\big].
\end{aligned}$$

We denote the last expression by H. We establish that H has the desired sign for each result in the proposition.

Proof of (i): If $N = 1$, it is immediate that $H \geq 0$, where the inequality is strict if and only if $\bar{\rho} \neq 0$.

Proof of (ii): Suppose $N \geq 2$. Using the conditions $\bar{\rho} \geq 1/2$ and $\sigma^2\bar{\rho} \geq N-1$, we have

$$\begin{aligned}
\bar{\rho}H &\geq (N-1)^2\big[(N-1)(2\bar{\rho}-1)-\bar{\rho}(1-\bar{\rho})N\big] \\
&\quad + 2\bar{\rho}^2N(N-1)(L-1)\big[(N-1)-(1-\bar{\rho})(N-1)\big] \\
&\quad + \bar{\rho}^2N^2(L-1)^2\big[(N-1)-\bar{\rho}(1-\bar{\rho})(N-2)\big] \\
&> -\bar{\rho}(1-\bar{\rho})N(N-1)^2+2\bar{\rho}^3N(N-1)^2 \\
&\geq 0,
\end{aligned}$$

and hence $H > 0$, as desired.

Proof of (iii): If $N \geq 2$ and $\bar{\rho} = 0$, it is immediate that $H < 0$.

Proof of (iv): Suppose $N \geq 3$, and $\sigma^2 \leq (1-\bar{\rho})(N-2)$. If $\rho \geq 0$, then it is immediate that $H < 0$ (note that $2\bar{\rho} - 1 < 1$). If $-[2(L-1)]^{-1} \leq \bar{\rho} < 0$, a different argument is needed. First, observe that

$$H \leq H_1(\bar{\rho}) := (N-1)^2\big[\sigma^2(2\bar{\rho}-1)-(1-\bar{\rho})N\big]+2\bar{\rho}N(N-1)(L-1)\big[\sigma^2\bar{\rho}-(1-\bar{\rho})(N-1)\big].$$

It is straightforward to check that $H_1''(\bar{\rho}) > 0$. Furthermore, $H_1(0) < 0$, and

$$\begin{aligned} H_1\big(-[2(L-1)]^{-1}\big) &\propto -(N-1)\left[\sigma^2 + N + \frac{2\sigma^2 + N}{2(L-1)}\right] + N\left[N - 1 + \frac{\sigma^2 + N - 1}{2(L-1)}\right] \\ &\propto -\sigma^2\big[2(N-1)L - N\big] \\ &< 0. \end{aligned}$$

Invoking the convexity of $H_1(\bar{\rho})$, we conclude that $H_1(\bar{\rho}) < 0$ if $-[2(L-1)]^{-1} \leq \bar{\rho} < 0$, and hence $H < 0$ for $\bar{\rho}$ in this interval. This completes the proof. □

The proposition describes how price informativeness is impacted by market size, as measured by L. If $N = 1$, price informativeness goes up with market size. This is the price informativeness result of Rostek and Weretka (2012, 2015) for the case of a constant commonality function $\bar{\rho}(L)$. However, when there are multiple informed agents in each group, price informativeness is decreasing in market size if $\bar{\rho} = 0$. In this case, adding another group adds noise to the price for the existing groups, while not adding any useful information. In fact, for relatively precise signals, price informativeness falls with market size for any nonnegative $\bar{\rho}$ (and also for a range of negative values of $\bar{\rho}$). In order to recover the Rostek and Weretka (2012, 2015) result for $N \geq 2$, we need correlations that are relatively high and signals that are relatively noisy.

Why does the relationship of price informativeness to market size change so dramatically when we go from $N = 1$ to $N \geq 2$? Consider the inference problem of an agent in group i. This agent seeks to learn θ_i from the equilibrium price, which is proportional to the sum of all signals in the economy. The part of the price that is potentially most informative about θ_i is the sum of the signals of other agents in group i (and there are some such agents only when $N \geq 2$). The signals of agents in groups $j \neq i$ cloud this information. Increasing the number of groups clouds it even further.

It seems counter-intuitive at first glance that an increase in market size lowers price informativeness when signals are relatively precise. This is because the accuracy of signals applies not only to the new signals that are added when the market grows in size, but also to the existing signals. If the existing signals already convey a lot of information, the scope of price discovery from the additional signals is lower.

A5 Welfare

The following result appears in Proposition 7.2:

Lemma A5.1 *Consider an $\mathcal{F}_1$-economy parametrized by $\lambda\eta_I$, $\lambda \geq 1$. Suppose $N_1^I \geq 4$. Then $\mathcal{U}_i^I(\lambda\eta_I) - \mathcal{U}_i^U(\lambda\eta_I)$ is strictly increasing in λ, for all $i \in L_I \cap L_U$.*

Proof Let $\Delta\mathcal{U}_i := \mathcal{U}_i^I - \mathcal{U}_i^U$. Note that $\mathcal{V}_i$ and G_i do not depend on λ, $\partial\phi^I/\partial\lambda > 0$, ϕ_i^U depends on λ only through ϕ^I (see (78) and (79)), and $F'(x) = 2(kx+1)^{-3}$, from (32). Hence, from (33), (34), (43), (79), (80) and (81), we have

$$\begin{aligned}
\frac{\partial(\Delta\mathcal{U}_i)}{\partial\lambda} &\propto G_i F'(\phi^I) - \left[G_i - (1-\mathcal{V}_i)\right] F'(\phi_i^U)\frac{\partial\phi_i^U}{\partial\phi^I} \\
&\geq G_i\left[F'(\phi^I) - F'(\phi_i^U)\frac{\partial\phi_i^U}{\partial\phi^I}\right] \\
&\propto F'(\phi^I) - F'(\phi_i^U)\frac{\partial\phi_i^U}{\partial\phi^I} \\
&= \frac{2}{(k\phi^I+1)^3} - \frac{2}{(k\phi_i^U+1)^3}\frac{(k\phi_i^U+1)\frac{\partial\Phi}{\partial\phi^I}}{2k\phi_i^U+b_i} \\
&\geq \frac{2}{(k\phi^I+1)^3} - \frac{2}{(k\phi^I+1)^2(2k\phi_i^U+b_i)}\frac{\partial\Phi}{\partial\phi^I} \\
&\propto (2k\phi_i^U+b_i) - (k\phi^I+1)\frac{\partial\Phi}{\partial\phi^I} \\
&\propto (2k\phi_i^U+b_i)^2 - (k\phi^I+1)^2\left(\frac{\partial\Phi}{\partial\phi^I}\right)^2 \\
&= b_i^2 + 4k\phi^I\frac{k\phi^I+2}{k\phi^I+1} - (k\phi^I+1)^2\left(1+\frac{1}{(k\phi^I+1)^2}\right)^2 \\
&= 4 + \left(k\phi^I\frac{k\phi^I+2}{k\phi^I+1}\right)^2 + \left(\frac{R_i^\top\eta_I}{R_1^\top\eta_I}\right)^2 \\
&\qquad - 2\frac{R_i^\top\eta_I}{R_1^\top\eta_I}\left(2 - k\phi^I\frac{k\phi^I+2}{k\phi^I+1}\right) - \left(k\phi^I+1+\frac{1}{k\phi^I+1}\right)^2 \\
&= \left(\frac{R_i^\top\eta_I}{R_1^\top\eta_I}\right)^2 - 2\frac{R_i^\top\eta_I}{R_1^\top\eta_I}\left(2 - k\phi^I\frac{k\phi^I+2}{k\phi^I+1}\right).
\end{aligned}$$

By the definition of an $\mathcal{F}_1$-economy, $R_j^\top\eta_I > 0$ for all $j \in L_I$. Hence, $\partial(\Delta\mathcal{U}_i)/\partial\lambda > 0$ if

$$k\phi^I\frac{k\phi^I+2}{k\phi^I+1} \geq 2. \tag{A11}$$

Using (36),

$$k\phi^I + 2 = \lambda\frac{\eta_I^\top R\eta_I}{R_1^\top\eta_I} \geq \frac{\eta_I^\top R\eta_I}{R_1^\top\eta_I} > N_1^I.$$

Since $k\phi^I(k\phi^I+2)/(k\phi^I+1)$ is strictly increasing in ϕ^I,

$$k\phi^I\frac{k\phi^I+2}{k\phi^I+1} > (N_1^I-2)\frac{N_1^I}{N_1^I-1}.$$

The right hand side of this inequality is increasing in N_1^I. Therefore, if $N_1^I \geq 4$, (A11) is satisfied. This completes the proof. □

A6 CARA Utility

In this section we consider an economy in which agents have constant absolute risk aversion, with a common risk aversion coefficient r, instead of linear utility with a quadratic holding cost as in the main model. In order to ensure that the conditional variance of the asset payoff is positive for all agents, we assume that the value of the asset for group i is $v_i := \theta_i + \zeta_i$, where the random variables $\{\zeta_i\}_{i\in L}$ are i.i.d. normal, with variance σ_ζ^2, and independent of all other random variables. The wealth of an agent in group i, given an asset position q_i, is $W_i = (v_i - p)q_i$. Informed agents in group i privately observe θ_i. Let

$$k^I := r\sigma_\zeta^2, \qquad k_i^U := k^I + r\sigma_\theta^2\left[1 - \frac{(R_i^\top \eta_I)^2}{\eta_I^\top R \eta_I}\right].$$

We show that, with the exception of Proposition 6.2, all the results in the main model continue to hold with minor modifications. Essentially, the only difference is that the parameter k is replaced by k^I for an informed agent and by k_i^U for an uninformed agent in group i.

We have the following analog of Proposition 3.1:

Proposition A6.1 (Demands and prices for given depths) *The depth parameters for informed agents are the same for all groups: $\phi_i^I = \phi^I$ for all $i \in L_I$. Given ϕ^I and $\{\phi_i^U\}_{i\in L_U}$, agents' demand functions are*

$$\begin{aligned} q_i^I &= \alpha^I(\theta_i - p), && i \in L_I, \\ q_i^U &= \frac{\phi_i^U}{k_i^U\phi_i^U + 1}\big[\mathbb{E}(\theta_i|p) - p\big] = -\alpha_i^U p, && i \in L_U, \end{aligned}$$

where

$$\begin{aligned} \alpha^I &= \Phi - \phi^I = \frac{\phi^I}{k^I\phi^I + 1}, \\ \alpha_i^U &= \Phi - \phi_i^U \\ &= \frac{\phi_i^U}{k_i^U\phi_i^U + 1}\left[1 - \frac{\sigma_{\theta_i p}}{\sigma_p^2}\right] \\ &= \frac{\phi_i^U}{k_i^U\phi_i^U + 1}\left[1 - \frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}(k^I\phi^I + 2)\right]. \end{aligned} \tag{A12}$$

The price function is given by

$$p = (k^I\phi^I + 2)^{-1}\eta_I^\top\theta,$$

and market depth is

$$\Phi = \phi^I\frac{k^I\phi^I + 2}{k^I\phi^I + 1}. \tag{A13}$$

Proof Consider an agent in group i. His information at the time of trade is $\mathcal{I}_i$, where $\mathcal{I}_i := (\theta_i, p)$ if he is informed, and $\mathcal{I}_i := p$ if he is uninformed. The depth parameter that he faces is ϕ_i, which is equal to ϕ_i^I if he is informed and ϕ_i^U if he is uninformed. Also associated with this agent is the parameter $k_i := r\mathrm{Var}(v_i|\mathcal{I}_i)$. He chooses q_i to maximize the mean-variance criterion U_i given by

$$\begin{aligned} U_i &:= \mathbb{E}(W_i|\mathcal{I}_i) - \frac{r}{2}\mathrm{Var}(W_i|\mathcal{I}_i) \\ &= \big[\mathbb{E}(\theta_i|\mathcal{I}_i) - p\big]q_i - \frac{r}{2}\mathrm{Var}(v_i|\mathcal{I}_i)q_i^2 \\ &= \big[\mathbb{E}(\theta_i|\mathcal{I}_i) - p\big]q_i - \frac{k_i}{2}q_i^2. \end{aligned} \tag{A14}$$

The first-order condition is

$$\mathbb{E}(\theta_i|\mathcal{I}_i) - p - \phi_i^{-1}q_i - k_i q_i = 0, \tag{A15}$$

which yields the optimal portfolio

$$q_i = \frac{\mathbb{E}(\theta_i|\mathcal{I}_i) - p}{k_i + \phi_i^{-1}} = \frac{\phi_i}{k_i\phi_i + 1}\big[\mathbb{E}(\theta_i|\mathcal{I}_i) - p\big]. \tag{A16}$$

For an informed agent ($i \in L_I$), we have

$$\mathbb{E}(\theta_i|\mathcal{I}_i) = \theta_i, \tag{A17}$$

and

$$k_i = r\mathrm{Var}(\theta_i + \zeta_i|\theta_i) = r\sigma_\zeta^2 = k^I, \tag{A18}$$

and hence

$$q_i^I = \frac{\phi_i^I}{k^I\phi_i^I + 1}(\theta_i - p).$$

Noting that the analysis in Section 3 of the paper that precedes the definition of equilibrium still applies, we can use the same reasoning as in the proof of Proposition 3.1 to show that $\phi_i^I = \phi^I$ for all $i \in L_I$. Thus we have the desired expressions for q_i^I and α^I.

The formulas for p and Φ follow from the same arguments as in the proof of Proposition 3.1. For an uninformed agent ($i \in L_U$),

$$\mathbb{E}(\theta_i|p) = \frac{\sigma_{\theta_i p}}{\sigma_p^2}p = \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k^I\phi^I + 2)p, \tag{A19}$$

and

$$k_i = r\mathrm{Var}(\theta_i + \zeta_i|p) = r\left[\sigma_\zeta^2 + \sigma_\theta^2 - \frac{\sigma_{\theta_i p}^2}{\sigma_p^2}\right] = k^I + r\sigma_\theta^2\left[1 - \frac{(R_i^\top\eta_I)^2}{\eta_I^\top R\eta_I}\right] = k_i^U, \tag{A20}$$

and hence $q_i^U = -\alpha_i^U p$, where α_i^U is given by (A12). $\square$

As in the main model, an equilibrium can be described in reduced form as depth parameters $(\phi^I, \phi_1^U, \dots, \phi_{L_U}^U) \in \mathbb{R}_{++}^{L_U+1}$ that solve the following equations (which are analogous to (19) and (20)):

$$\sum_{i\in L_U} N_i^U\left[\phi^I\frac{k^I\phi^I + 2}{k^I\phi^I + 1} - \phi_i^U\right] = \frac{\phi^I}{k^I\phi^I + 1}\big[(k^I\phi^I + 2) - N^I\big], \tag{A21}$$

$$\phi^I\frac{k^I\phi^I + 2}{k^I\phi^I + 1} = \phi_i^U\left[\frac{1 - \frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}(k^I\phi^I + 2)}{k_i^U\phi_i^U + 1} + 1\right], \qquad i \in L_U. \tag{A22}$$

Thus the price function and the equations for the depth parameters take the same form as in the main model, with the parameter k being replaced by k^I for informed agents, and by k_i^U for uninformed agents in group i. Moreover, these parameters are exogenous, so our existence result, Proposition 3.2, still applies. Proposition 4.1 on price informativeness also holds. Using (22), we have

$$k_i^U = k^I + r\sigma_\theta^2(1 - \mathcal{V}_i). \tag{A23}$$

The results on depths and slopes need to be modified.

Proposition A6.2 (Depths) *The depth parameters ϕ^I and $\{\phi_i^U\}_{i\in L_U}$ satisfy the following properties:*

i. $\phi_i^U > \phi^I$ *for all* $i \in L_U$*.*

ii. $\phi_i^U = \phi_j^U$ *if* $\mathcal{V}_i = \mathcal{V}_j$*. Furthermore, if* $\sigma_\zeta^2 \geq \sigma_\theta^2 \eta_I^\top R \eta_I / 8$*, then* $\phi_i^U = \phi_j^U$ *if and only if* $\mathcal{V}_i = \mathcal{V}_j$*, and* $\phi_i^U > \phi_j^U$ *if and only if* $\mathcal{V}_i > \mathcal{V}_j$*.*

iii. If $N^U = 0$*, then* $k^I\phi^I + 2 = N^I$*. If* $N^U \geq 1$*, then* $k^I\phi^I + 2 < N^I + N^U$*.*

Proof From (A22),

$$\phi_i^U \frac{k_i^U \phi_i^U + 2}{k_i^U \phi_i^U + 1} - \phi^I \frac{k^I \phi^I + 2}{k^I \phi^I + 1} = \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k^I\phi^I + 2)\frac{\phi_i^U}{k_i^U \phi_i^U + 1}.$$

Note that the left-hand side is nonnegative, and $x(kx+2)/(kx+1)$ is strictly increasing in x and strictly decreasing in k. Since $k_i^U > k^I$, we can conclude that $\phi_i^U > \phi^I$ for all $i \in L_U$. The result in part (i) follows.

From the above equation,

$$k_i^U \phi_i^U + 2 - \left[k_i^U + (\phi_i^U)^{-1}\right]\phi^I \frac{k^I\phi^I + 2}{k^I\phi^I + 1} = \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k^I\phi^I + 2).$$

Since $k_i^U = k^I + r\sigma_\theta^2(1 - \mathcal{V}_i)$, it follows that $\phi_i^U = \phi_j^U$ if $\mathcal{V}_i = \mathcal{V}_j$. In order to establish the result that depths are ranked by price informativeness, we fix an equilibrium of a given economy (in particular, we fix ϕ^I, σ_p^2 and $\eta_I^\top R \eta_I$), and consider different hypothetical values of $R_i^\top \eta_I$, and hence of β_i, defined by

$$\beta_i := \frac{\sigma_{\theta_i p}}{\sigma_p^2} = \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k^I\phi^I + 2). \tag{A24}$$

A higher value of β_i is associated with a higher value of $\mathcal{V}_i$. From (A13) and (A22), ϕ_i^U solves

$$k_i^U (\phi_i^U)^2 + b_i \phi_i^U - \Phi = 0, \tag{A25}$$

where

$$b_i = 2 - \beta_i - k_i^U \Phi, \tag{A26}$$

and

$$\begin{aligned} k_i^U &= k^I + r\sigma_\theta^2 \left[1 - \beta_i^2 \frac{\eta_I^\top R \eta_I}{(k^I\phi^I + 2)^2}\right] \\ &= k^I + r(\sigma_\theta^2 - \beta_i^2 \sigma_p^2). \end{aligned}$$

Differentiating (A25) with respect to β_i, we get

$$2k_i^U \phi_i^U \frac{\partial \phi_i^U}{\partial \beta_i} + (\phi_i^U)^2 \frac{\partial k_i^U}{\partial \beta_i} + b_i \frac{\partial \phi_i^U}{\partial \beta_i} + \phi_i^U \frac{\partial b_i}{\partial \beta_i} = 0,$$

so that

$$\begin{aligned}
\frac{\partial \phi_i^U}{\partial \beta_i} &= -\frac{\phi_i^U}{2k_i^U\phi_i^U + b_i}\left[\phi_i^U\frac{\partial k_i^U}{\partial \beta_i} + \frac{\partial b_i}{\partial \beta_i}\right] \\
&= \frac{\phi_i^U}{2k_i^U\phi_i^U + b_i}\left[1 + (\Phi - \phi_i^U)\frac{\partial k_i^U}{\partial \beta_i}\right] \\
&= \frac{\phi_i^U}{2k_i^U\phi_i^U + b_i}\left[1 - 2r\alpha_i^U\beta_i\sigma_p^2\right] \\
&= \frac{\phi_i^U}{2k_i^U\phi_i^U + b_i}\left[1 - \frac{2r\phi_i^U(1-\beta_i)\beta_i\sigma_p^2}{k_i^U\phi_i^U + 1}\right].
\end{aligned}$$

From (A25), $k_i^U\phi_i^U + b_i > 0$. Also, $\phi_i^U/(k_i^U\phi_i^U + 1) < 1/k_i^U < 1/k^I$, $(1-\beta_i)\beta_i \leq 1/4$, and $\sigma_p^2 = \sigma_\theta^2\eta_I^\top R\eta_I(k^I\phi^I + 2)^{-2} < \sigma_\theta^2\eta_I^\top R\eta_I/4$. Therefore,

$$\begin{aligned}
\frac{\partial \phi_i^U}{\partial \beta_i} &\propto 1 - \frac{2r\phi_i^U(1-\beta_i)\beta_i\sigma_p^2}{k_i^U\phi_i^U + 1} \\
&> 1 - \frac{r\sigma_\theta^2\eta_I^\top R\eta_I}{8k^I} \\
&\propto \sigma_\zeta^2 - \frac{\sigma_\theta^2\eta_I^\top R\eta_I}{8}.
\end{aligned}$$

The result in part (ii) follows. The proof of part (iii) is analogous to that of Proposition 4.2 (iii). □

Proposition A6.3 (Slopes) *The slope parameters α^I and $\{\alpha_i^U\}_{i\in L_U}$ satisfy the following properties:*

i. $\alpha^I > 0$.

ii. $\alpha_i^U < \alpha^I$ for all $i \in L_U$.

iii. $\alpha_i^U = \alpha_j^U$ if $\mathcal{V}_i = \mathcal{V}_j$. Furthermore, if $\sigma_\zeta^2 \geq \sigma_\theta^2\eta_I^\top R\eta_I/8$, then $\alpha_i^U = \alpha_j^U$ if and only if $\mathcal{V}_i = \mathcal{V}_j$, and $\alpha_i^U < \alpha_j^U$ if and only if $\mathcal{V}_i > \mathcal{V}_j$.

The slope parameters also satisfy properties (iv)–(vii) in Proposition 4.3.

Proposition A6.3 follows from Proposition A6.2 and from the arguments in Proposition 4.3. There are two differences in the results on depths and slopes compared to those in the main model. First, in order to rank depths and slopes by price informativeness we need a lower bound on σ_ζ^2 (this ensures that k_i^U is not too large relative to k^I). This way of ranking depths and slopes is useful for interpretation, but is not needed for any other results. Second, the inequalities in Proposition A6.2 (i) and (iii) and Proposition A6.3 (ii) are strict because $k_i^U > k^I$ for all $i \in L_U$. This second difference also accounts for the following modification of Lemma 4.4:

Lemma A6.4 (Naive economy) *If all uninformed agents are naive, they have the same depth and slope parameters: $\phi_i^U = \phi^U > \phi^I$, and $\alpha_i^U = \alpha^U < \alpha^I$, for all $i \in L_U$.*

The analog of Proposition 5.1 is as follows:

Proposition A6.5 (Competitive equilibrium) *In a competitive economy with the mass of agents given by* $\{N_i^I, N_i^U\}_{i\in L}$*, the price function is*

$$\hat{p} = \gamma^{-1}\eta_I^\top\theta,$$

where

$$\gamma := \frac{N^I + \sum_{i\in L_U} N_i^U \frac{k^I}{k_i^U}}{1 + \sum_{i\in L_U} N_i^U \frac{k^I}{k_i^U}\frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}}, \tag{A27}$$

and the slope parameters are

$$\begin{aligned}\hat{\alpha}^I &= (k^I)^{-1},\\ \hat{\alpha}_i^U &= (k_i^U)^{-1}\left[1 - \frac{\sigma_{\theta_i\hat{p}}}{\sigma_{\hat{p}}^2}\right]\\ &= (k_i^U)^{-1}\left[1 - \frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}\gamma\right], \qquad i\in L_U.\end{aligned}$$

The slope parameters satisfy all the properties in Proposition A6.3.

We need the following analog of Lemma A3.1:

Lemma A6.6 *In the economy parametrized by* $(\lambda\eta_I, \lambda\eta_U)$*,* $\lambda\geq 1$*, we have*

$$\lim_{\lambda\to\infty}\frac{k^I\phi^I(\lambda)}{\lambda} = \gamma,$$

where γ *is defined by (A27).*

Proof From (A22), ϕ_i^U solves

$$k_i^U(\phi_i^U)^2 + b_i(\phi^I;\lambda)\phi_i^U - \phi^I\frac{k^I\phi^I+2}{k^I\phi^I+1} = 0,$$

where

$$b_i(\phi^I;\lambda) = 2 - \frac{R_i^\top\eta_I}{\lambda\eta_I^\top R\eta_I}(k^I\phi^I+2) - k_i^U\phi^I\frac{k^I\phi^I+2}{k^I\phi^I+1}.$$

The solution is given by

$$\phi_i^U = g_i(\phi^I;\lambda) = \frac{-b_i(\phi^I;\lambda) + \sqrt{b_i^2(\phi^I;\lambda) + 4k_i^U\phi^I\frac{k^I\phi^I+2}{k^I\phi^I+1}}}{2k_i^U}. \tag{A28}$$

From (A21),

$$0 = \frac{k^I\phi^I+2}{k^I\phi^I+1} - \frac{N^I}{k^I\phi^I+1}\lambda + \sum_{i\in L_U} N_i^U\left[\frac{g_i(\phi^I;\lambda)}{\phi^I} - \frac{k^I\phi^I+2}{k^I\phi^I+1}\right]\lambda \tag{A29}$$

$$= \frac{k^I\phi^I+2}{k^I\phi^I+1} - \frac{N^I + \sum_{i\in L_U} N_i^U\frac{k^I}{k_i^U}}{k^I\phi^I+1}\lambda + \sum_{i\in L_U} N_i^U\left[\frac{g_i(\phi^I;\lambda)}{\phi^I} - \frac{k^I\phi^I+2}{k^I\phi^I+1} + \frac{k^I}{k_i^U(k^I\phi^I+1)}\right]\lambda$$

$$= \frac{k^I\phi^I+2}{k^I\phi^I+1} - \frac{N^I + \sum_{i\in L_U} N_i^U\frac{k^I}{k_i^U}}{k^I\phi^I+1}\lambda + \sum_{i\in L_U} N_i^U H_i(\phi^I;\lambda), \tag{A30}$$

where

$$H_i(\phi^I;\lambda) := h_i(\phi^I;\lambda) - \frac{k^I\phi^I+2}{k^I\phi^I+1}\lambda + \frac{k^I}{k_i^U(k^I\phi^I+1)}\lambda,$$
$$h_i(\phi^I;\lambda) := \frac{g_i(\phi^I;\lambda)\lambda}{\phi^I} = -\frac{b_i(\phi^I;\lambda)\lambda}{2k_i^U\phi^I} + \sqrt{\left[\frac{b_i(\phi^I;\lambda)\lambda}{2k_i^U\phi^I}\right]^2 + \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\lambda^2}.$$

Note that $h_i(\phi^I;\lambda)$ is strictly positive and satisfies

$$\begin{aligned}
0 &= [h_i(\phi^I;\lambda)]^2 + \frac{b_i(\phi^I;\lambda)\lambda}{k_i^U\phi^I}h_i(\phi^I;\lambda) - \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\lambda^2 \\
&= h_i(\phi^I;\lambda)\left[h_i(\phi^I;\lambda) + \frac{2\lambda}{k_i^U\phi^I} - \frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}\left(\frac{k^I}{k_i^U}+\frac{2}{k_i^U\phi^I}\right) - \frac{k^I\phi^I+2}{k^I\phi^I+1}\lambda\right] - \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\lambda^2 \qquad \text{(A31)} \\
&= h_i(\phi^I;\lambda)\left[H_i(\phi^I;\lambda) - \frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}\left(\frac{k^I}{k_i^U}+\frac{2}{k_i^U\phi^I}\right) + \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\lambda\right] - \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\lambda^2.
\end{aligned}$$

Dividing both sides of this equation by $h_i(\phi^I;\lambda)$, and noting that $\lambda/h_i(\phi^I;\lambda) = \phi^I/g_i(\phi^I;\lambda)$, we obtain

$$H_i(\phi^I;\lambda) = \frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}\left(\frac{k^I}{k_i^U}+\frac{2}{k_i^U\phi^I}\right) + \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\left(\frac{\phi^I}{g_i(\phi^I;\lambda)}-1\right)\lambda.$$

Using the arguments in Proposition 5.2, we can show that $\phi^I \to \infty$ as $\lambda \to \infty$. From (A28),

$$\frac{g_i(\phi^I;\lambda)}{\phi^I} = -\frac{b_i(\phi^I;\lambda)}{2k_i^U\phi^I} + \sqrt{\left[\frac{b_i(\phi^I;\lambda)}{2k_i^U\phi^I}\right]^2 + \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}}.$$

Since $b_i(\phi^I;\lambda)/\phi^I \to -k_i^U$, we see that $g_i(\phi^I;\lambda)/\phi^I \to 1$. Moreover λ/ϕ^I is bounded (we prove this below). Therefore, $H_i(\phi^I;\lambda) \to \frac{k^I}{k_i^U}\frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}$. Now the result follows from (A30).

It remains to show that λ/ϕ^I is bounded. From (A31),

$$\begin{aligned}
0 &= h_i(\phi^I;\lambda)\left[h_i(\phi^I;\lambda) - \lambda - \frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}\left(\frac{k^I}{k_i^U}+\frac{2}{k_i^U\phi^I}\right) + \frac{2\lambda}{k_i^U\phi^I} - \frac{\lambda}{k^I\phi^I+1}\right] - \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\lambda^2 \\
&= h_i(\phi^I;\lambda)\left[h_i(\phi^I;\lambda) - \lambda - \frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}\left(\frac{k^I}{k_i^U}+\frac{2}{k_i^U\phi^I}\right) + \frac{k^I-k_i^U}{k_i^U(k^I\phi^I+1)}\lambda\right] \\
&\quad + \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\left[h_i(\phi^I;\lambda)-\lambda\right]\lambda.
\end{aligned}$$

Dividing both sides of this equation by $h_i(\phi^I;\lambda)$, and noting that $\lambda/h_i(\phi^I;\lambda) = \phi^I/g_i(\phi^I;\lambda)$, we obtain

$$h_i(\phi^I;\lambda) - \lambda = \left[1 + \frac{k^I\phi^I+2}{k_i^U\phi^I(k^I\phi^I+1)}\frac{\phi^I}{g_i(\phi^I;\lambda)}\right]^{-1}\left[\frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I}\left(\frac{k^I}{k_i^U}+\frac{2}{k_i^U\phi^I}\right) - \frac{k^I-k_i^U}{k_i^U(k^I\phi^I+1)}\lambda\right].$$

From (A29),

$$\frac{k^I\phi^I+2}{k^I\phi^I+1} - \frac{N^I+N^U}{k^I\phi^I+1}\lambda + \sum_{i\in L_U} N_i^U\left[h_i(\phi^I;\lambda)-\lambda\right] = 0.$$

From the above two equations, we can show by contradiction that λ/ϕ^I is bounded. This completes the proof. $\square$

Using Lemma A6.6, we can show that Proposition 5.2 holds. Propositions 5.3 and 5.4 also hold. The proofs are along the same lines as those in the paper.

Next, we turn to welfare analysis. It is convenient to use the following monotonic transformation of ex ante expected utility for an agent in group i:

$$\mathcal{U}_i := \frac{1}{2r}\Big(\big[\mathbb{E}(\exp(-rW_i)\big]^{-2} - 1\Big), \tag{A32}$$

where $W_i = (v_i - p)q_i$. We define

$$F(k, x) := \frac{x(kx+2)}{(kx+1)^2}, \tag{A33}$$

which is the same as the definition of the function F in the paper (see (32)), except that here we allow F to depend on k in addition to x. Gains from trade for group i are $G_i := \sigma^2_{\theta_i - p}/\sigma^2_\theta$ as in the main model.

Lemma A6.7 (Utilities) *Ex ante utilities are given by*

$$\begin{aligned} \mathcal{U}_i^I &= \frac{\sigma_\theta^2}{2} F(k^I, \phi^I) G_i, && i \in L_I, \\ \mathcal{U}_i^U &= \frac{\sigma_\theta^2}{2} F(k_i^U, \phi_i^U)\big[G_i - (1 - \mathcal{V}_i)\big], && i \in L_U. \end{aligned} \tag{A34}$$

Proof For an agent in group i,

$$\begin{aligned} \mathbb{E}\big[-\exp(-rW_i)\big] &= -\mathbb{E}\big[\mathbb{E}(\exp(-rW_i)|\mathcal{I}_i)\big] \\ &= -\mathbb{E}\big[\exp(-rU_i)\big], \end{aligned}$$

where U_i is the mean-variance criterion given by (A14). Using (A14)–(A16) and (A33), we have

$$\begin{aligned} U_i &= \frac{1}{2\phi_i}(k_i\phi_i + 2)q_i^2 \\ &= \frac{1}{2}F(k_i, \phi_i)\big[\mathbb{E}(\theta_i|\mathcal{I}_i) - p\big]^2. \end{aligned}$$

Now we invoke the fact that if $X \sim N(0, \sigma^2)$, then $\mathbb{E}\big(e^{-\frac{1}{2}X^2}\big) = (1+\sigma^2)^{-\frac{1}{2}}$. We have

$$\begin{aligned} \mathbb{E}\big[-\exp(-rW_i)\big] &= -\mathbb{E}\left[\exp\left(-\frac{1}{2}rF(k_i,\phi_i)\big[\mathbb{E}(\theta_i|\mathcal{I}_i) - p\big]^2\right)\right] \\ &= -\left[1 + rF(k_i,\phi_i)\mathbb{E}[\mathbb{E}(\theta_i|\mathcal{I}_i) - p]^2\right]^{-\frac{1}{2}}, \end{aligned}$$

and therefore (from (A32)):

$$\mathcal{U}_i = \frac{1}{2}F(k_i,\phi_i)\mathbb{E}\big[\mathbb{E}(\theta_i|\mathcal{I}_i) - p\big]^2.$$

Using (A17)–(A20), we obtain:

$$\mathcal{U}_i^I = \frac{1}{2}F(k^I, \phi^I)\mathbb{E}(\theta_i - p)^2,$$
$$\mathcal{U}_i^U = \frac{1}{2}F(k_i^U, \phi_i^U)\left[1 - \frac{\sigma_{\theta_i p}}{\sigma_p^2}\right]^2 \sigma_p^2.$$

The desired expression for $\mathcal{U}_i^I$ is immediate. For $\mathcal{U}_i^U$ the argument is the same as in the proof of Lemma 6.1. □

Thus equilibrium utilities are the same as in the main model (up to a monotonic transformation) except that the parameter k is replaced by k^I for the informed, and by k_i^U for the uninformed in group i. We do not have a result that is analogous to Proposition 6.2. However, Example 6.1 is still valid (k_1^U converges to k^I as $\sigma_\nu^2 \to 0$, so that $\lim_{\sigma_\nu^2\to 0} F(k_1^U, \phi_1^U) = \lim_{\sigma_\nu^2\to 0} F(k^I, \phi_1^U) > \lim_{\sigma_\nu^2\to 0} F(k^I, \phi^I)$). Lemma 7.1 holds with k replaced by k^I; thus ϕ^I solves

$$\beta_1 = \frac{R_1^\top \eta_I}{\eta_I^\top R \eta_I}(k^I\phi^I + 2) = 1. \tag{A35}$$

It is straightforward to check that Propositions 7.2 and 7.3, and Lemma 7.4 also hold. Next, we verify that Proposition 7.5 holds (we restate this result for convenience):

Proposition A6.8 (Welfare) *Consider an $\mathcal{F}_1$-economy with two groups. Suppose $\rho \leq 1/2$ and $N_1^I \leq N_2^I/3$. Then $\mathcal{U}_1^U = 0$ for all N_1^I, and the utility of all other agents is strictly decreasing in N_1^I.*

Proof We will show that $\partial\mathcal{U}_2^U/\partial N_1^I < 0$. The other welfare effects follow from the arguments in the proof of Proposition 7.5. From this proof, we recall that $\partial\mathcal{V}_2/\partial N_1^I < 0$, and $\partial G_2/\partial N_1^I < 0$. From (A23), we have $\partial k_2^U/\partial N_1^I \propto -\partial\mathcal{V}_2/\partial N_1^I > 0$, and from (A33), $\partial F(x,k)/\partial k < 0$, and $\partial F(x,k)/\partial x > 0$. Hence, using the utility expression given by (A34), it suffices to show that $\partial\phi_2^U/\partial N_1^I < 0$.

From (A24), (A25), (A26) and (A35), ϕ_2^U solves

$$k_2^U(\phi_2^U)^2 + b_2\phi_2^U - \Phi = 0, \tag{A36}$$

where

$$b_2 = 2 - \frac{R_2^\top \eta_I}{R_1^\top \eta_I} - k_2^U \Phi.$$

Implicitly differentiating (A36) with respect to N_1^I, we obtain:

$$\begin{aligned}
\frac{\partial\phi_2^U}{\partial N_1^I} &= \frac{1}{2k_2^U\phi_2^U + b_2}\left[\frac{\partial\Phi}{\partial N_1^I} - \left(\frac{\partial k_2^U}{\partial N_1^I}\phi_2^U + \frac{\partial b_2}{\partial N_1^I}\right)\phi_2^U\right] \\
&= \frac{1}{2k_2^U\phi_2^U + b_2}\left[\frac{\partial\Phi}{\partial N_1^I} - \left(\frac{\partial k_2^U}{\partial N_1^I}\phi_2^U - \frac{\partial\frac{R_2^\top\eta_I}{R_1^\top\eta_I}}{\partial N_1^I} - \frac{\partial k_2^U}{\partial N_1^I}\Phi - k_2^U\frac{\partial\Phi}{\partial N_1^I}\right)\phi_2^U\right] \\
&= \frac{1}{2k_2^U\phi_2^U + b_2}\left[(1 + k_2^U\phi_2^U)\frac{\partial\Phi}{\partial N_1^I} + \phi_2^U\frac{\partial\frac{R_2^\top\eta_I}{R_1^\top\eta_I}}{\partial N_1^I} + (\Phi - \phi_2^U)\phi_2^U\frac{\partial k_2^U}{\partial N_1^I}\right].
\end{aligned}$$

From (A36), $2k_2^U\phi_2^U + b_2 > k_2^U\phi_2^U + b_2 = \Phi/\phi_2^U > 0$. From (86), $\partial\Phi/\partial N_1^I \le 0$, and from (87), $\partial\frac{R_2^\top\eta_I}{R_1^\top\eta_I}/\partial N_1^I < 0$. From (A12), (A24) and (A35), and the assumption that $N_1^I \le N_2^I/3$,

$$\Phi - \phi_2^U \propto 1 - \frac{R_2^\top\eta_I}{R_1^\top\eta_I} \propto R_1^\top\eta_I - R_2^\top\eta_I = (1-\rho)(N_1^I - N_2^I) < 0.$$

As noted earlier, $\partial k_2^U/\partial N_1^I > 0$. Using all these facts, we conclude that $\partial\phi_2^U/\partial N_1^I < 0$. □

Finally, it is easy to check that Propositions 8.1 and 8.2 hold.

A7 An Extended Model

In this section we extend the model in the paper to allow for agents who are either well-informed or poorly informed. To simplify our calculations we suppose that there are only two groups, 1 and 2, with independent values, and all agents in group 2 are well-informed. In particular, for each group i, $i = 1, 2$, there are N_i^I well-informed traders who observe a signal $s_i^I = \theta_i + \epsilon_i^I$. In addition, there are N^U poorly informed traders in group 1 who observe a signal $s^U = s_1^I + \epsilon^U = \theta_1 + \epsilon_1^I + \epsilon^U$. We assume that (i) $N_1^I \ge 1, N_2^I \ge 1$ and $N^U \ge 1$; (ii) $\theta_1, \theta_2, \epsilon_1^I, \epsilon_2^I$ and ϵ^U are mutually independent joint normal random variables with zero mean; and (iii) $\mathrm{Var}(\theta_i) = \sigma_\theta^2$, and $\mathrm{Var}(\epsilon_i^I) = \sigma_{\epsilon_I}^2$, for $i = 1, 2$. We denote the variance of ϵ^U by $\sigma_{\epsilon_U}^2$.

Let ϕ_i^I be the depth parameter for well-informed agents in group i, and ϕ^U the depth parameter for poorly informed agents in group 1. Let

$$\Omega := \frac{N_1^I\sigma_{\epsilon_U}^2}{(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2)}.$$

The following proposition characterizes the linear equilibrium for given depths.

Proposition A7.1 (Equilibrium for given depths) *The depth parameters for well-informed agents are the same for both groups: $\phi_1^I = \phi_2^I = \phi^I$. Given ϕ^I and ϕ^U, agents' demand functions are*

$$\begin{aligned} q_i^I &= \mu^I s_i^I - \alpha^I p, \qquad i = 1, 2, \\ q_1^U &= \mu^U s^U - \alpha^U p, \end{aligned}$$

where

$$\alpha^I = \frac{\phi^I}{k\phi^I + 1}, \tag{A37}$$

$$\mu^I = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_I}^2}\frac{\phi^I}{k\phi^I + 1},$$

$$\alpha^U = \frac{\phi^U}{k\phi^U + 1}\left[1 - \Omega(k\phi^I + 2)\right], \tag{A38}$$

$$\mu^U = \frac{(N_2^I)^2\sigma_\theta^2}{\left[(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2)\right]\left[k + (\phi^U)^{-1}\right] + N_1^I N^U\sigma_{\epsilon_U}^2\left[k + (\phi^I)^{-1}\right]}.$$

The price function is

$$p = \Phi^{-1}\left[\mu^I(N_1^I s_1^I + N_2^I s_2^I) + N^U \mu^U s^U\right], \tag{A39}$$

where Φ *is market depth, given by*

$$\Phi = \phi^I \frac{k\phi^I + 2}{k\phi^I + 1}. \tag{A40}$$

Proof We look for an equilibrium in linear strategies of the form

$$q_i^I(s_i^I, p) = \mu_i^I s_i^I - \alpha_i^I p, \tag{A41}$$
$$q_1^U(s^U, p) = \mu^U s^U - \alpha^U p. \tag{A42}$$

The market-clearing condition is

$$N_1^I \mu_1^I s_1^I + N_2^I \mu_2^I s_2^I + N^U \mu^U s^U - \Phi p = 0,$$

where

$$\Phi := N_1^I \alpha_1^I + N_2^I \alpha_2^I + N^U \alpha^U. \tag{A43}$$

The group-specific depth parameters are related to Φ as in the main model:

$$\phi_i^I = \Phi - \alpha_i^I, \qquad \phi^U = \Phi - \alpha^U. \tag{A44}$$

Agents' optimal strategies are given by

$$q_i^I(s_i^I, p) = \frac{\mathbb{E}(\theta_i | s_i^I, p) - p}{k + (\phi_i^I)^{-1}}, \tag{A45}$$
$$q_1^U(s^U, p) = \frac{\mathbb{E}(\theta_1 | s^U, p) - p}{k + (\phi^U)^{-1}}. \tag{A46}$$

We have

$$\mathbb{E}(\theta_i | s_i^I, p) = \mathbb{E}(\theta_i | s_i^I) = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_I}^2} s_i^I. \tag{A47}$$

Therefore, from (A41), (A45) and (A47),

$$\alpha_i^I = \frac{1}{k + (\phi_i^I)^{-1}} = \frac{\phi_i^I}{k\phi_i^I + 1},$$
$$\mu_i^I = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_I}^2} \alpha_i^I.$$

Moreover,

$$\Phi = \alpha_i^I + \phi_i^I = \frac{\phi_i^I}{k\phi_i^I + 1} + \phi_i^I = \phi_i^I \frac{k\phi_i^I + 2}{k\phi_i^I + 1},$$

which is increasing in ϕ_i^I. It follows that ϕ_i^I is the same for both groups, and hence so are α_i^I and μ_i^I. Henceforth, we drop the i subscripts on these variables. Also,

$$\begin{aligned}\mathbb{E}(\theta_1 | s^U, p) &= \mathbb{E}\left[\theta_1 | s^U, \Phi^{-1}(N_1^I \mu^I s_1^I + N_2^I \mu^I s_2^I + N^U \mu^U s^U)\right] \\ &= \mathbb{E}(\theta_1 | s^U, s^I),\end{aligned} \tag{A48}$$

where

$$s^I = (\mu^I)^{-1}[\Phi p - N^U \mu^U s^U] \tag{A49}$$
$$= N_1^I s_1^I + N_2^I s_2^I$$
$$= N_1^I(\theta_1 + \epsilon_1^I) + N_2^I(\theta_2 + \epsilon_2^I).$$

We have

$$\mathbb{E}(\theta_1 | s^U, s^I) = b^U s^U + b^I s^I, \tag{A50}$$

where

$$b^U = \frac{\text{Cov}(\theta_1, s^U)\text{Var}(s^I) - \text{Cov}(\theta_1, s^I)\text{Cov}(s^U, s^I)}{\text{Var}(s^U)\text{Var}(s^I) - [\text{Cov}(s^U, s^I)]^2}$$
$$= \frac{(N_2^I)^2\sigma_\theta^2}{(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2)},$$
$$b^I = \frac{\text{Cov}(\theta_1, s^I)\text{Var}(s^U) - \text{Cov}(\theta_1, s^U)\text{Cov}(s^U, s^I)}{\text{Var}(s^U)\text{Var}(s^I) - [\text{Cov}(s^U, s^I)]^2}$$
$$= \frac{N_1^I \sigma_\theta^2 \sigma_{\epsilon_U}^2}{\left[(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2)\right]\left[\sigma_\theta^2 + \sigma_{\epsilon_I}^2\right]}.$$

Therefore, from (A42), (A46), (A48), (A49) and (A50),

$$\alpha^U = \frac{1 - (\mu^I)^{-1} b^I \Phi}{k + (\phi^U)^{-1}}$$
$$= \frac{\phi^U}{k\phi^U + 1}\left[1 - b^I \frac{\sigma_\theta^2 + \sigma_{\epsilon_I}^2}{\sigma_\theta^2}(k\phi^I + 2)\right]$$
$$= \frac{\phi^U}{k\phi^U + 1}\left[1 - \frac{N_1^I \sigma_{\epsilon_U}^2}{(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2)}(k\phi^I + 2)\right],$$

and

$$\mu^U = \frac{b^U - (\mu^I)^{-1} b^I N^U \mu^U}{k + (\phi^U)^{-1}}$$
$$= \frac{b^U}{k + (\phi^U)^{-1} + (\mu^I)^{-1} b^I N^U}$$
$$= \frac{(N_2^I)^2\sigma_\theta^2}{\left[(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2)\right]\left[k + (\phi^U)^{-1}\right] + N_1^I N^U \sigma_{\epsilon_U}^2\left[k + (\phi^I)^{-1}\right]}.$$

This completes the proof. □

Note that the coefficients $(\alpha^I, \mu^I, \alpha^U, \mu^U)$ converge to the corresponding expressions in Proposition 3.1 when $\sigma_{\epsilon_I}^2 = 0$ and $\sigma_{\epsilon_U}^2 \to \infty$.

Proposition A7.1 gives us prices and demand functions in terms of the depth parameters ϕ^I and ϕ^U. Using (A40), (A43) and (A44), we obtain

$$N^U\left[\phi^I \frac{k\phi^I + 2}{k\phi^I + 1} - \phi^U\right] = \frac{\phi^I}{k\phi^I + 1}\left[(k\phi^I + 2) - N^I\right], \tag{A51}$$

where $N^I := N_1^I + N_2^I$. From (A38), (A40) and (A44), we have

$$\phi^I \frac{k\phi^I + 2}{k\phi^I + 1} - \phi^U = \frac{\phi^U}{k\phi^U + 1} \left[1 - \Omega(k\phi^I + 2)\right]. \tag{A52}$$

An equilibrium can then be described in reduced form as depths $(\phi^I, \phi^U) \in \mathbb{R}^2_{++}$ that solve (A51) and (A52). These equations are the same as the equilibrium equations (19) and (20), for the case of $L_I = L = \{1, 2\}$ and $L_U = \{1\}$, except that $R_1^\top \eta_I / \eta_I^\top R \eta_I$ is replaced by Ω.

Proposition A7.2 (Equilibrium characterization) *There is a unique equilibrium. It has the following properties:*

i. $\phi^U > \phi^I$*;*

ii. $\alpha^I > 0$*;*

iii. $\alpha^U < \alpha^I$*;*

iv. $\alpha^U > 0$ *if* $N_1^I \leq N_2^I$*.*

Proof The arguments below are identical to the corresponding arguments in the main paper, with $R_1^\top \eta_I / \eta_I^\top R \eta_I$ replaced by Ω.

Proof of existence: The existence proof is the same as for Proposition 3.2. In the final step of the proof we need to establish that $f'(0) < 0$. Noting that

$$\Omega < \frac{N_1^I}{(N_1^I)^2 + (N_2^I)^2} \leq \frac{N_1^I}{(N_1^I)^2 + 1} \leq \frac{1}{2}, \tag{A53}$$

we see that

$$\begin{aligned} f'(0) &= -(N^I - 2) + N^U \left[(1 - \Omega)^{-1} - 2\right] \\ &< -(N^I - 2) + N^U \left[\left(1 - \frac{1}{2}\right)^{-1} - 2\right] \\ &= -(N^I - 2) \\ &\leq 0. \end{aligned}$$

Uniqueness of equilibrium follows from the specification of the trading game, as in Proposition 3.2.

Proof of (i)–(iv): To show that $\phi^U > \phi^I$, we use the same argument as in the proof of Proposition 4.2 (i), noting that $\Omega > 0$. The inequality $\alpha^U < \alpha^I$ is then immediate from (A44). From (A37), we have $\alpha^I > 0$. Finally, analogous to the proof of Proposition 4.2 (vi), we see that $\alpha_1^U > 0$ if and only if $N^I \Omega < 1$. Under the assumption that $N_1^I < N_2^I$, we have

$$N^I \Omega < N^I \frac{N_1^I}{(N_1^I)^2 + (N_2^I)^2} \leq 1.$$

This completes the proof. ☐

The following result is the analog of Proposition 5.1. As in the main paper, we distinguish the parameters of a competitive economy with a "hat".

Proposition A7.3 (Competitive equilibrium) *In a competitive economy with the mass of agents given by* (N_1^I, N_2^I, N^U)*, the slope parameters are*

$$\begin{aligned}
\hat{\alpha}^I &= k^{-1}, \\
\hat{\mu}^I &= k^{-1}\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_I}^2}, \\
\hat{\alpha}^U &= k^{-1}\frac{1 - N^I\Omega}{1 + N^U\Omega}, \\
\hat{\mu}^U &= k^{-1}\frac{(N_2^I)^2\sigma_\theta^2}{(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2) + N_1^I N^U \sigma_{\epsilon_U}^2}.
\end{aligned}$$

The slope parameters satisfy all the properties in Proposition A7.2.

Proof The market-clearing condition is

$$N_1^I \hat{\mu}_1^I s_1^I + N_2^I \hat{\mu}_2^I s_2^I + N^U \hat{\mu}^U s^U - \hat{\Phi} p = 0,$$

where

$$\hat{\Phi} := N_1^I \hat{\alpha}_1^I + N_2^I \hat{\alpha}_2^I + N^U \hat{\alpha}^U.$$

Agents' portfolio choices are given by (A45) and (A46), but with zero price impact. The conditional expectations are the same as in the imperfectly competitive case. Hence, we obtain:

$$\begin{aligned}
\hat{\alpha}^I &= k^{-1}, \\
\hat{\mu}^I &= k^{-1}\frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_{\epsilon_I}^2}, \\
\hat{\alpha}^U &= k^{-1}\big[1 - (\hat{\mu}^I)^{-1} b^I \hat{\Phi}\big] \\
&= k^{-1}(1 - \Omega k \hat{\Phi}),
\end{aligned}$$

and

$$\begin{aligned}
\hat{\mu}^U &= k^{-1}\left[b^U - (\hat{\mu}^I)^{-1} b^I N^U \hat{\mu}^U\right] \\
&= \frac{b^U}{k + (\hat{\mu}^I)^{-1} b^I N^U} \\
&= k^{-1}\frac{(N_2^I)^2\sigma_\theta^2}{(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2) + N_1^I N^U \sigma_{\epsilon_U}^2}.
\end{aligned}$$

Therefore,

$$\hat{\Phi} = k^{-1}\big[N^I + N^U(1 - \Omega k \hat{\Phi})\big],$$

so that

$$k\hat{\Phi} = \frac{N^I + N^U}{1 + N^U\Omega}.$$

It follows that

$$\begin{aligned}
\hat{\alpha}^U &= k^{-1}\left[1 - \frac{(N^I + N^U)\Omega}{1 + N^U\Omega}\right] \\
&= k^{-1}\frac{1 - N^I\Omega}{1 + N^U\Omega}.
\end{aligned}$$

It is easy to check that the slope parameters satisfy properties (ii)–(iv) in Proposition A7.2. $\square$

Next, we provide convergence and limit results analogous to those in Propositions 5.2, 5.3 and 5.4.

Proposition A7.4 (Convergence) *We have the following convergence results:*

i. $\lim_{\lambda\to\infty}\mathcal{E}(\lambda\eta_I,\lambda N^U)=\hat{\mathcal{E}}(\eta_I,N^U)$*, and* ϕ^I *and* ϕ^U *are strictly increasing in* λ*;*

ii. $\lim_{\lambda\to\infty}\mathcal{E}(\lambda\eta_I,N^U)=\lim_{\lambda\to\infty}\hat{\mathcal{E}}(\lambda\eta_I,N^U)$*, and* ϕ^I *and* ϕ^U *are strictly increasing in* λ*. Also,* $\lim_{N_\ell^I\to\infty}\mathcal{E}(\eta_I,N^U)=\lim_{N_\ell^I\to\infty}\hat{\mathcal{E}}(\eta_I,N^U)$*;*

iii. There exist strictly positive scalars $\underline{\kappa}$ *and* $\bar{\kappa}$ *such that* $\{\phi^I,\phi^U\}\subset[\underline{\kappa},\bar{\kappa}]$ *for all* $N^U\geq 1$*, and* $\phi^I(N^U)-\phi^I(\check{N}^U)\propto\alpha^U(\phi^I(\check{N}^U))$*, for all* $N^U>\check{N}^U\geq 1$*. Furthermore,* $\lim_{N^U\to\infty}\alpha^U=0$ *and* $\lim_{N^U\to\infty}N^U\alpha^U<\infty$*.*

Proof These results follow from the same arguments as in the proofs of Propositions 5.2, 5.3 and 5.4, with $R_1^\top\eta_I/\eta_I^\top R\eta_I$ replaced by Ω. In the proof of Proposition 5.4, the assumption that $N_i^I\geq 2$ is needed to establish that $\lim_{\phi^I\to 0}g_i(\phi^I)/\phi^I<2$ (see (62)). Here we can use (A53):

$$\lim_{\phi^I\to 0}\frac{g_1(\phi^I)}{\phi^I}=(1-\Omega)^{-1}<\left[1-\frac{1}{2}\right]^{-1}=2.$$

Thus we do not need to assume that $N_1^I\geq 2$. $\square$

Finally, we turn to price informativeness.

Proposition A7.5 (Price informativenesss) *Price informativeness for group 1 is higher (and that for group 2 is lower) in the imperfectly competitive economy compared to the corresponding perfectly competitive economy.*

Proof Using (21) and (A39), we can calculate price informativeness in the imperfectly competitive economy:

$$\mathcal{V}_1=\frac{(N_1^I+N^U\mu)^2\sigma_\theta^2}{\left[(N_1^I+N^U\mu)^2+(N_2^I)^2\right](\sigma_\theta^2+\sigma_{\epsilon_I}^2)+(N^U\mu)^2\sigma_{\epsilon_U}^2},$$
$$\mathcal{V}_2=\frac{(N_2^I)^2\sigma_\theta^2}{\left[(N_1^I+N^U\mu)^2+(N_2^I)^2\right](\sigma_\theta^2+\sigma_{\epsilon_I}^2)+(N^U\mu)^2\sigma_{\epsilon_U}^2},$$

where

$$\mu:=\frac{\mu^U}{\mu^I}=\frac{(N_2^I)^2(\sigma_\theta^2+\sigma_{\epsilon_I}^2)}{\left[(N_1^I)^2\sigma_{\epsilon_U}^2+(N_2^I)^2(\sigma_\theta^2+\sigma_{\epsilon_I}^2+\sigma_{\epsilon_U}^2)\right]\frac{k+(\phi^U)^{-1}}{k+(\phi^I)^{-1}}+N_1^IN^U\sigma_{\epsilon_U}^2}.$$

For the corresponding competitive economy, the price informativeness measures $\hat{\mathcal{V}}_1$ and $\hat{\mathcal{V}}_2$ are given by the same expressions as for $\mathcal{V}_1$ and $\mathcal{V}_2$ above, but with μ replaced by $\hat{\mu}$, where

$$\hat{\mu}:=\frac{\hat{\mu}^U}{\hat{\mu}^I}=\frac{(N_2^I)^2(\sigma_\theta^2+\sigma_{\epsilon_I}^2)}{(N_1^I)^2\sigma_{\epsilon_U}^2+(N_2^I)^2(\sigma_\theta^2+\sigma_{\epsilon_I}^2+\sigma_{\epsilon_U}^2)+N_1^IN^U\sigma_{\epsilon_U}^2}.$$

Clearly, $\partial \mathcal{V}_2/\partial \mu < 0$. Furthermore,

$$\begin{aligned}\frac{\partial \mathcal{V}_1}{\partial \mu} &\propto (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2) - N_1^I N^U \mu \sigma_{\epsilon_U}^2 \\ &\propto \left[(N_1^I)^2\sigma_{\epsilon_U}^2 + (N_2^I)^2(\sigma_\theta^2 + \sigma_{\epsilon_I}^2 + \sigma_{\epsilon_U}^2)\right]\frac{k + (\phi^U)^{-1}}{k + (\phi^I)^{-1}} \\ &> 0.\end{aligned}$$

Since $\phi^U > \phi^I$, we have $\mu > \hat{\mu}$, and hence $\mathcal{V}_1 > \hat{\mathcal{V}}_1$ and $\mathcal{V}_2 < \hat{\mathcal{V}}_2$. □

For a discussion of the results in this section, see Section 9 of the paper.

A8 Numerical Computations

In this section we provide an algorithm for calculating the depth parameter ϕ^I, which determines all other equilibrium variables. We also do some simulations on the correlation matrix R to demonstrate the flexibility of our model.

The equation for ϕ^I is $f(\phi^I) = 0$, where the function f is defined by (44). In the proof of Proposition 3.2 we show that $f(0) = 0$, $f'(0) < 0$ and $\lim_{\phi^I \to \infty} f(\phi^I) = \infty$. We define the iteration sequences $\{x_t\}_{t=0}^\infty$ and $\{y_t\}_{t=0}^\infty$ as follows.

First, we choose a sufficiently large value of ϕ_0^I such that $f(\phi_0^I) > 0$, and set $x_0 = y_0 = \phi_0^I$. Next, if $f(x_0/2) < 0$, we set $y_1 = x_0/2$, and if $f(x_0/2) > 0$, we set $x_1 = x_0/2$. If $f(x_1/2) > 0$, we set $x_2 = x_1/2$, and continue in this manner until the first t_1 such that $f(x_0/2^{t_1}) < 0$, whereupon we set $y_1 = x_{t_1} = x_0/2^{t_1}$.

If $f((x_{t_1} + x_{t_1-1})/2) > 0$, we set $y_2 = (x_{t_1} + x_{t_1-1})/2$, and if $f((x_{t_1} + x_{t_1-1})/2) < 0$, we set $x_{t_1+1} = (x_{t_1} + x_{t_1-1})/2$. If $f((x_{t_1+1} + x_{t_1-1})/2) < 0$, we set $x_{t_1+2} = (x_{t_1+1} + x_{t_1-1})/2$, and continue in this manner until the first $t_2 > t_1$ such that $f((x_{t_2-1} + x_{t_1-1})/2) > 0$, whereupon we set $y_2 = x_{t_2} = (x_{t_2-1} + x_{t_1-1})/2$.

Similarly, we can find $\{y_t\}_{t\geq 3}$ such that $f(y_3) < 0, f(y_4) > 0, f(y_5) < 0, \ldots$. Clearly $y^* := \lim_{t\to\infty} y_t$ exists, and $f(y^*) = 0$. Thus y^* is the desired value of ϕ^I.

In the paper we assume that the correlation matrix R is positive definite, and that $R_i^\top \eta_I \geq 0$ for all i. We do not impose the equicommonality assumption of Rostek and Weretka (2012, 2015) (see Section A4). We now numerically investigate how price informativeness and depth are affected when equicommonality is imposed on R.

Since R is positive definite, it can be written as $R = CC^\top$, where C is a lower-triangular matrix with positive diagonal elements (Cholesky decomposition). Moreover, the diagonal elements of R are equal to one, implying that each row of C has unit length. In order to ensure that $R_i^\top \eta_I \geq 0$ for all i we assume in addition that each element of C below the diagonal is positive.

For simplicity, we consider the case of $L = 3$. Then the Cholesky factor C takes the form

$$C = \begin{pmatrix} 1 & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{pmatrix},$$

where all the entries on or below the diagonal are positive, $c_{21}^2 + c_{22}^2 = 1$, and $c_{31}^2 + c_{32}^2 + c_{33}^2 =$

1. This gives us

$$R = CC^\top = \begin{pmatrix} 1 & c_{21} & c_{31} \\ c_{21} & 1 & c_{21}c_{31} + c_{22}c_{32} \\ c_{31} & c_{21}c_{31} + c_{22}c_{32} & 1 \end{pmatrix}.$$

We randomly sample the matrix R by independently choosing c_{21} from the interval $(0, 0.1)$ with a uniform distribution, and each of c_{31}, c_{32} and c_{33} from the interval $(0, 1)$ with a uniform distribution, subject to the constraint $c_{31}^2 + c_{32}^2 + c_{33}^2 = 1$. The scalar c_{22} is determined by the relation $c_{21}^2 + c_{22}^2 = 1$. For any given matrix R, we have a corresponding matrix $\hat{R}$ which has the same form as R, with the same value of c_{21}, but which additionally satisfies the equicommonality condition: $c_{21} = c_{31} = c_{21}c_{31} + c_{22}c_{32}$.

For each R and $\hat{R}$, we calculate price informativeness and depth (for informed agents) for the following parameter values: $k = 1$, $\eta_I = (2, 4, 6)$, and $\eta_U = (50, 30, 10)$. Depth depends on both η_I and η_U, while price informativeness depends only on η_I. The figures below plot expected price informativeness and the expected value of the depth parameter ϕ^I for both R and $\hat{R}$, based on 100,000 realizations of R and $\hat{R}$. Recall that ϕ^I pins down all other depth parameters, including market depth Φ which is monotonically increasing in ϕ^I. The plots show that, in the economy under consideration, imposing equicommonality reduces price informativeness for all groups and increases market liquidity.

Figure A1: Expected price informativeness $\mathcal{V}_i$ for group $i = 1, 2, 3$

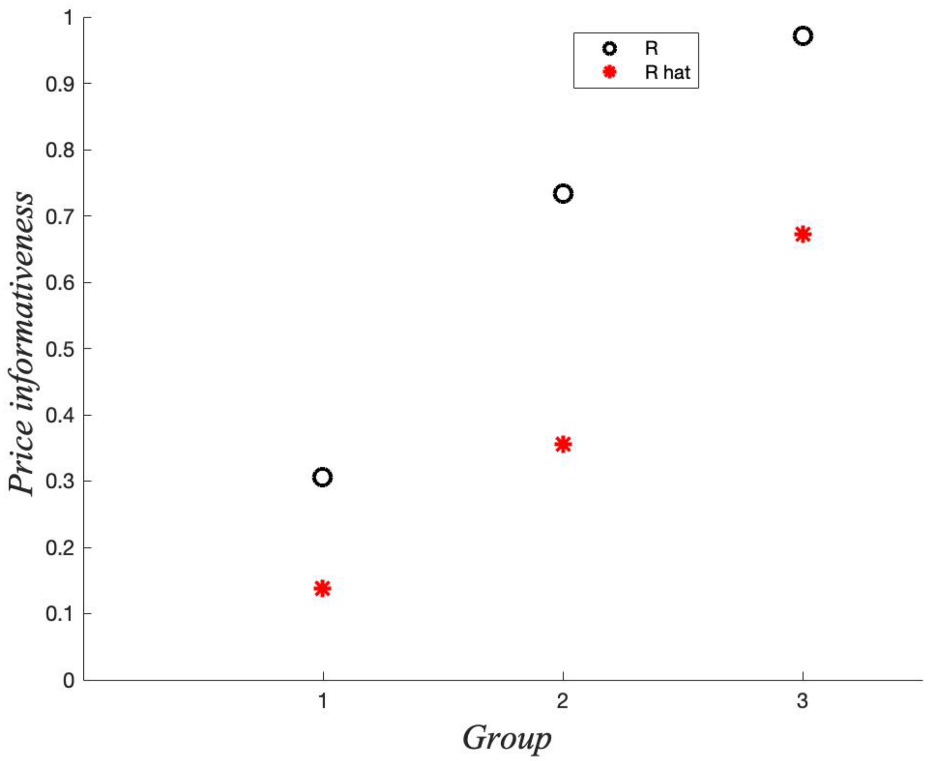
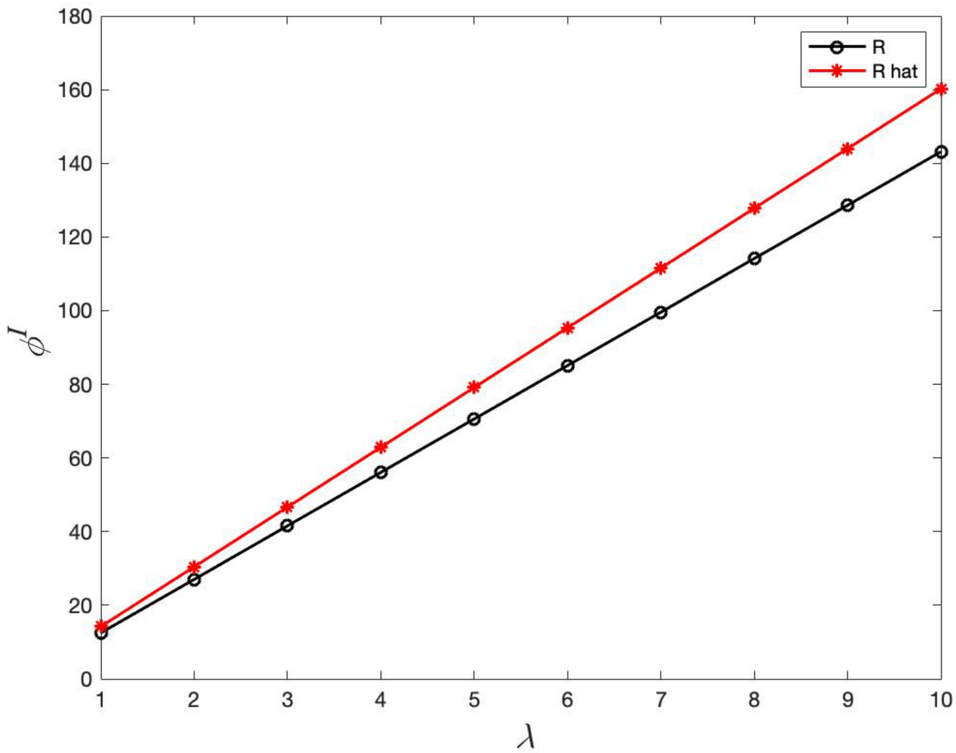


Figure A2: Expected ϕ^I for the economy parametrized by $(\lambda\eta_I, \lambda\eta_U)$, $\lambda \geq 1$



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