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Information, market power and welfare ☆

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Abstract

We study the market for a risky asset in which traders are heterogeneous both in terms of their value for the asset and the information that they have about this value. Traders behave strategically and use the equilibrium price to extract information that is relevant to them. Due to adverse selection, uninformed traders are less willing than the informed to provide liquidity. We evaluate the impact of a change in the size or composition of the investor population on price informativeness, liquidity and welfare, with applications to the rise of passive investing and the adoption of ESG standards.
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1. Introduction

A number of recent developments in financial markets have been driven by major shifts in the size or composition of the investor population. Examples include the rise of passive investment strategies at the expense of active research-led investment, the increasing importance of non-financial considerations such as ESG (environmental, social and governance) criteria, and trading driven by sentiment on social media.[1] These developments involve large traders with significant price impact, and pronounced changes not only in the mix of informed and uninformed traders, but also in the relative proportions of investors with different trading motives.

What are the implications of these changes for market quality, in particular for liquidity, price informativeness and the welfare of market participants? In order to address these questions, we study a model in which the investor base is heterogeneous in terms of both information and trading motives. The diversity in trading motives in turn invites a closer investigation of the price discovery role of financial markets, wherein prices convey information not only about fundamentals but also about other variables of interest (such as ESG performance).[2]

Our model features a single risky asset. There are several groups of agents distinguished by their value for the asset. For group i, this value is a random variable θ_i. Some agents in group i are privately informed; these agents know θ_i. The remaining agents in group i have no private information and rely on the price to infer information about θ_i. The values $\{\theta_i\}$ are joint normally distributed, with an arbitrary pattern of correlation.

Heterogeneity in values can be due to different hedging or liquidity needs, or because of differing investment opportunities or goals.[3] We interpret θ_1 as the monetary payoff of the risky asset. Thus agents in group 1 are pure speculators who care only about this payoff. Groups $i \neq 1$ can be thought of as investors who combine the asset with other trading, investment or structuring activities which result in a value that differs from θ_1. Alternatively, these agents may have objectives that lead them to sacrifice financial returns, for example due to ESG concerns.

All agents are rational and strategic; there are no noise traders. The trading protocol is a uniform-price double auction. The auction takes the form of a demand submission game wherein each agent submits a demand schedule, which is a function of his private signal (if he has one) and the price. Agents take account of their price impact and incorporate the informational content of the price into their bids. We study linear Bayesian Nash equilibria of this demand submission game.

[1] The Financial Times (FT, 2022) reports that passive fund ownership of US stocks overtook active ownership for the first time in 2021. According to Bloomberg (2021), ESG assets are on track to exceed \$50 trillion by 2025, representing more than a third of projected total global assets under management. Riedl and Smeets (2017) and Barber et al. (2021) provide evidence of ESG investors pursuing non-financial goals. On social media driven trading see, for example, Aloosh et al. (2023).

[2] While the vast literature on price informativeness in financial markets has focused on informativeness about future cash flows, a number of empirical studies present evidence of asset prices conveying other types of information, such as hedging activity (de Roon et al., 2000, Kang et al., 2020) or ESG performance (Ng and Rezaee, 2020). Huij et al. (2023) suggest how investors can use a firm's stock price to infer private information about its exposure to climate-related risks.

[3] Vives (2011) discusses heterogeneous values in Treasury and electricity auctions. In Rostek and Weretka (2012, 2015) values depend on group affiliations or on the geographic location of traders. In Friedman and Heinle (2016) and Goldstein et al. (2022) investors with ESG concerns have a different value for the asset than traditional investors. Rahi and Zigrand (2018) show how diversity in values can be microfounded by adding hedgers to a model along the lines of Grossman and Stiglitz (1980) or Hellwig (1980). Rahi (2021) provides examples of interdependent values in a production economy with uncertain cost or demand. Glebkin and Kuong (2023) show how differences in trading speed can account for heterogeneous values.

The equilibrium price is a linear combination of all the values $\{\theta_i\}$, and hence does not fully reveal any individual θ_i. Indeed, price informativeness is typically not the same for different groups. For any group, a lower price is an indicator of lower value, and this effect is stronger if prices are more informative. Thus higher price informativeness for group i is associated with greater adverse selection for this group, which manifests itself in the form of greater bid shading and more inelastic demand functions relative to other groups.

We analyze the effect of market size, as measured by the number of traders, on information aggregation and competitiveness. First, price informativeness (for any group) does not depend on the size of the economy per se, but on the relative number of informed agents in different groups. If we increase the number of agents in all groups without changing their relative proportions, price informativeness is unaffected. Second, price-taking behavior is obtained if there is a large number of agents, but this does not require a large number in all groups. What is crucial is that there are many informed agents in at least one group. A large number of uninformed agents does not result in price-taking behavior. This is because any liquidity that uninformed agents provide is limited by adverse selection.

One aspect of convergence to competitive equilibrium as the number of informed agents goes to infinity is that market depth goes to infinity (or price impact goes to zero). Under further assumptions, we show that depth and welfare go up monotonically as the number of informed agents goes up in all groups in the same proportion. But neither depth nor welfare is monotone in the number of informed agents in a given group. It is possible for an increase in the number of informed speculators to make all agents worse off.

With imperfect competition, agents not only have price impact but this impact is greater for informed agents, due to adverse selection which impedes liquidity provision by the uninformed. The adverse selection effect can be so strong that informed agents are worse off relative to the uninformed even if information is costless.

We close the paper with a discussion of the developments in financial markets that we mentioned in the first paragraph. In our model, a rise in passive investment reduces price informativeness about fundamentals. It also reduces market depth if non-fundamental trading is not too high. It is nevertheless possible for an increase in passive investment to be Pareto improving. An increase in the proportion of ESG investors leads to prices that convey less information about future cash flows and more about ESG performance. The impact on market liquidity depends on the number of informed ESG investors. Liquidity goes down when this number is low, and goes up when the number is high. Once there is a sufficiently high number of informed ESG investors, any further increase is Pareto improving. Finally, an influx of sentiment traders, as in the recent meme stock frenzy, leaves price informativeness unchanged. We cite empirical evidence in support of some of these results in Section 8.

In order for a model to serve as a useful vehicle for assessing the effect of passive investing or ESG investing on price discovery and liquidity (where liquidity is measured by price impact), it must have the following elements: (a) imperfect competition, (b) agents who differ in the quality of their information (for passive investing), and (c) agents who differ in the value that they derive from holding the asset (for ESG investing). Our model has all of these ingredients. As we explain below, there is no other model in the literature that is sufficiently flexible or tractable for this purpose.

Before discussing the literature in detail, we would like to emphasize two points of a general nature that are highlighted by our paper.

First, how the "noise" in prices is modeled is important. Our model, in which the noise arises from diverse motivations for trade, has very different comparative statics properties from a conventional noise trader model. Our view is that heterogeneous values are a more natural way of underpinning a partially revealing equilibrium than exogenous noise trade. Even though values are exogenous in our model, trades are not, and this is what accounts for the difference in our results. At any rate, our analysis shows that the usual assumption of noise trade is not an innocuous modeling device; it has material consequences for results that inform our understanding of price informativeness and liquidity in asset markets.

Second, a signal for one agent can be noise for another. Thus there should be no presumption that an increase in the number of informed agents improves price discovery. In our model the opposing forces of signal and noise are exactly offsetting: a proportional increase in the number of agents in all groups leaves price informativeness unchanged.

Related literature: The papers that are most closely related to ours are Kyle (1989), Vives (2011) and Rostek and Weretka (2012, 2015). We discuss these in turn and explain why our model produces different results, and how it is better equipped to handle the applications we have in mind.

Kyle (1989) studies an imperfectly competitive economy in which informed and uninformed traders compete in demand functions, with noise traders to prevent prices from being fully revealing. The analog of Kyle's model in our setting is as follows. Suppose there are two groups, with values θ_1 and θ_2, where θ_1 is the monetary payoff of the asset while θ_2 incorporates a liquidity or hedging motive. There are both informed and uninformed agents in group 1; these are the speculators in Kyle's model. All agents in group 2 are informed (they know θ_2); these agents, who we will refer to as hedgers, play the role of noise traders except that their trades are based on optimizing behavior.

Kyle finds that the information content of prices is lower with imperfect competition, and is increasing in the number of uninformed agents. In contrast, in our model, price informativeness does not depend on the degree of competition or the number of uninformed traders. Kyle's results rely crucially on the assumption of exogenous noise trade.[4] When he compares an imperfectly competitive economy to the corresponding competitive economy, the noise trade is assumed to be the same in both economies. In our setting, the ability to trade without price impact applies to all traders, including the hedgers who inject noise into prices from the perspective of the speculators. In Kyle's model, more uninformed traders make the market more liquid, thereby stimulating informed trade and making prices more informative. This is not the case in our model. While an increase in the number of uninformed speculators may increase liquidity, this is exploited by both informed speculators and hedgers, so that there is no impact on price informativeness.

In terms of our applications, ESG investing cannot be handled by a common values model such as that of Kyle (1989). In principle, the Kyle (1989) model can be brought to bear on the issue of passive investing, but it does not yield any comparative statics results for depth or welfare. Our model is more tractable than Kyle's since informed traders in our model are perfectly informed, while those in Kyle's model are not. We achieve this tractability even though we eschew the simplicity of a noise trader setting and instead allow for multiple groups of agents with their own value for the asset. We discuss this further in Section 2.2.

Vives (2011) introduces heterogeneous values in the double auction setting of Kyle (1989), dispensing with the need for noise traders. The correlation between values is the same for any pair of bidders. Equilibrium is "privately revealing" in the sense that, for any agent, the price together with his own private information is a sufficient statistic for the information of all agents.

[4] This also applies to results that build on Kyle (1989), such as those in García and Sangiorgi (2011).

Price informativeness is increasing in the number of agents. We will compare this result to ours when we discuss the Rostek and Weretka (2012, 2015) model below.

Vives (2011) finds that, in an imperfectly competitive economy, adverse selection increases illiquidity as measured by price impact. We expand on this theme by showing that the illiquidity effect is stronger for those agents who learn less from the price. It is most pronounced for informed agents (who do not learn from the price), thereby reducing their incentive to acquire information in the first place. Due to the private revelation property, the price-taking equilibrium in Vives' model is ex post efficient. This is not the case in our setting; our welfare analysis involves a comparison of second-best outcomes.

Rostek and Weretka (2012, 2015) extend the Vives (2011) model to allow for a more general correlation of values. Prices are not privately revealing as in Vives (2011). In the baseline case in which the average correlation between values is independent of market size, they show that prices convey more information in a larger economy.[5] This is a generalization of the price informativeness result of Vives (2011) mentioned above, and stands in contrast to our result that price informativeness is independent of market size. The difference in results is due to specific assumptions and a different notion of market size. Our model features multiple agents who have the same value for the asset, while in Vives (2011) and Rostek and Weretka (2012, 2015), or V-R-W for short, each agent has his own idiosyncratic value. An increase in market size in our model involves a higher number of agents in all groups (where agents in a given group have the same value for the asset), keeping the number of groups fixed. In V-R-W, there is only one agent in each group, and a bigger economy consists of more groups. We discuss this point in greater detail in Section 5.

In V-R-W the analysis is restricted to a symmetric setting in which all agents submit the same demand function (i.e. with the same weight on their private signal and on the price). In our model, strategies vary across agents depending on their information, including the information that they glean from prices (which differs across agents). This allows us to study the impact of increased market participation by a subset of agents. For example, we find that the liquidity provided by uninformed agents is limited by adverse selection to a greater degree than the liquidity provided by informed agents. The heterogeneity of strategies in our model also opens up the possibility of applications that involve a change in the composition of the investor population, as in passive investing and ESG investing. The V-R-W model cannot be used for such applications.[6]

There is a large literature on strategic foundations for competitive rational expectations equilibrium. Notable contributions in an auction setting with interdependent values include Milgrom (1981), Pesendorfer and Swinkels (2000) and Reny and Perry (2006). These papers restrict bids to a single unit. Closer to our paper is the literature on divisible-good auctions with interdependent values. Limit results can be found in Vives (2011, 2014) and Rostek and Weretka (2012, 2015), where convergence to competitive equilibrium is obtained as the number of agents grows without bound, and in Kyle and Lee (2022), in which convergence requires that the speculative

[5] In the Rostek and Weretka (2012, 2015) model, price informativeness, depth and welfare can change in an essentially arbitrary way, depending on how the average correlation between values changes with market size. The case in which the average correlation is constant provides the clearest point of comparison with our paper.

[6] A few papers study variants of the Vives (2011) model that feature some heterogeneity in bidder strategies, but in settings that are different from ours. In Glebkin and Kuong (2023) there are two groups, one of which consists of price-taking agents. Manzano and Vives (2021) have two groups that differ in the precision of agents' private signals; the equilibrium is privately revealing as in Vives (2011). In Rostek and Yoon (2019) correlations between values can be arbitrary, as in our model, but all agents are assumed to have private information of the same precision.

motive become negligible as well.[7] Our convergence results differ from these, in particular with regard to price informativeness. Unlike the papers cited in this paragraph, we do not impose any restrictions on the correlation matrix of bidder values.

Rostek and Yoon (2020) provide a general overview of the literature on uniform-price double auctions in a linear-normal setting. A number of papers study competitive equilibria with interdependent values, sidestepping the difficulties that arise when agents have market power and act strategically: Vives (2014) studies a perfectly competitive version of Vives (2011); Rahi and Zigrand (2018) and Rahi (2021) analyze learning externalities in information production.[8]

2. The model

There is a single risky asset in zero net supply. There are several groups of agents distinguished by their value for the asset, which is random ex ante. We use L to denote both the number of groups and the index set for these groups. The asset value for an agent in group $i \in L$ is θ_i. Agents in group i may be informed or uninformed; an informed agent privately observes θ_i. Thus agents who belong to group i share the same value for the asset but do not have the same information about this value; they either know the realization of θ_i or they have no information about it. Agents in group j, $j \neq i$, have a different value for the asset, θ_j. Just like agents in group i, group j agents are either informed (they know the realization of θ_j) or are uninformed (they have no information about θ_j). We interpret θ_1 as the monetary payoff of the asset, so that agents in group 1 are pure speculators. This is only for purposes of interpretation, however, and has no bearing on the formal analysis.

The random vector $\theta := (\theta_i)_{i \in L}$ is normally distributed with zero mean, and $\mathrm{Var}(\theta_i)$ is the same for all i. We denote by R the correlation matrix of θ, with ij'th element $\rho_{ij} := \mathrm{corr}(\theta_i, \theta_j)$; in the two-group case we drop the subscripts and write ρ_{12} simply as ρ. We assume that R is positive definite.

The payoff of an agent in group i is $W_i := (\theta_i - p)q - (k/2)q^2$, where p is the asset price, and q is the number of units of the asset bought by the agent. The scalar k is positive and can be interpreted as an inventory or holding cost parameter, or proxy for risk aversion.

The equilibrium price is determined in a trading game as follows. Each agent submits a demand function that is linear in his private signal (if he has one) and in the price,[9] whereupon the "auctioneer" finds a price at which excess demand is zero, and allocates to each agent the quantity demanded by him at that price. If there are multiple market-clearing prices, the price with the lowest absolute value is chosen (the positive value in case of ties). If there is no market-clearing price, no trade takes place.[10]

[7] In the noise trader setting of Kyle (1989), there is convergence to competitive equilibrium if the noise trade grows fast enough as the number of informed speculators goes to infinity. Refinements of this result can be found in García and Urošević (2013) and Kovalenkov and Vives (2014).

[8] Other papers in which agents have interdependent values and equilibrium prices convey information include Bergemann et al. (2021) and Heumann (2021), who introduce multidimensional signals into the Vives (2011) model, Yoon (2019), in which agents choose between an OTC market and an exchange, Babus and Kondor (2018), in which dealers engage in bilateral trading on a network, Bernhardt and Taub (2015) on learning about common and private values in a duopoly, and Du and Zhu (2017) on the optimal frequency of trading. Kyle et al. (2018) discuss the similarities between a model with interdependent values and one with overconfident traders who agree to disagree.

[9] As in Kyle (1989), we can consider strategies that are more general but lead to the same linear equilibrium.

[10] These assumptions ensure that the demand submission game is well-defined. Similar assumptions have been employed in the literature; see Kyle (1989) and Vayanos (1999).

For group $i \in L$, the number of informed traders is N_i^I and the number of uninformed traders is N_i^U, with $N_i^I + N_i^U \geq 1$ for all i. Let $L_I := \{i \in L | N_i^I \geq 1\}$ and $L_U := \{i \in L | N_i^U \geq 1\}$. Thus L_I indexes the groups that have at least one informed agent, and L_U indexes those that have at least one uninformed agent. We will use the notation L_I and L_U to also denote the cardinality of the sets L_I and L_U, respectively. We assume that $2 \leq L_I \leq L$. We put no restriction on L_U; thus $0 \leq L_U \leq L$. We denote the total number of informed and uninformed agents in the economy by N^I and N^U, respectively, i.e. $N^I := \sum_{i \in L} N_i^I$ and $N^U := \sum_{i \in L} N_i^U$. It will sometimes be convenient to use the shorthand notation η_I for the vector $(N_i^I)_{i \in L}$ and η_U for the vector $(N_i^U)_{i \in L}$. All vectors are column vectors by default. We assume that $R_i^\top \eta_I \geq 0$ for all i.[11] As we shall see, this is equivalent to $\mathrm{Cov}(\theta_i, p) \geq 0$ for all i.

In our model, price informativeness is group-specific. The price is fully revealing for group i if θ_i can be inferred from the price. Our assumption that $L_I \geq 2$ (there are at least two informed agents with different values for the asset) ensures that the price is not fully revealing for any group.

For random variables x and y, we denote the covariance of x and y by σ_{xy}, and the variance of x by σ_x^2. Given our assumption that the variance of θ_i is the same for all i, we write $\mathrm{Var}(\theta_i)$ as σ_θ^2.

2.1. Discussion of the assumptions

We assume that (a) agents differ in their value for the asset, (b) there are, in general, multiple agents who share the same value, and (c) agents are either fully informed or completely uninformed about their value.

Heterogeneity in values is necessary for trade under asymmetric information. Most of the literature assumes that there are only two values, the monetary payoff of the asset and a value that is not explicitly modeled but is designed to capture all other motives for trade (lumped together as “noise trade”). We allow an arbitrary number of values, from which we derive the optimal trades of all agents. Values other than monetary payoff can be due to hedging motives or ESG concerns; see the Introduction (including footnotes 1, 2 and 3) for an extended discussion of this.

While an explicit microfoundation for values other than monetary payoff is clearly desirable, a model with exogenously specified values has the advantage of tractability. Even though values are exogenous in our model, trades are not. All agents respond to endogenous variables such as prices and liquidity, affecting these in turn. This leads to results that differ in important ways from those in the noise trader setting of Kyle (1989).

That many traders share the same value is natural when we think of groups such as pure speculators, hedgers engaged in the same production activity, or investors pursuing common social goals. The assumption of full or no information allows us to build a tractable framework in which the information of agents is heterogeneous. Many of our results extend to a more general setting in which agents have either a precise, but not perfect, signal or a coarse signal about their value for the asset (see Section 9).

Our preference assumptions (risk-neutrality with a quadratic holding cost) deliver a simple quadratic objective function. This specification is increasingly common in the literature; see, for

[11] Essentially this assumption requires that the correlations $\{\rho_{ij}\}$ are not too negative. For example, if $L = L_I = 2$, it is equivalent to the condition that $\rho \geq -\min\{N_2^I/N_1^I, N_1^I/N_2^I\}$.

example, Vives (2011, 2014), Rostek and Weretka (2012, 2015), Yoon (2019), Bergemann et al. (2021), Chen and Duffie (2021), Heumann (2021), Manzano and Vives (2021), and Glebkin and Kuong (2023). All our results, with the exception of Proposition 6.2, extend to a setting with negative exponential utility (constant absolute risk aversion), with only minor modifications. Example 6.1 generalizes to this setting as well. See Section A6 of the Online Appendix for details.

2.2. *Relationship to the literature*

In order to see more formally how our model fits into the literature, it is useful to describe the following "super-model".[12] Suppose there are L groups of agents. The value for the asset is θ_i for agents in group i. This group has N_i^I informed agents and N_i^U uninformed agents. We use the symbols L, N_i^I and N_i^U to also denote the corresponding index set; thus $i \in L$ is group i, and $n \in N_i^I$ is an informed agent in group i. Agent $n \in N_i^I$ observes a private signal of the form $\theta_i + \epsilon_{in}$, where ϵ_{in} is idiosyncratic noise independent of all other random variables. The precision of this information is the same for all informed agents: $\mathrm{Var}(\epsilon_{in}) = \sigma_\epsilon^2$ for all $n \in N_i^I$, $i \in L$. Uninformed agents have no private information. In addition to informed and uninformed agents in each of the L groups, there is exogenous noise trade with variance σ_ξ^2. All random variables are joint normally distributed. We assume that $\mathrm{Var}(\theta_i) = \sigma_\theta^2$ for all $i \in L$.

The Kyle (1989) model is obtained by setting $L = 1$ (common values). The Rostek and Weretka (2012, 2015) model is obtained by setting $N_i^I = 1$, $N_i^U = 0$, and $\sigma_\xi^2 = 0$ (one informed trader in each group, no uninformed traders, no noise trade). They also impose a restriction on the correlations between values (the "equicommonality" assumption). Our model is obtained by setting $\sigma_\epsilon^2 = \sigma_\xi^2 = 0$ (the informed traders have perfect information and there is no noise trade). Our model can be thought of as combining the heterogeneous information setting of Kyle (1989) (agents differ in the precision of their information) with the heterogeneous values setting of Rostek and Weretka (2012, 2015).

While the "super-model" in its full generality is not tractable, the special cases described in the previous paragraph are. In the case of our model, the restriction that $\sigma_\xi^2 = 0$ is immaterial since noise trade serves no purpose in a setting with heterogeneous values. However, given that we do have heterogeneous values, if we allow σ_ϵ^2 to be positive, the model becomes too complex to yield any useful results. Conversely, due to the restriction that $\sigma_\epsilon^2 = 0$, our model is analytically simpler than that of Kyle (1989), in spite of the additional complication of heterogeneous values. For example, we obtain an explicit solution for price informativeness and, under the assumption of free entry of uninformed speculators, a closed-form solution for all equilibrium variables. There is no closed-form solution for any equilibrium variable in the Kyle (1989) model, even when there is free entry.[13]

There is one other tractable case that we discuss in Section 5, and analyze in detail in Section A4 of the Online Appendix, where we set $N_i^I = N$ and $N_i^U = 0$ for all $i \in L$. This is a generalization of the Rostek and Weretka (2012, 2015) model to allow for multiple informed traders who have the same value for the asset.

[12] Since the goal here is to compare our main model with other models in the literature, we describe a simple "super-model" that does not include the extension in Section 9 as a special case.

[13] In the model with free entry in Kyle (1989), market depth and equilibrium utilities depend on a price informativeness parameter φ_I, which solves a fifth-degree polynomial equation (equation (65) in Kyle's paper).

3. Equilibrium

We denote the demand functions of informed and uninformed agents in group i by $q_i^I(p,\theta_i)$ and $q_i^U(p)$, respectively. Given our linearity assumption, these functions take the form

$$\begin{aligned} q_i^I(p,\theta_i) &= \mu_i\theta_i - \alpha_i^I p, \qquad i \in L_I, \\ q_i^U(p) &= -\alpha_i^U p, \qquad i \in L_U, \end{aligned} \tag{1}$$

for some scalars μ_i, α_i^I and α_i^U. Hence, aggregate demand $D(p,\theta)$ is given by

$$\begin{aligned} D(p,\theta) &= \sum_{i\in L_I} N_i^I q_i^I(p,\theta_i) + \sum_{i\in L_U} N_i^U q_i^U(p) \\ &= \sum_{i\in L_I} N_i^I(\mu_i\theta_i - \alpha_i^I p) - \sum_{i\in L_U} N_i^U \alpha_i^U p \\ &= \sum_{i\in L_I} N_i^I \mu_i\theta_i - \Phi p, \end{aligned} \tag{2}$$

where

$$\Phi := \sum_{i\in L_I} N_i^I \alpha_i^I + \sum_{i\in L_U} N_i^U \alpha_i^U. \tag{3}$$

We assume that $\Phi > 0$; we will verify below that this assumption is always satisfied. Thus Φ is the absolute value of the slope of the aggregate demand function (for any given θ). The market-clearing condition is $D(p,\theta) = 0$. If there is an exogenous market order z, the market-clearing condition becomes $\sum_{i\in L_I} N_i^I \mu_i\theta_i - \Phi p + z = 0$, so that $\Phi = (\partial p/\partial z)^{-1}$. Thus we can interpret Φ as the overall depth of the market, which we will call *market depth*.

Agents behave strategically, taking into account the impact of their bids on the equilibrium price. Given the market-clearing condition, an informed agent in group i understands that if he buys q units of the asset, the equilibrium price is determined by the equation $q + D(p,\theta) - (\mu_i\theta_i - \alpha_i^I p) = 0$. Hence, from (2), the inverse demand function p_i^I that this agent faces is

$$p_i^I(q) = (\phi_i^I)^{-1} q + (\phi_i^I)^{-1}\left[\sum_{j\in L_I} N_j^I \mu_j\theta_j - \mu_i\theta_i\right], \tag{4}$$

where

$$\phi_i^I = \Phi - \alpha_i^I. \tag{5}$$

Similarly, the inverse demand function that an uninformed agent in group i faces is obtained from the equation $q + D(p,\theta) + \alpha_i^U p = 0$. It is given by

$$p_i^U(q) = (\phi_i^U)^{-1} q + (\phi_i^U)^{-1} \sum_{i\in L_I} N_i^I \mu_j\theta_i, \tag{6}$$

where

$$\phi_i^U = \Phi - \alpha_i^U. \tag{7}$$

Thus $(\phi_i^I)^{-1}$ and $(\phi_i^U)^{-1}$ are the price impact parameters, and ϕ_i^I and ϕ_i^U the corresponding depth parameters, for informed and uninformed agents in group i, respectively. Depth differs across agents depending on their group and on whether they are informed or not. This is because the residual demand function that an agent faces depends on his own contribution to net aggregate demand. We will restrict attention to equilibria at which the depth parameters of informed and uninformed agents in every group are strictly positive.[14]

Definition 3.1 *(Equilibrium).* A profile of demand schedules $\{\{q_i^I\}_{i\in L_I}, \{q_i^U\}_{i\in L_U}\}$, and price p, is a Bayesian Nash equilibrium of the trading game if $q_i^I(p,\theta_i)$ maximizes

$$\mathbb{E}(W_i|\theta_i, p) = \left[\theta_i - p_i^I(q)\right]q - \frac{k}{2}q^2, \tag{8}$$

$q_i^U(p)$ maximizes

$$\mathbb{E}(W_i|p) = \left[\mathbb{E}(\theta_i|p) - p_i^U(q)\right]q - \frac{k}{2}q^2, \tag{9}$$

and the market clears:

$$\sum_{i\in L_I} N_i^I q_i^I(p,\theta_i) + \sum_{i\in L_U} N_i^U q_i^U(p) = 0.$$

An equilibrium can be described in terms of slope and depth parameters, $\{\mu_i, \alpha_i^I, \phi_i^I\}_{i\in L_I}$ and $\{\alpha_i^U, \phi_i^U\}_{i\in L_U}$. These parameters pin down agents' demand functions, the equilibrium price function, and market depth. As we shall see below, the slopes for informed agents reduce to a single parameter α^I, with $\mu_i = \alpha_i^I = \alpha^I$ for all $i \in L_I$. This is because informed agents have the same holding cost k and do not learn from prices. The symmetry in slopes is reflected in depths as well: $\phi_i^I = \phi^I$ for all $i \in L_I$. Uninformed agents, on the other hand, do learn from prices and the amount they learn differs across groups. As a result, their slopes and depths are group-dependent.[15]

We begin by characterizing demands and prices for given depths ϕ^I and $\{\phi_i^U\}_{i\in L_U}$. All proofs are in Appendix A.

Proposition 3.1 *(Demands and prices for given depths). The depth parameters for informed agents are the same for all groups: $\phi_i^I = \phi^I$ for all $i \in L_I$. Given ϕ^I and $\{\phi_i^U\}_{i\in L_U}$, agents' demand functions are*

$$q_i^I = \alpha^I(\theta_i - p), \qquad i \in L_I, \tag{10}$$

$$q_i^U = \frac{\phi_i^U}{k\phi_i^U + 1}\left[\mathbb{E}(\theta_i|p) - p\right] = -\alpha_i^U p, \qquad i \in L_U, \tag{11}$$

where

$$\alpha^I = \Phi - \phi^I = \frac{\phi^I}{k\phi^I + 1}, \tag{12}$$

$$\alpha_i^U = \Phi - \phi_i^U \tag{13}$$

[14] In fact, depths must be strictly positive at any equilibrium at which the trades of all agents are finite.

[15] For comparison, there are two depth parameters in the Kyle (1989) model, one for the informed and one for the uninformed, while in the Rostek and Weretka (2012, 2015) model there is a single depth parameter that applies to all agents.

$$= \frac{\phi_i^U}{k\phi_i^U + 1}\left[1 - \frac{\sigma_{\theta_i p}}{\sigma_p^2}\right] \tag{14}$$

$$= \frac{\phi_i^U}{k\phi_i^U + 1}\left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k\phi^I + 2)\right]. \tag{15}$$

The price function is given by

$$p = (k\phi^I + 2)^{-1}\eta_I^\top \theta, \tag{16}$$

and market depth is

$$\Phi = \phi^I \frac{k\phi^I + 2}{k\phi^I + 1}. \tag{17}$$

The price function takes a very simple form. It depends only on the number of informed agents across groups, $\{N_i^I\}_{i\in L}$, and on ϕ^I, the depth parameter for informed agents. The depth parameters for uninformed agents, $\{\phi_i^U\}_{i\in L_U}$, do not explicitly appear in the price function. Market depth Φ is positive since ϕ^I is positive, and is increasing in ϕ^I.

Proposition 3.1 gives us prices and demand functions in terms of depths. In order to complete our equilibrium characterization, we need to calculate the depths. Substituting for α^I and Φ in (3) gives us

$$\sum_{i\in L_U} N_i^U \alpha_i^U = \frac{\phi^I}{k\phi^I + 1}\big[(k\phi^I + 2) - N^I\big]. \tag{18}$$

From (13), (15), (17) and (18), we obtain the following system of equations:

$$\sum_{i\in L_U} N_i^U \left[\phi^I \frac{k\phi^I + 2}{k\phi^I + 1} - \phi_i^U\right] = \frac{\phi^I}{k\phi^I + 1}\big[(k\phi^I + 2) - N^I\big], \tag{19}$$

$$\phi^I \frac{k\phi^I + 2}{k\phi^I + 1} = \phi_i^U \left[\frac{1 - \frac{R_i^\top \eta_I}{\eta_I^\top R\eta_I}(k\phi^I + 2)}{k\phi_i^U + 1} + 1\right], \qquad i \in L_U. \tag{20}$$

An equilibrium can be described in reduced form as a vector of depths $(\phi^I, \phi_1^U, \dots, \phi_{L_U}^U) \in \mathbb{R}_{++}^{L_U+1}$ that solves (19) and (20). This equation system has a simple solution if there are no uninformed traders (the set L_U is empty) and $N^I \geq 3$. Then we have $k\phi^I + 2 = N^I$ and hence $p = (N^I)^{-1}\eta_I^\top \theta$, from (16). In general, however, there is no closed-form solution.

Proposition 3.2 *(Equilibrium existence). For any given distribution of agents $\{N_i^I, N_i^U\}_{i\in L}$ satisfying $N^I \geq 3$ and $R_i^\top \eta_I / \eta_I^\top R\eta_I \leq 1/2$ for all $i \in L_U$, there exists a unique equilibrium. It is completely characterized by ϕ^I; for $i \in L_U$, $\phi_i^U = g_i(\phi^I)$, where g_i is a strictly increasing function.*

Given the characteristics of the economy, described by the correlation matrix R and the distribution of agents across groups $\{N_i^I, N_i^U\}_{i\in L}$, there exists a positive solution ϕ^I to the equation system (19)–(20). The value of ϕ^I in turn pins down $\phi_i^U = g_i(\phi^I)$ for all $i \in L_U$, and also market depth Φ, from (17). Uniqueness of equilibrium follows from the specification of the trading game. If there are multiple solutions for ϕ^I, the trading game stipulates that the highest solution be chosen, since this corresponds to the price with the lowest absolute value (due to (16); see also footnote 10). It also corresponds to the highest level of ϕ_i^U, for each $i \in L_U$, and of Φ. We provide an algorithm for calculating ϕ^I in Section A8 of the Online Appendix.

Proposition 3.2 requires that $R_i^\top \eta_I / \eta_I^\top R \eta_I \leq 1/2$ for all $i \in L_U$. Sufficient conditions for this to hold can be deduced from the following lemma:

Lemma 3.3. *Suppose one of the following conditions is satisfied: (i) $N_i^I \geq 2$; (ii) $N_i^I \geq 1$ and $R \geq 0$; or (iii) $\rho_{ij} = \rho$ for all $i \neq j$. Then $R_i^\top \eta_I / \eta_I^\top R \eta_I \leq 1/2$.*

4. Adverse selection, liquidity and bid shading

In this section we study the connection between learning from prices and adverse selection. Adverse selection in turn impacts liquidity and bid shading. Our model provides an explicit link between adverse selection and price informativeness, which differs across agents.

We use the terms liquidity and depth interchangeably. It will be clear from the context if we are referring to depth for a specific agent (e.g. ϕ^I for informed agents or ϕ_i^U for uninformed agents in group i) or for the market as a whole (as measured by Φ). We also speak more informally of the elasticity of an agent's demand function as a measure of his willingness to provide liquidity. In our linear setting, bid shading by an agent means that his demand function is less elastic than in a perfectly liquid market with no informational frictions.

Uninformed agents make inferences from the price about their value for the asset. We use the following measure of *price informativeness* for group i:

$$\mathcal{V}_i := \frac{\mathrm{Var}(\theta_i) - \mathrm{Var}(\theta_i | p)}{\mathrm{Var}(\theta_i)}. \tag{21}$$

Since $N_i^I \geq 1$ for at least two groups, it follows from the price function (16) that $\mathcal{V}_i \in [0, 1)$; prices are partially revealing for each group. In the next proposition we collect some results about price informativeness from Rahi and Zigrand (2018). We say that $A \propto B$ if A and B have the same sign ($A = cB$, for some $c > 0$).

Proposition 4.1 *(Price informativeness). Given $\eta_I := (N_i^I)_{i \in L}$, price informativeness for group i is*

$$\mathcal{V}_i = \frac{(R_i^\top \eta_I)^2}{\eta_I^\top R \eta_I}. \tag{22}$$

Furthermore,

$$\frac{\partial \mathcal{V}_i}{\partial N_i^I} \propto R_i^\top \eta_I. \tag{23}$$

Given our assumption that $R_i^\top \eta_I \geq 0$ for all i, price informativeness for each group is increasing in the number of informed agents in that group. Price informativeness does not depend on the number of uninformed agents in any group. Moreover, $\mathcal{V}_i$ is homogeneous of degree zero in η_I; if we scale the number of informed agents up or down, keeping fixed their relative proportions across groups, price informativeness is unaffected. We will revisit these properties in the context of the literature in our discussion of Proposition 5.2.

Now we provide a detailed characterization of the depths ϕ^I and $\{\phi_i^U\}_{i\in L_U}$, and the slopes α^I and $\{\alpha_i^U\}_{i\in L_U}$, and relate them to price informativeness. Since depths and slopes are two sides of the same coin, due to the relations $\alpha^I = \Phi - \phi^I$ and $\alpha_i^U = \Phi - \phi_i^U$ (equations (12) and (13)), we state our results on both and then discuss them together.

Proposition 4.2 *(Depths). The depth parameters ϕ^I and $\{\phi_i^U\}_{i\in L_U}$ satisfy the following properties:*

i. $\phi_i^U \ge \phi^I$ *for all* $i \in L_U$, *and* $\phi_i^U = \phi^I$ *if and only if* $\mathcal{V}_i = 0$.
ii. $\phi_i^U = \phi_j^U$ *if and only if* $\mathcal{V}_i = \mathcal{V}_j$, *and* $\phi_i^U > \phi_j^U$ *if and only if* $\mathcal{V}_i > \mathcal{V}_j$.
iii. *If* $N^U = 0$, *then* $k\phi^I + 2 = N^I$. *If* $N^U \ge 1$, *then* $k\phi^I + 2 \le N^I + N^U$, *with equality if and only if* $\mathcal{V}_i = 0$ *for all* $i \in L_U$.

Proposition 4.3 *(Slopes). The slope parameters α^I and $\{\alpha_i^U\}_{i\in L_U}$ satisfy the following properties:*

i. $\alpha^I > 0$.
ii. $\alpha_i^U \le \alpha^I$ *for all* $i \in L_U$, *and* $\alpha_i^U = \alpha^I$ *if and only if* $\mathcal{V}_i = 0$.
iii. $\alpha_i^U = \alpha_j^U$ *if and only if* $\mathcal{V}_i = \mathcal{V}_j$, *and* $\alpha_i^U < \alpha_j^U$ *if and only if* $\mathcal{V}_i > \mathcal{V}_j$.
iv. *Suppose* $L_U = L$. *Then* $\alpha_i^U = 0$ *for all* i *if and only if* $\mathcal{V}_i = \mathcal{V}_j$ *for all* i, j.
v. *Suppose* $L_I = L_U = L$, *and* $\mathcal{V}_i \ne \mathcal{V}_j$ *for some* i, j. *Then* $\min_{i\in L} \alpha_i^U < 0 < \max_{i\in L} \alpha_i^U$.
vi. $\alpha_i^U > 0$ *if*

$$\frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(N^I + N^U) < 1. \tag{24}$$

vii. *Suppose* $L_U = \{1\}$. *Then* $\alpha_1^U > 0$ *if and only if* $\frac{R_1^\top \eta_I}{\eta_I^\top R \eta_I} N^I < 1$.

In order to interpret these results, it is useful to compare the economy to one in which agents are "naive" in the sense that they ignore the information contained in prices. If an uninformed agent in group i is naive, he behaves as though $\mathcal{V}_i = 0$. The following observation is immediate from Propositions 4.2 and 4.3, and equation (12).

Lemma 4.4 *(Naive economy). If all uninformed agents are naive, $k\phi^I + 2 = N^I + N^U$. Furthermore, $\phi_i^U = \phi^I$ and $\alpha_i^U = \alpha^I = \phi^I/(k\phi^I + 1)$, for all $i \in L_U$.*

In an economy with naive agents the slope and depth parameters are the same for all groups and the same for informed and uninformed agents. Note that $\alpha^I < \lim_{\phi^I\to\infty} \alpha^I = k^{-1}$, i.e. the common slope parameter is lower than what would arise in a perfectly liquid naive economy (we will discuss competitive equilibrium in detail later; see Proposition 5.1). Thus there is some bid shading in a naive economy due to imperfect competition, but none due to adverse selection.

Now we ask what happens when we introduce adverse selection through learning from prices. All informed agents have the same (positive) slope parameter α^I, and the corresponding depth parameter ϕ^I. But these parameters are lower than in the economy with naive agents; from Proposition 4.2 (iii), $k\phi^I + 2 < N^I + N^U$ if prices are informative for at least one group. The additional

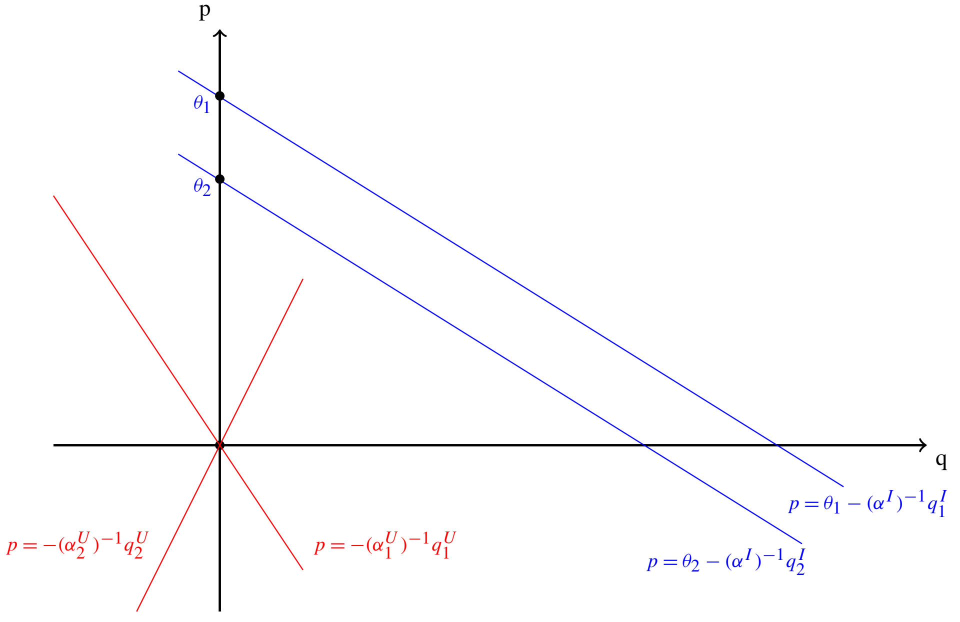

Fig. 1. Inverse demand functions.

bid shading by informed agents, beyond that in the naive economy, is due to lower liquidity provision by uninformed agents. For an uninformed agent, a lower price is bad news about his value for the asset (since $\mathrm{Cov}(\theta_i, p) \propto R_i^\top \eta_I \geq 0$ for all i). Learning from prices induces him to reduce his quantity response to a lower price. Indeed, this learning effect can be so strong that he buys less when the price falls. Thus adverse selection induces uninformed agents to shade their bids, the more so the more they learn from prices. This in turn implies that they provide less liquidity to informed agents, so the latter have greater price impact (ϕ^I is lower).

In Fig. 1, we show inverse demand functions for the case of two groups, with a nonzero number of informed and uninformed agents in both groups (see (10) and (11)). In a naive economy, the blue curves are flatter and the red curves (that pass through the origin) are parallel to the blue curves. When uninformed agents learn from prices, demand becomes more inelastic for all agents but more so for the uninformed. If price informativeness is the same for both groups, we have $\alpha_1^U = \alpha_2^U = 0$, and demands are perfectly inelastic for all uninformed agents. If price informativeness differs for the two groups, the demand curve of the less informed group is downward sloping while that of the more informed group is upward sloping.

More generally, suppose there are L groups and $L_I = L_U = L$. Then the following statements are equivalent: (a) $\mathcal{V}_i > \mathcal{V}_j$, (b) $\phi_i^U > \phi_j^U$, and (c) $\alpha_i^U < \alpha_j^U$. Among uninformed agents, those who learn the least from prices have the most elastic demand and contribute the most to liquidity provision. The ones who learn the most have an upward sloping demand curve; these agents use up liquidity instead of providing it. Agents who have the most elastic demand are also those whose counterparties have less elastic, or even upward sloping, demands. As a result, any deviation by the former from their equilibrium demand at any given price requires a greater price adjustment in order for the market to absorb it. Thus uninformed agents for whom price informativeness is the lowest, by virtue of having the most elastic demands also have the greatest price impact, or lowest depth. Conversely, agents who learn the most from prices have upward sloping demands and the least price impact, or highest depth.

If $L_I = L_U = L$, uninformed agents in at least one group have upward sloping demand functions because it is not possible to bound price informativeness for all groups. Price informativeness for group i depends on the relative weight of θ_i in the price function. This relative weight cannot be low for all i. Condition (24) for the demand function of the uninformed in group i to be downward sloping cannot be satisfied for all i (multiplying both sides of (24) by N_i^I and adding up, we get $N^I + N^U < N^I$, a contradiction). However, if the condition $L_I = L_U = L$ does not hold, it is possible that all demands are downward sloping. For ease of interpretation, suppose $\rho_{ij} = \rho$ for all $i \neq j$. Then (24) becomes $N_i^I < (N^I + N^U)^{-1} \sum_{j \in L} (N_j^I)^2$, which is satisfied for all $i \in L_U$ if $\{N_i^I\}_{i \in L_U}$ are small relative to $\{N_j^I\}_{j \notin L_U}$, and N^U is also small relative to N^I.

In part (vii) of Proposition 4.3, we consider the case where there are uninformed agents in group 1 only. Given our interpretation of θ_1 as the monetary value of the asset, agents in group 1 are pure speculators, who can be informed or uninformed, while agents in other groups, all of whom are informed about their own value, are "noise traders" from the perspective of the speculators. If $\rho_{ij} = \rho$ for all $i \neq j$, the uninformed have downward sloping demands if and only if $N_1^I < (N^I)^{-1} \sum_{i \in L} (N_i^I)^2$; in the two-group case, this condition is simply $N_1^I < N_2^I$. In other words, the demand functions of uninformed speculators are downward sloping if the number of informed speculators is low relative to the number of "noise traders".

From the expression for α_i^U given by (14), we see that the parameter that measures adverse selection for uninformed agents in group i is the regression coefficient of θ_i on p, given by

$$\beta_i := \frac{\sigma_{\theta_i p}}{\sigma_p^2} = \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} (k\phi^I + 2). \tag{25}$$

We refer to β_i as the *price sensitivity* for group i. At a given equilibrium, $\beta_i > \beta_j$ if and only if $\mathcal{V}_i > \mathcal{V}_j$. This allows us to rank depths and slopes by price informativeness.[16] It is worth emphasizing that bid shading and price impact for an agent in group i depend on the sensitivity of θ_i to p, not on how much he knows about θ_i. If $\mathcal{V}_i = 0$, $\alpha_i^U = \alpha^I$ and $\phi_i^U = \phi^I$; an uninformed agent who learns nothing shades his bid no more than an informed agent, and has the same price impact. On the other hand, if $\mathcal{V}_i$ is close to one, an uninformed agent shades his bid more, and has a lower price impact, than an informed agent, even though the two agents have almost the same information in equilibrium.

We have used an economy with naive agents as a benchmark for our economy. Another instructive benchmark is the full-information economy in which all agents in group i observe θ_i, for all i. Let $N := N^I + N^U$ be the total number of agents, informed or uninformed. If all agents are informed ($N^I = N$), then $k\phi^I + 2 = N$, by Proposition 4.2 (iii). If some of the N agents are uninformed but naive, we again have $k\phi^I + 2 = N$, by Lemma 4.4. On the other hand, if there are uninformed agents who extract information from prices, we have $k\phi^I + 2 < N$ (using Proposition 4.2 (iii) once again). These observations imply that ϕ^I, and therefore market depth Φ, is the same in the full-information economy and the naive economy, but is lower if there are some agents who are rational and uninformed. Learning from prices leads to more bid shading,

[16] From (14), α_i^U is the product of two terms: $(1 - \beta_i)$ which captures the direct learning effect, and $\phi_i^U/(k\phi_i^U + 1)$ which reflects the effect of learning on depth. If $\mathcal{V}_i > \mathcal{V}_j$, then the direct learning term is lower for i while the depth term is higher. The former dominates, so that $\alpha_i^U < \alpha_j^U$.

and hence lower market depth, than in an economy with no informational frictions. It is in this sense that adverse selection impacts liquidity in our setting.

5. Market size, price informativeness and liquidity

In this section we study the impact of an increase in market size, as measured by the number of agents, on price informativeness and liquidity. Some of the results are as one would expect. A bigger economy is more liquid and converges to a (perfectly) competitive limit as the number of agents grows without bound. Other results differ from those in the literature. A proportional increase in the number of agents in all groups leaves price informativeness unchanged, which is in contrast to the results in Vives (2011) and Rostek and Weretka (2012, 2015). We trace this discrepancy to different assumptions, and highlight the more general takeaway that private information can manifest itself as a signal or as noise in the price, so that more information does not imply higher price informativeness in general. Our price informativeness results differ from those in Kyle (1989) as well, and we attribute this to the assumption of noise trade in Kyle's model.

We show that the driver of higher liquidity and convergence to competitive equilibrium is a larger population of informed agents in one or more groups. The liquidity provided by uninformed agents is constrained by adverse selection and the economy remains imperfectly competitive even when there is an unbounded number of uninformed agents.

Our competitive benchmark is the economy described in Section 2 but with a continuum of informed and uninformed agents in each group, reinterpreting N_i^I and N_i^U as the mass (rather than the number) of informed and uninformed agents in group i. Thus each agent has zero price impact, or equivalently all individual depths are infinite. We begin by characterizing the equilibrium of a competitive economy, denoting the price function by $\hat{p}$ and the slope parameters by $\hat{\alpha}^I$ and $\{\hat{\alpha}_i^U\}_{i\in L_U}$ in order to distinguish them from the price function and slope parameters of the imperfectly competitive economy discussed so far.

Proposition 5.1 *(Competitive equilibrium). In a competitive economy with the mass of agents given by $\{N_i^I, N_i^U\}_{i\in L}$, the price function is*

$$\hat{p} = \gamma^{-1}\eta_I^\top \theta, \tag{26}$$

where

$$\gamma := \frac{N^I + N^U}{1 + \sum_{i\in L_U} N_i^U \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}}, \tag{27}$$

and the slope parameters are

$$\hat{\alpha}^I = k^{-1}, \tag{28}$$

$$\hat{\alpha}_i^U = k^{-1}\left[1 - \frac{\sigma_{\theta_i \hat{p}}}{\sigma_{\hat{p}}^2}\right] \tag{29}$$

$$= k^{-1}\left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}\gamma\right], \qquad i \in L_U. \tag{30}$$

The slope parameters satisfy all the properties in Proposition 4.3.

Comparing (28) with (29), we see that uninformed agents shade their bids when they learn from prices. This is due to adverse selection just as in the imperfectly competitive case. We provide a fuller discussion of this point after Proposition 7.2.

Recall that $\eta_I := (N_i^I)_{i\in L}$ and $\eta_U := (N_i^U)_{i\in L}$. It will be convenient to refer to the equilibrium of the imperfectly competitive economy and the equilibrium of the corresponding competitive economy by $\mathcal{E}(\eta_I, \eta_U)$ and $\hat{\mathcal{E}}(\eta_I, \eta_U)$, respectively, where

$$\mathcal{E}(\eta_I, \eta_U) := \Big(p, (\mathcal{V}_i)_{i\in L}, \alpha^I, \phi^I, (\alpha_i^U, \phi_i^U)_{i\in L_U}\Big),$$
$$\hat{\mathcal{E}}(\eta_I, \eta_U) := \Big(\hat{p}, (\mathcal{V}_i)_{i\in L}, \hat{\alpha}^I, \hat{\phi}^I, (\hat{\alpha}_i^U, \hat{\phi}_i^U)_{i\in L_U}\Big).$$

For notational convenience, we suppress the dependence of the parameters describing an equilibrium on (η_I, η_U). The parameters $\hat{p}, \hat{\alpha}^I$, and $\{\hat{\alpha}_i^U\}_{i\in L_U}$ are given by Proposition 5.1, while $\hat{\phi}^I = \infty$, and $\hat{\phi}_i^U = \infty$ for all $i \in L_U$.

Note that price informativeness for each group is the same for the two economies; even though p and $\hat{p}$ are not equal, they are both proportional to $\eta_I^\top \theta$ (compare (16) and (26)). This is in contrast to the result in Kyle (1989) that prices reveal less information when competition is imperfect. The reason is that in Kyle (1989), informed speculators trade less aggressively in the imperfectly competitive economy, while the noise trade is the same by assumption. In our setting, price impact leads to less aggressive trading by informed agents in all groups, speculators as well as agents who trade for other reasons.

If $N^U \geq 1$, we define $\phi^U := (N^U)^{-1} \sum_{i\in L_U} N_i^U \phi_i^U$; thus ϕ^U is the (weighted) average depth parameter for uninformed agents. We parametrize the economy by a scalar $\lambda \geq 1$, which scales the number of agents.

Proposition 5.2 *(Convergence I). The equilibrium $\mathcal{E}$ converges to $\hat{\mathcal{E}}$ as the number of agents increases in fixed proportion:* $\lim_{\lambda\to\infty} \mathcal{E}(\lambda\eta_I, \lambda\eta_U) = \hat{\mathcal{E}}(\eta_I, \eta_U)$. *Price informativeness does not depend on λ, and ϕ^I and ϕ^U are strictly increasing in λ.*

We interpret the original economy as one for which $\lambda = 1$. As we increase λ, the number of informed and uninformed agents in each group goes up, but their relative proportions remain the same. The proposition says that the limiting equilibrium is competitive. Depths go up as the economy grows in size but price informativeness remains unchanged (the latter is immediate from (22)).

It is instructive to compare the price informativeness result in Proposition 5.2 to those in Vives (2011) and Rostek and Weretka (2012, 2015), or V-R-W for short. In the V-R-W model, in the baseline case in which the average correlation between values does not change with market size, prices are more informative in a larger economy. The divergent results on price informativeness are due to three key differences between the V-R-W setting and ours. First, agents in our model have either complete or no information about their value for the asset, while agents in V-R-W have noisy information of the same precision. Second, we have multiple agents in each group (N_i^I informed agents and N_i^U uninformed agents in group i) while V-R-W have a single agent per group. Third, an increase in market size in our model means scaling up the number of agents in each group, while keeping the number of groups L fixed, while in V-R-W market size is measured by L.

In order to gain more insight into how the relationship between market size and price discovery depends on the informational environment, we study an extension of the V-R-W model in Section A4 of the Online Appendix, where we allow for N agents in each group; the V-R-W model is obtained by setting $N = 1$. If $N \geq 2$, we find that price informativeness can go up or down with market size, and this is true regardless of how market size is measured, by N or by L. The reason is that a larger pool of signals, while potentially more informative, also adds to the noise in the price. When $N \geq 2$, an agent in group i is primarily interested in inferring the signals of other agents in his own group, because these agents have information about his value for the asset. Adding more agents to other groups clouds this information.

The general point is that, from the perspective of a given agent in the economy, private information of other agents can enter into prices in a way that contributes to or impedes price discovery. In our model, price informativeness for group i is increasing in the number of informed agents in group i, but this effect is offset by the "noise" in prices resulting from a higher number of informed agents in other groups. If the economy is scaled up, the opposing forces of "signal" and "noise" are exactly offsetting and price informativeness is unaffected. When signals are noisy, the same forces are at play, but the overall effect depends on the parameters.

Next, we show that the market becomes perfectly liquid, or infinitely deep, for all agents even if we fix the number of uninformed agents in each group, increasing (in fixed proportion across groups) only the number of informed agents. In fact, the market becomes perfectly liquid even if we only let the number of informed agents in a single group go to infinity, keeping fixed not only the number of uninformed agents in each group, but also the number of informed agents in all other groups. The limiting equilibrium in these cases is not the competitive equilibrium described in Proposition 5.1. Rather, it coincides with the corresponding limit of the competitive equilibrium.

Proposition 5.3 *(Convergence II). We have the following convergence results:*

i. $\lim_{\lambda\to\infty} \mathcal{E}(\lambda\eta_I, \eta_U) = \lim_{\lambda\to\infty} \hat{\mathcal{E}}(\lambda\eta_I, \eta_U)$. *Price informativeness does not depend on* λ, *and* ϕ^I *and* ϕ^U *are strictly increasing in* λ.
ii. *Suppose* $R_\ell \geq 0$. *Then,* $\lim_{N_\ell^I\to\infty} \mathcal{E}(\eta_I, \eta_U) = \lim_{N_\ell^I\to\infty} \hat{\mathcal{E}}(\eta_I, \eta_U)$.[17]

While price impact goes to zero for all agents when the number of informed agents (in any group) goes to infinity, this is not the case when the number of uninformed agents becomes large. When we consider the effect of a change in N_i^U on ϕ^I and α_i^U, we write $\phi^I(N_i^U)$ and $\alpha_i^U(\phi^I(N_i^U))$ to make this dependence explicit. Note that α_i^U depends on ϕ^I but not directly on N_i^U.

Proposition 5.4 *(Uninformed trades). Suppose* $N_i^I \geq 2$ *and* $R_i^\top \eta_I > 0$ *for all* i. *Then:*

i. *There exist strictly positive scalars* $\underline{\kappa}$ *and* $\bar{\kappa}$ *such that* $\{\phi^I, \phi_1^U, \ldots, \phi_{L_U}^U\} \subset [\underline{\kappa}, \bar{\kappa}]$ *for all* $(N_1^U, \ldots, N_{L_U}^U) \in \mathbb{R}_+^{L_U}$.
ii. $\phi^I(N_i^U) - \phi^I(\breve{N}_i^U) \propto \alpha_i^U(\phi^I(\breve{N}_i^U))$, *for all* $N_i^U > \breve{N}_i^U \geq 1$.
iii. $\lim_{N_i^U\to\infty} \alpha_i^U = 0$ *and* $\lim_{N_i^U\to\infty} N_i^U \alpha_i^U < \infty$.

17 We need to assume that $R_\ell \geq 0$, otherwise our standing assumption that $R_i^\top \eta_I \geq 0$ for all i will be violated for large N_ℓ^I.

Thus the depth parameter ϕ^I is a bounded function of N_i^U for all i, and it is also bounded away from zero. These properties are inherited by $\{\phi_i^U\}_{i\in L_U}$, as well as by market depth Φ, as these are pinned down by ϕ^I. Any change in ϕ^I is accompanied by a change in $\{\phi_i^U\}_{i\in L_U}$ and Φ in the same direction. If α_i^U is (initially) positive, entry of uninformed agents into group i improves market liquidity, but since α_i^U converges to zero, the market remains illiquid to some degree (all the depth parameters are bounded) even when entry of these agents is unrestricted. If α_i^U is (initially) negative, greater market participation by uninformed agents in group i lowers market liquidity; these agents absorb liquidity rather than providing it. In Fig. 1, as N_i^U increases, the inverse demand functions of uninformed agents in group i eventually become steeper, converging to the vertical axis as N_i^U goes to infinity. The relative positions of the inverse demand functions for uninformed agents in different groups are the same for all N_i^U. This is because the slope parameters $\{\alpha_j^U\}_{j\in L_U}$ are ranked by price informativeness (Proposition 4.3 (iii)), which does not depend on N_i^U.

The last observation about price informativeness being invariant with respect to N_i^U highlights the point that we made earlier about adverse selection being measured by price sensitivity, given by (25). An increase in N_i^U leaves price informativeness $\mathcal{V}_i$ unchanged, but it does affect price sensitivity β_i through its effect on ϕ^I. If $\alpha_i^U > 0$, ϕ^I goes up, increasing β_i and driving α_i^U to zero as N_i^U grows without bound. While liquidity improves, it is limited by adverse selection. If $\alpha_i^U < 0$, an increase in N_i^U reduces liquidity due to the upward sloping demands of these agents, even though this is offset to some extent by a reduction in β_i.

Our results on the impact of uninformed trade on depth and price informativeness differ markedly from those in the Kyle (1989) model. In Kyle (1989), prices become more informative as the number of uninformed agents increases. This is because of higher depth, to which informed speculators respond by trading more aggressively, while the noise trade is the same by assumption. In our model, an increase in the number of uninformed traders does not necessarily raise market depth. Moreover, even in the case where depth goes up, all agents trade more aggressively, not just informed speculators but also agents with other trading motives. Consequently, price informativeness is unaffected.

The novel feature of our model that allows us to develop the insights in this section is heterogeneity in both values and private information. In the Kyle (1989) model there is heterogeneity in private information but in a common values setting with noise trade. As we have seen, exogenous noise trade is not a mechanical modeling device; it leads to fundamentally different conclusions about the impact on price discovery of imperfect competition or of uninformed trade. The interdependent values setting of Rostek and Weretka (2012, 2015) does away with the need for noise trade, but it assumes that there is only one agent in each group, and that all agents have information of the same precision. The additional flexibility of our model, with many asymmetrically informed agents in each group, allows us to unlock new results, as exemplified by Propositions 5.3 and 5.4.

6. Welfare

In this section we lay the groundwork for our welfare analysis. A key finding is that the utility differential between informed and uninformed agents is lower with imperfect competition, and can even be negative. This is due to adverse selection, which manifests itself as lower liquidity for the informed. That informed agents can be worse off compared to the uninformed in the same economy is a novel result.

We can calculate ex ante utilities by plugging the demand function of each agent into his objective function (given by (8) or (9)). In order to interpret the resulting expressions some definitions will be useful. As in Rahi (2021), we define the *gains from trade* for group i by

$$G_i := \frac{\sigma^2_{\theta_i - p}}{\sigma^2_\theta}. \tag{31}$$

Agents in group i have more profitable trading opportunities the greater the distance between their own value θ_i and the market value p. Indeed, if $p = \theta_i$, there are no gains from trade for these agents and their optimal trade is zero. We define the function $F : (0, \infty) \to (0, \infty)$ by

$$F(x) := \frac{x(kx+2)}{(kx+1)^2}. \tag{32}$$

It is easy to check that F is strictly increasing. We denote the ex ante utilities of informed and uninformed agents in group i by $\mathcal{U}_i^I$ and $\mathcal{U}_i^U$, respectively.

Lemma 6.1 *(Utilities). Ex ante utilities are given by*

$$\mathcal{U}_i^I = \frac{\sigma^2_\theta}{2} F(\phi^I) G_i, \qquad i \in L_I, \tag{33}$$

$$\mathcal{U}_i^U = \frac{\sigma^2_\theta}{2} F(\phi_i^U)\big[G_i - (1 - \mathcal{V}_i)\big], \qquad i \in L_U. \tag{34}$$

Note that $\phi_i^U \geq \phi^I$, and hence $F(\phi_i^U) \geq F(\phi^I)$, with equality if and only if $\mathcal{V}_i = 0$ (see Proposition 4.2 (i)). Comparing the utilities of informed and uninformed agents in the same group, we see that privileged information is a double-edged sword. If there is no information leakage ($\mathcal{V}_i = 0$), informed agents are unambiguously better off. If prices reveal some information, however, adverse selection kicks in and liquidity (as measured by depth) is lower for the informed. As we shall see below in Example 6.1, the adverse impact on liquidity can outweigh the informational advantage of informed agents so that they are worse off relative to the uninformed, even if information is costless.

We denote the *utility differential* between the informed and uninformed in group i by $\Delta\mathcal{U}_i := \mathcal{U}_i^I - \mathcal{U}_i^U$. Henceforth, when we refer to $\mathcal{U}_i^I$, $\mathcal{U}_j^U$ or $\Delta\mathcal{U}_\ell$, it is understood that $i \in L_I$, $j \in L_U$ and $\ell \in L_I \cap L_U$, respectively. We denote the corresponding competitive equilibrium variables with a "hat".

Proposition 6.2 *(Utility differentials). Utility differentials satisfy the following properties:*

i. $\Delta\mathcal{U}_i < \Delta\mathcal{U}_j$ *if and only if* $\mathcal{V}_i > \mathcal{V}_j$.
ii. *There exists a threshold level of price informativeness* $\mathcal{V}^*$ *such that* $\Delta\mathcal{U}_i > 0$ *if and only if* $\mathcal{V}_i < \mathcal{V}^*$.
iii. $\Delta\mathcal{U}_i < \Delta\hat{\mathcal{U}}_i = \sigma^2_\theta (2k)^{-1}(1 - \mathcal{V}_i)$.

Parts (i) and (ii) of Proposition 6.2 provide a comparison of utility differentials for different groups at a given equilibrium. The utility differential is lower for groups with higher price informativeness. In fact, informed agents are worse off relative to uninformed agents in the same group if price informativeness exceeds a certain threshold level. Part (iii) says that the utility differential is lower with imperfect competition than with perfect competition. This is because agents not only have price impact when the market is imperfectly competitive, but this impact is greater for informed agents. In a competitive economy, agents can trade in an infinitely deep market with no price impact, and information always has positive value (though this value declines with price informativeness).

Proposition 6.2 (ii), and our discussion of Lemma 6.1, suggest that informed agents can be worse off compared to the uninformed due to adverse selection. In the following example this is indeed the case.

Example 6.1. Suppose there are three groups, with $N_1^I \geq 2$, $N_2^I = N_3^I \geq 2$, $N_i^U \geq 1$ for all i, and

$$\theta_1 = \tilde{\theta}_1, \quad \theta_2 = a\tilde{\theta}_2 + \nu_1, \quad \theta_3 = -a\tilde{\theta}_2 + \nu_2,$$

where $\{\tilde{\theta}_1, \tilde{\theta}_2, \nu_1, \nu_2\}$ are mutually independent normal random variables with zero mean, $0 < a < 1$, and

$$\sigma^2_{\tilde{\theta}_1} = \sigma^2_{\tilde{\theta}_2} = 1, \quad \sigma^2_{\nu_1} = \sigma^2_{\nu_2} = \sigma^2_\nu, \quad a^2 + \sigma^2_\nu = 1.$$

Hence $\rho_{12} = \rho_{13} = 0$ and $\rho_{23} = -a^2$, so that $R_1^\top \eta_I = N_1^I$, and $R_2^\top \eta_I = R_3^\top \eta_I = N_2^I(1 - a^2)$. By Proposition 3.2, there exists a unique equilibrium (the assumption that $N_i^I \geq 2$ ensures that $R_i^\top \eta_I / \eta_I^\top R \eta_I \leq 1/2$, by Lemma 3.3).

We consider limits as σ^2_ν goes to zero, and hence a goes to 1. From (22), we see that

$$\mathcal{V}_1 = \frac{(R_1^\top \eta_I)^2}{\eta_I^\top R \eta_I} = \frac{(N_1^I)^2}{(N_1^I)^2 + 2(N_2^I)^2(1 - a^2)},$$

which converges to 1 as σ^2_ν goes to 0. Thus, in the limit, prices become perfectly informative for group 1. We claim that, for group 1, gains from trade do not vanish, and depth for the uninformed remains strictly higher than that for the informed:

Claim: (i) $\lim_{\sigma^2_\nu \to 0} G_1 > 0$; and (ii) $\lim_{\sigma^2_\nu \to 0} \phi_1^U > \lim_{\sigma^2_\nu \to 0} \phi^I$.

The proof of the claim is in Appendix A. Using the claim, it follows from Lemma 6.1 that $\mathcal{U}_1^I < \mathcal{U}_1^U$ for sufficiently small σ^2_ν. The informed have lower ex ante utility than the uninformed because the informed have greater price impact even as their informational advantage vanishes. ‖

The result that the informed can be disadvantaged relative to the uninformed should be distinguished from the Hirshleifer effect (Hirshleifer, 1971), which refers to a welfare loss due to more information. Here we are comparing the welfare of informed and uninformed agents at a given equilibrium rather than providing a comparative static with respect to information. Our result is due to an adverse depth effect; it does not arise in a competitive economy.

7. Free entry of uninformed speculators

In our model, price informativeness depends only on exogenous parameters (the correlation matrix R and the number of informed traders in each group), and all other equilibrium variables can be pinned down as functions of the depth parameter ϕ^I. However, in general, there is no closed-form solution for ϕ^I. In this section, we study a limiting case of our economy in which there is free entry of uninformed speculators. For this limit economy, there is an explicit solution for ϕ^I, and hence an explicit solution for all equilibrium variables. This enables us to derive comparative statics for depth and welfare which we use in the applications in Section 8. Unlike our economy with free entry, the one in Kyle (1989) does not admit a closed-form solution, even for price informativeness (see footnote 13).

It is natural to think of free entry of uninformed traders who are motivated only by speculation ("retail investors"), while traders of other types operate in a more specialized environment with barriers to entry. Given our interpretation of θ_1 as the monetary payoff of the asset, uninformed speculators are uninformed agents who belong to group 1.

Definition 7.1 *($\mathcal{F}_1$-economy).* Suppose $N_1^I \geq 2$ and $R_i^\top \eta_I > 0$ for all $i \in L_I$. Then we refer to the limiting economy as $N_1^U \to \infty$ as an $\mathcal{F}_1$-economy.

The symbol $\mathcal{F}$ serves as a mnemonic for "free entry", and the subscript 1 indicates that there is free entry of uninformed agents into group 1.

Lemma 7.1. *An $\mathcal{F}_1$-economy has a unique equilibrium with $\phi^I > 0$ and $\alpha_1^U = 0$. The equilibrium price is given by*

$$p = \mathbb{E}(\theta_1 | p) = \frac{R_1^\top \eta_I}{\eta_I^\top R \eta_I} \eta_I^\top \theta, \tag{35}$$

and ϕ^I solves

$$\beta_1 = \frac{R_1^\top \eta_I}{\eta_I^\top R \eta_I}(k\phi^I + 2) = 1. \tag{36}$$

Free entry of uninformed speculators wipes out their trading profits. In the limiting economy, $\alpha_1^U = 0$. Thus each uninformed speculator trades a zero amount and his equilibrium utility $\mathcal{U}_1^U$ is zero. From (11), the equilibrium price is given by $p = \mathbb{E}(\theta_1 | p)$, and hence the regression coefficient of θ_1 on p, which is equal to the price sensitivity β_1 defined in (25), is equal to 1. Equation (36) can be interpreted as a "zero-profit" condition.

Uninformed speculators enter the market as long as there are profits to be made. If $\beta_1 < 1$, then $\alpha_1^U > 0$ (see equation (15)), i.e. their demand functions are downward sloping. As more of them enter, they provide higher liquidity to other traders. If $\beta_1 > 1$, on the other hand, we have $\alpha_1^U < 0$. In this case, an increase in the number of uninformed speculators drains liquidity from the market. An equilibrium with free entry is established when ϕ^I satisfies (36). This equation gives us a simple solution for ϕ^I that, like price informativeness, depends only on R and η_I.

For an $\mathcal{F}_1$-economy, we can strengthen Proposition 5.2 on convergence to competitive equilibrium to monotone convergence (Proposition 5.2 shows monotone convergence for ϕ^I and $\phi^U := (N^U)^{-1} \sum_{i \in L_U} N_i^U \phi_i^U$, but not for the individual depths $\{\phi_i^U\}_{i \in L_U}$ or for agents' utilities). As in the general case, we distinguish variables for a competitive $\mathcal{F}_1$-economy with a "hat". The competitive depth parameters for all agents are equal to infinity.

Proposition 7.2 *(Monotone convergence). Consider an $\mathcal{F}_1$-economy parametrized by $\lambda \eta_I$, $\lambda \geq 1$. The price function $p(\lambda \eta_I)$ and the vector*

$$\Xi(\lambda \eta_I) := \Big(\alpha^I, \phi^I, (|\alpha_i^U|, \phi_i^U)_{i \in L_U}, (\mathcal{U}_i^I)_{i \in L_I}, (\mathcal{U}_i^U)_{i \in L_U}\Big)$$

do not depend on N_i^U, $i \neq 1$. *We have* $p(\lambda\eta_I) = p(\eta_I) = \hat{p}(\eta_I)$, $\Xi(\lambda\eta_I)$ *is increasing in* λ, *and* $\lim_{\lambda\to\infty} \Xi(\lambda\eta_I) = \hat{\Xi}(\eta_I)$.[18] *Furthermore, if* $N_1^I \geq 4$, *then* $\mathcal{U}_i^I(\lambda\eta_I) - \mathcal{U}_i^U(\lambda\eta_I)$ *is strictly increasing in* λ, *for all* $i \in L_I \cap L_U$.

The depth parameter ϕ^I does not depend on N_i^U, $i \neq 1$; this is a consequence of the zero-profit condition (36). Hence the same property holds for p and Ξ. If a change in N_i^U, $i \neq 1$, disturbs (36), the number of uninformed speculators adjusts until (36) is restored.

As the number of informed agents goes up in all groups in the same proportion, the price function remains unchanged, the individual depth parameters increase monotonically to infinity, and demand functions become more responsive to the price. Thus increased competition leads to lower price impact but has no effect on prices. All agents are better off as a result: utilities increase monotonically, converging to their competitive equilibrium values. The utility of the informed increases at a faster rate than that of the uninformed. These welfare effects are driven entirely by depth; since p does not depend on λ, gains from trade and price informativeness are not affected by market size.

An $\mathcal{F}_1$-economy provides a useful way to understand the connection between bid shading, adverse selection and liquidity. Consider first the case of perfect competition. Then agents can trade with no price impact and bid shading is purely a consequence of adverse selection. Comparing (28) and (29), uninformed agents shade their bids if they learn from prices. The magnitude of bid shading for group i is measured by $\sigma_{\theta_i \hat{p}}/\sigma_{\hat{p}}^2$. In the case of imperfect competition, we can compare (12) and (14). Since $p = \hat{p}$, we see that the adverse selection component of bid shading, which is measured by $\sigma_{\theta_i p}/\sigma_p^2$ for group i, is the same as in the corresponding competitive economy. But there is an additional effect due to price impact, for both informed and uninformed agents. This depth effect reduces α^I and $|\alpha_i^U|$ below their competitive levels.

While depth and welfare increase monotonically if the number of informed agents goes up in the same proportion for all groups, they are not in general monotone in the number of informed agents in a given group.

Proposition 7.3 *(Depth). We have the following results on the depth parameter* ϕ^I *of an* $\mathcal{F}_1$-*economy:*

i. ϕ^I *is strictly convex in* N_i^I *for all* $i \in L_I$.
ii. $\partial\phi^I/\partial N_1^I > 0$ *if and only if* $\mathcal{V}_1 > 1/2$.
iii. *Consider* $i \in L_I$, $i \neq 1$. *Suppose* $\rho_{mj} = \rho$ *for all* $m \neq j$, *and* $N_1^I + N_i^I \geq N_j^I$ *for all* $j \neq 1, i$. *Then* $\partial\phi^I/\partial N_i^I > 0$.[19]

The relationship between ϕ^I and N_1^I is U-shaped. For given $\{N_j^I\}_{j\in L}$, ϕ^I is determined by the zero-profit condition (36). A simple calculation shows that, keeping ϕ^I fixed, price sensitivity for uninformed speculators, β_1, is increasing in N_1^I if $\mathcal{V}_1 < 1/2$, and decreasing in N_1^I if $\mathcal{V}_1 > 1/2$. Hence, for low values of N_1^I, an increase in N_1^I leads to a higher β_1, inducing uninformed speculators to tilt their demand functions so that they are upward sloping ($\alpha_1^U < 0$; see

[18] If $\alpha_j^U = 0$ (this must be the case for $j = 1$), then $\alpha_j^U = \hat{\alpha}_j^U = 0$ and $\mathcal{U}_j^U = \hat{\mathcal{U}}_j^U = 0$ for all λ. Monotonicity in λ is strict for all other elements of Ξ.

[19] In the two-group case, $\partial\phi^I/\partial N_i^I > 0$ for $i \neq 1$, since the condition on N_i^I is trivially satisfied.

(15)). More uninformed speculators enter, reducing ϕ^I (due to their upward sloping demands; see Proposition 5.4 (ii)), and thereby restoring the zero-profit condition. For large values of N_1^I, an increase in N_1^I lowers β_1 so that α_1^U becomes positive, and entry of uninformed speculators raises ϕ^I. Thus the U-shaped relationship between ϕ^I and N_1^I is attributable to the hump-shaped relationship between adverse selection, as measured by β_1, and N_1^I.

Since market depth Φ is strictly increasing in ϕ^I (from (17)), the monotonicity results for ϕ^I in Proposition 7.3 also hold for Φ. From Lemma 6.1, welfare depends on depth and gains from trade. On the latter, we have the following result:

Lemma 7.4 *(Gains from trade). Consider an $\mathcal{F}_1$-economy. Suppose $\rho_{ij} = \rho$ for all $i \neq j$. Then*

$$G_i = (1 - \mathcal{V}_i) + \frac{(1-\rho)^2 (N_i^I - N_1^I)^2}{\eta_I^\top R \eta_I}. \tag{37}$$

In particular,

$$G_1 = 1 - \mathcal{V}_1. \tag{38}$$

From (31), G_i is a measure of the distance between θ_i and p. The first term of (37) indicates that this distance is inversely related to price informativeness. Indeed, it is intuitive to think of price informativeness for group i as being high when p is close to θ_i and hence G_i is low. This intuition is correct for group 1. For other groups, however, it is incomplete, and we need to take account of the second term in (37). This term captures the distance between θ_i and p in terms of the distance between N_i^I and N_1^I, the number N_1^I being key in determining the equilibrium price given by (35).

We now bring together our results on depth and gains from trade to show that an increase in the number of informed speculators, N_1^I, can make all agents worse off. The following proposition is for the case of two groups; under stronger assumptions, it can be generalized to arbitrarily many groups.

Proposition 7.5 *(Welfare). Consider an $\mathcal{F}_1$-economy with two groups. Suppose $\rho \leq 1/2$ and $N_1^I \leq N_2^I/3$. Then $\mathcal{U}_1^U = 0$ for all N_1^I, and the utility of all other agents is strictly decreasing in N_1^I.*

Under the conditions of the proposition, an increase in N_1^I leads to lower welfare for all agents (other than uninformed speculators, whose utility is zero for any level of N_1^I due to free entry). This is a consequence of lower depth as well as lower gains from trade. The depth effect comes from the downward sloping part of the U-shaped relationship between ϕ^I and N_1^I discussed earlier. Gains from trade for group 1 are lower because $G_1 = 1 - \mathcal{V}_1$ and $\mathcal{V}_1$ is increasing in N_1^I. For group 2, price informativeness falls as N_1^I increases but, for low values of N_1^I relative to N_2^I, gains from trade are nevertheless lower due to the second term in (37).[20]

[20] Glebkin and Kuong (2023) provide a welfare result that complements ours. In their model, one group consists of N "large" agents who know their value for the asset, while the other group consists of a continuum of "small" agents who have noisy information about their own value. They show that an increase in N can lower the sum of agents' utilities. This result requires some restrictions on the joint distribution of values and signals which do not hold in our model.

By combining the asymmetric information setting of Kyle (1989) with the heterogeneous values setting of Rostek and Weretka (2012, 2015), our model provides a framework for assessing the impact of higher market participation on liquidity and welfare. All agents have a well-defined objective function, so that a full welfare analysis can be carried out. Proposition 7.2 shows that a proportional increase in market size raises market liquidity and is Pareto improving. A higher number of informed agents in one group on the other hand, has an ambiguous effect on liquidity (Proposition 7.3), and can make all agents worse off (Proposition 7.5). This last result contributes to the ongoing policy debates on the social value of informed speculation.

8. Applications

We began this paper by identifying two important developments in the investment world in recent years, the rise of passive investment and the adoption of ESG standards. In this section we apply our model to these developments. The applications showcase the flexibility and tractability of our framework. While our findings on price informativeness have been partially anticipated in the literature, the results on liquidity and welfare are new.

Recall that we interpret θ_1 as the monetary payoff of the asset. An $\mathcal{F}_1$-economy is one in which there is free entry of uninformed speculators. The informativeness of the price about future cash flows is $\mathcal{V}_1$. Market depth is given by Φ.

The last three decades have seen a substantial increase in passive investment at the expense of active investment strategies (FT, 2022). In our model, this corresponds to investors in group 1 switching from informed to uninformed. Let M be the number of agents who make this switch.

Proposition 8.1 *(Passive investing). We have the following results on passive investing, which is measured by M:*

i. *Suppose $\mathcal{V}_1 > 0$. Then $\partial\mathcal{V}_1/\partial M < 0$.*
ii. *In an $\mathcal{F}_1$-economy, $\partial\Phi/\partial M < 0$ if and only if $\mathcal{V}_1 > 1/2$.*
iii. *Consider an $\mathcal{F}_1$-economy with two groups. Suppose $\rho \le 1/2$ and $N_1^I \le N_2^I/3$. Then an increase in M leads to a Pareto improvement.*

Part (i) of the proposition is immediate from Proposition 4.1. It says that an increase in passive investment lowers the informativeness of the price about future cash flows. This is consistent with the evidence presented in Sammon (2022).

Part (ii) follows from Proposition 7.3 (ii) and (17). It says that, in an economy with free entry of uninformed speculators, market depth falls as more speculators switch from informed to uninformed, provided non-fundamental trading is not too high ($\mathcal{V}_1 > 1/2$). This is consistent with the findings of Haddad et al. (2022), who document a 15% decrease in the elasticity of aggregate demand for individual stocks due to the rise of passive investing in the last 20 years.

Part (iii) states conditions under which an increase in passive investment is Pareto improving. It follows from Proposition 7.5 and the observation that, in an economy with free entry of uninformed speculators, an increase in M has the same effect as a decrease in N_1^I, the number of informed speculators.

In the Kyle (1989) model with free entry, price informativeness for informed speculators is increasing in the number of informed speculators, and hence decreasing in passive investment. However, Kyle's result requires the assumption of free entry, while our result does not; in fact, price informativeness in our model does not depend on the number of uninformed speculators. On depth and welfare, it is difficult to derive any comparative statics in the Kyle (1989) model, since there is no closed-form solution to equilibrium variables (see footnote 13).

For our second application, suppose there are only two kinds of investors, pure speculators (group 1) and ESG investors (group 2). The value of the asset for group 2, θ_2, incorporates a measure of ESG performance. This is how ESG concerns have been modeled in the literature, when ESG performance is unknown ex ante (see, for example, Friedman and Heinle (2016) and Goldstein et al. (2022)). In our model, some ESG investors are informed about θ_2; they conduct research on ESG performance, as well as on fundamentals, insofar as θ_1 and θ_2 are correlated. Other ESG investors are uninformed about θ_2. This is analogous to pure speculators being either informed or uninformed about the monetary payoff θ_1.

The observed phenomenon of many active funds adopting ESG standards (see, for example, BlackRock (2023)) corresponds in our model to some informed agents in group 1 switching to become informed agents in group 2. We denote the number of such agents by E.

Proposition 8.2 *(ESG investing). Suppose there are two groups and $\rho \geq 0$. Then we have*

i. $\partial\mathcal{V}_1/\partial E < 0$, *and* $\partial\mathcal{V}_2/\partial E > 0$.
ii. *In an $\mathcal{F}_1$-economy, $\partial\Phi/\partial E < 0$ if $N_2^I \leq N_1^I/3$, and $\partial\Phi/\partial E > 0$ if $N_2^I \geq N_1^I/2$.*
iii. *Consider an $\mathcal{F}_1$-economy. Suppose $2N_1^I/N_2^I \leq -1 + \sqrt{1 + 4(1-\rho)}$. Then an increase in E leads to a Pareto improvement.*

Thus our model predicts that an increase in ESG investment in an asset reduces how informative the asset price is about future cash flows, and increases how informative it is about ESG performance. This is simply a consequence of more informed trading based on ESG considerations, at the expense of informed trading arising from purely financial motives. In an economy with free entry of uninformed speculators, an increase in the number of informed ESG investors reduces market liquidity when the number of these investors is low, and increases market liquidity when their number is high. This is due to the U-shaped relationship between liquidity and informed speculation, with the downward sloping part arising when the speculative component of informed trading is relatively low (see Proposition 7.3). Finally, in an economy with free entry, an increase in the number of informed ESG investors makes all agents better off provided there is already a large number of these investors (for example, if $\rho \in [0, 1/4]$, the condition is $N_2^I \geq 2N_1^I$). Informed speculators are strictly better off when their number is depleted because of higher liquidity and lower leakage of information through prices, while uninformed speculators remain at zero utility due to free entry. ESG investors also benefit from higher liquidity.

Proposition 8.2 illustrates the usefulness of generalizing the Rostek and Weretka (2012, 2015) framework to allow for multiple agents who share the same value for the asset. The results on price informativeness also hold in a competitive setting, for example that of Rahi and Zigrand (2018) or Goldstein et al. (2022). But a competitive model cannot be used to draw any conclusions about price impact.

We can also use our model to shed light on the meme stock frenzy that started in January 2021. Suppose group 2 consists of "sentiment" traders, in contrast to the speculators in group 1 who care only about future cash flows. The remaining groups are motivated by considerations such as hedging or ESG performance. Influencers like Keith Gill in the GameStop saga correspond to investors who are informed about θ_2. Most sentiment traders are uninformed, however, and our model predicts that an increase in their number N_2^U has no effect on price informativeness for any group, including price informativeness about future cash flows (see Proposition 4.1), in line with the evidence provided by Aloosh et al. (2023).

The impact of an influx of uninformed investors on market depth depends in our model on whether these investors use or ignore the information contained prices. If they learn from prices, the depth effect is small (Proposition 5.4). If they do not, they can have a substantial impact; see our discussion of naive agents in Section 4. Ozik et al. (2021) document a sizable attenuation of stock market illiquidity in the US at the beginning of the Covid-19 pandemic due to a surge of retail investors. This is consistent in our model with these investors being uninformed but not extracting information from prices.

9. An extended model

For analytical tractability we assume in this paper that agents are either perfectly informed or completely uninformed about their value for the asset. In Section A7 of the Online Appendix, we extend the model to allow for traders who are either well-informed or poorly informed. Well-informed traders have precise, but not perfect, information about their value (precise enough to obviate any need to extract information from prices), while the signal of the poorly informed is coarser. Our main model can be seen as a limiting case of the extended model.

We show that most of the results of the paper continue to hold in this more general setting. In particular, poorly informed traders have more inelastic demand functions than the well-informed and have lower price impact. Convergence to competitive equilibrium obtains as the number of well-informed agents goes to infinity, but the contribution of poorly informed agents to liquidity remains limited as their number grows. Interestingly, the last result holds even if the information of the poorly informed is only slightly worse than that of the well-informed. The key here is that poorly informed agents learn from the price but the well-informed do not. As a result, the trades of poorly informed agents are constrained by adverse selection, while those of the well-informed are not.

Our results on price informativeness, on the other hand, do not carry over to the extended model. In particular, price informativeness is not invariant to the degree of competition. In our main model, the uninformed do not contribute to price discovery, but in the extended model, the poorly informed do. Moreover, since the poorly informed have lower price impact than the well-informed, they exert a disproportionately high influence on price informativeness. Of course, this wedge in price impact only arises with imperfect competition. We show that, if there are poorly informed agents in only one group, price informativeness for this group is higher in an imperfectly competitive economy compared to the corresponding perfectly competitive economy.

10. Conclusion

In this paper we study price discovery, liquidity and welfare in a fairly general model incorporating heterogeneity in both trading motives and information. Prices reflect information about multiple facets of the economic environment that are of interest to the diverse investor pool. Agents have rational expectations and behave strategically. The model is well-suited for evaluating the impact of the changing size and composition of the investor population in financial markets.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix A. Proofs

Proof of Proposition 3.1. Using (4) and (8), the first-order condition for an informed agent in group i is $\theta_i - p_i^I(q) - (\phi_i^I)^{-1}q - kq = 0$. The second-order condition, $k + 2(\phi_i^I)^{-1} > 0$, is satisfied. Noting that $p_i^I(q_i^I) = p$, we obtain the optimal portfolio:

$$q_i^I = \frac{\phi_i^I}{k\phi_i^I + 1}(\theta_i - p). \tag{39}$$

Comparing this expression for q_i^I with (1), we see that $\mu_i = \alpha_i^I = \phi_i^I/(k\phi_i^I + 1)$. From (5),

$$\Phi = \phi_i^I + \alpha_i^I = \phi_i^I + \frac{\phi_i^I}{k\phi_i^I + 1}.$$

Since the right-hand side of this equation is increasing in ϕ_i^I, and is equal to the same value Φ for all i, ϕ_i^I must be the same for all i, and so must α_i^I. Letting $\phi_i^I = \phi^I$ and $\alpha_i^I = \alpha^I$ for all $i \in L_I$ gives us (12), from which (17) also follows.

Similarly, using (6) and (9), we can derive the optimal portfolio for an uninformed agent in group i:

$$q_i^U = \frac{\phi_i^U}{k\phi_i^U + 1}\left[\mathbb{E}(\theta_i|p) - p\right] = -\frac{\phi_i^U}{k\phi_i^U + 1}\left[1 - \frac{\sigma_{\theta_i p}}{\sigma_p^2}\right]p, \tag{40}$$

thus establishing (14). Equation (13) follows from (7). Using the market-clearing condition $D(p,\theta) = 0$, and noting that $\mu_i = \alpha^I$, we have $p = \Phi^{-1}\alpha^I \sum_{i \in L_I} N_i^I \theta_i = \Phi^{-1}\alpha^I \eta_I^\top \theta$. The price function given by (16) now follows from (12) and (17). Using this price function, we have $\sigma_{\theta_i p}/\sigma_p^2 = (R_i^\top \eta_I / \eta_I^\top R \eta_I)(k\phi^I + 2)$. Substituting this expression into (14) gives us (15). □

Proof of Proposition 3.2. Equation (20) can be written as

$$k(\phi_i^U)^2 + b_i\phi_i^U - \phi^I \frac{k\phi^I + 2}{k\phi^I + 1} = 0, \tag{41}$$

where

$$b_i := 2 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k\phi^I + 2) - k\phi^I \frac{k\phi^I + 2}{k\phi^I + 1}, \tag{42}$$

for $i \in L_U$. Since ϕ^I and $\{\phi_i^U\}_{i \in L_U}$ must be strictly positive, the only admissible solution to (41) is

$$\phi_i^U = g_i(\phi^I) := \frac{-b_i + \sqrt{b_i^2 + 4k\phi^I \frac{k\phi^I+2}{k\phi^I+1}}}{2k}. \tag{43}$$

Substituting for ϕ_i^U in (19), we get an equation which involves only the variable ϕ^I:

$$f(\phi^I) := \frac{\phi^I}{k\phi^I + 1}\left[(k\phi^I + 2) - N^I\right] + \sum_{i\in L_U} N_i^U \left[g_i(\phi^I) - \phi^I \frac{k\phi^I + 2}{k\phi^I + 1}\right] = 0. \tag{44}$$

We have

$$g_i(0) = 0, \tag{45}$$

$$g_i'(0) = \left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}\right]^{-1}, \tag{46}$$

$$\lim_{\phi^I\to\infty} \frac{g_i(\phi^I)}{\phi^I} = 1 + \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}, \tag{47}$$

and consequently, $f(0) = 0$, and

$$f'(0) = -(N^I - 2) + \sum_{i\in L_U} N_i^U \left[\left(1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}\right)^{-1} - 2\right],$$

$$\lim_{\phi^I\to\infty} \frac{f(\phi^I)}{\phi^I} = 1 + \sum_{i\in L_U} N_i^U \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}.$$

Since $\lim_{\phi^I\to\infty} f(\phi^I)/\phi^I > 0$, we have $\lim_{\phi^I\to\infty} f(\phi^I) = \infty$. Moreover, since $R_i^\top \eta_I/\eta_I^\top R \eta_I \le 1/2$ for all $i \in L_U$, and $N^I \ge 3$,

$$f'(0) \le -(N^I - 2) + \sum_{i\in L_U} N_i^U \left[\left(1 - \frac{1}{2}\right)^{-1} - 2\right] = -(N^I - 2) < 0.$$

Therefore, by the continuity of f, there exists $\phi^I > 0$ such that $f(\phi^I) = 0$. Substituting this ϕ^I into (43), we get a positive solution $g_i(\phi^I)$ for ϕ_i^U, $i \in L_U$. It is apparent from (41) that g_i is strictly increasing in ϕ^I.

Uniqueness of equilibrium follows from the specification of the trading game. See the discussion following the statement of the proposition. □

Proof of Lemma 3.3. See the Online Appendix, Section A2. □

Proof of Proposition 4.2. *Proof of (i):* From (20),

$$\phi_i^U \frac{k\phi_i^U + 2}{k\phi_i^U + 1} - \phi^I \frac{k\phi^I + 2}{k\phi^I + 1} = \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k\phi^I + 2)\frac{\phi_i^U}{k\phi_i^U + 1}. \tag{48}$$

Since the left-hand side is nonnegative, and $x(kx + 2)/(kx + 1)$ is strictly increasing in x, the result follows.

Proof of (ii): We can rewrite (48) as

$$k\phi_i^U + 2 - \big[k + (\phi_i^U)^{-1}\big]\phi^I \frac{k\phi^I + 2}{k\phi^I + 1} = \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k\phi^I + 2).$$

The left-hand side of this equation is strictly increasing in ϕ_i^U, for given ϕ^I. The result follows.

Proof of (iii): If $N^U = 0$ ($L_U = \varnothing$), then $k\phi^I + 2 = N^I$ from (18). Now suppose $N^U \geq 1$ ($L_U \neq \varnothing$). From (12), (13), and part (i) of this proposition, we have $\alpha_i^U \leq \alpha^I$ for all $i \in L_U$, and $\alpha_i^U = \alpha^I$ if and only if $\mathcal{V}_i = 0$. Hence, from (12) and (18), $\alpha^I[(k\phi^I + 2) - N^I] = \sum_{i \in L_U} N_i^U \alpha_i^U \leq N^U \alpha^I$, with equality if and only if $\mathcal{V}_i = 0$ for all $i \in L_U$. This proves the result. □

Proof of Proposition 4.3. *Proof of (i):* This is immediate from (12).

Proof of (ii) and (iii): From (12) and (13), $\alpha^I - \alpha_i^U = \phi_i^U - \phi^I$. Hence, statements (ii) and (iii) are equivalent to statements (i) and (ii) of Proposition 4.2, respectively.

Proof of (iv): Suppose $L_U = L$. If $\alpha_i^U = 0$ for all i, then $R_i^\top \eta_I$ must be the same for all i from (15). It follows that $\mathcal{V}_i$ is the same for all i. Conversely, if $\mathcal{V}_i$ is the same for all i, then so is α_i^U from part (iii). We denote the common value of α_i^U across all groups i by α^U. Since $\mathcal{V}_i$ is the same for all i, so is $R_i^\top \eta_I$, and hence $R_i^\top \eta_I / \eta_I^\top R \eta_I = 1/N^I$. From (15), $\alpha^U \propto 1 - (1/N^I)(k\phi^I + 2) \propto N^I - (k\phi^I + 2)$. On the other hand, from (18), $\alpha^U \propto (k\phi^I + 2) - N^I$. It follows that $\alpha^U = 0$ (and $k\phi^I + 2 = N^I$).

Proof of (v): Suppose $L_I = L_U = L$. Let i_0 and j_0 be groups with the lowest and highest price informativeness, respectively, i.e. $R_{i_0}^\top \eta_I = \min_{i \in L} R_i^\top \eta_I$ and $R_{j_0}^\top \eta_I = \max_{i \in L} R_i^\top \eta_I$. Since price informativeness is not the same for all groups, $R_{i_0}^\top \eta_I < R_{j_0}^\top \eta_I$. If $R_{i_0}^\top \eta_I > 0$, then using the assumption that $L_I = L$, and hence $N_i^I \geq 1$ for all $i \in L$,

$$\frac{R_{i_0}^\top \eta_I}{\eta_I^\top R \eta_I} = \frac{R_{i_0}^\top \eta_I}{\sum_i N_i^I R_i^\top \eta_I} < \frac{R_{i_0}^\top \eta_I}{\sum_i N_i^I R_{i_0}^\top \eta_I} = \frac{1}{N^I}.$$

It follows that, whether $R_{i_0}^\top \eta_I$ is positive or equal to zero,

$$\frac{R_{i_0}^\top \eta_I}{\eta_I^\top R \eta_I} < \frac{1}{N^I}. \tag{49}$$

Using an analogous argument,

$$\frac{R_{j_0}^\top \eta_I}{\eta_I^\top R \eta_I} > \frac{1}{N^I}. \tag{50}$$

It suffices to show that it is impossible that $\alpha_i^U \geq 0$ for all i, or that $\alpha_i^U \leq 0$ for all i. We establish this by contradiction. Suppose $\alpha_i^U \geq 0$ for all i. Then, from (18), $k\phi^I + 2 \geq N^I$. Consequently, using (50), $1 - (R_{j_0}^\top \eta_I / \eta_I^\top R \eta_I)(k\phi^I + 2) < 0$. Hence, from (15), $\alpha_{j_0}^U < 0$, a contradiction. Similarly, if $\alpha_i^U \leq 0$ for all i, then $k\phi^I + 2 \leq N^I$. Using (49), $1 - (R_{i_0}^\top \eta_I / \eta_I^\top R \eta_I)(k\phi^I + 2) > 0$. Hence, from (15), $\alpha_{i_0}^U > 0$, a contradiction.

Proof of (vi): From (15) and Proposition 4.2 (iii), we see that

$$\alpha_i^U \propto 1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(k\phi^I + 2) \geq 1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}(N^I + N^U).$$

The result follows.

Proof of (vii): Suppose $L_U = \{1\}$. From (15) and (18), we see that $\alpha_1^U > 0$ if and only if

$$N^I < k\phi^I + 2 < \frac{\eta_I^\top R \eta_I}{R_1^\top \eta_I},$$

and $\alpha_1^U \leq 0$ if and only if

$$\frac{\eta_I^\top R \eta_I}{R_1^\top \eta_I} \leq k\phi^I + 2 \leq N^I.$$

Since ϕ^I must satisfy one (and only one) of these conditions, the result follows. □

Proof of Proposition 5.1. Solving for agents' portfolio choices, analogous to (39) and (40) but with zero price impact, we obtain the slope coefficients $\hat{\alpha}^I$ and $\hat{\alpha}_i^U$ given by (28) and (29), respectively. Using the market-clearing condition,

$$\sum_{i \in L_I} N_i^I \hat{\alpha}^I (\theta_i - \hat{p}) - \sum_{i \in L_U} N_i^U \hat{\alpha}_i^U \hat{p} = 0,$$

the equilibrium price is

$$\hat{p} = \hat{\Phi}^{-1} \hat{\alpha}^I \eta_I^\top \theta, \tag{51}$$

where

$$\hat{\Phi} := N^I \hat{\alpha}^I + \sum_{i \in L_U} N_i^U \hat{\alpha}_i^U. \tag{52}$$

Therefore, $\sigma_{\theta_i \hat{p}} / \sigma_{\hat{p}}^2 = (R_i^\top \eta_I / \eta_I^\top R \eta_I) \hat{\Phi} (\hat{\alpha}^I)^{-1}$, so that, from (28) and (29),

$$\hat{\alpha}_i^U = k^{-1} \left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} \hat{\Phi} (\hat{\alpha}^I)^{-1} \right]. \tag{53}$$

Plugging this expression into (52), and using (28),

$$\hat{\Phi}(\hat{\alpha}^I)^{-1} = N^I + \sum_{i \in L_U} N_i^U \left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} \hat{\Phi}(\hat{\alpha}^I)^{-1} \right],$$

which gives us

$$\hat{\Phi}(\hat{\alpha}^I)^{-1} = \gamma, \tag{54}$$

where γ is defined by (27). Equation (26) now follows from (51), and equation (30) from (53).

Finally, we verify that the slope parameters satisfy all the properties in Proposition 4.3. Parts (i), (ii) and (iii) are obvious. For part (iv), observe that if $\hat{\alpha}_i^U = 0$ for all i, then $\mathcal{V}_i = \mathcal{V}_j$ for all i, j, by part (iii). Conversely, if $\mathcal{V}_i = \mathcal{V}_j$ for all i, j, then $R_i^\top \eta_I / \eta_I^\top R \eta_I = (N^I)^{-1}$ for all i, so that $\gamma = N^I$ from (27). Plugging these values into (30), we see that $\hat{\alpha}_i^U = 0$ for all i. For part (v), we use the same argument as in the proof of Proposition 4.3 (v). Equations (49) and (50) still apply. If $\hat{\alpha}_i^U \geq 0$ for all i, then, from (30), $R_i^\top \eta_I / \eta_I^\top R \eta_I \leq \gamma^{-1}$ for all i. Hence, from (27), $\gamma^{-1} \leq (N^I)^{-1}$. Taken together, we have $R_i^\top \eta_I / \eta_I^\top R \eta_I \leq (N^I)^{-1}$ for all i, contradicting (50). A similar argument shows that we cannot have $\hat{\alpha}_i^U \leq 0$, for all i, either. For part (vi), the proof is analogous to that for Proposition 4.3 (vi), using (27) and (30). For part (vii), we use (30), (52) and (54), and the same argument as in the proof of Proposition 4.3 (vi), except that $k\phi^I + 2$ is replaced by γ. □

Proof of Proposition 5.2. From (44), ϕ^I satisfies the following equation for all λ (for notational ease, we suppress the dependence of ϕ^I on λ):

$$\frac{k\phi^I + 2}{k\phi^I + 1} - \frac{\lambda N^I}{k\phi^I + 1} + \lambda \sum_{i \in L_U} N_i^U \left[\frac{g_i(\phi^I;\lambda)}{\phi^I} - \frac{k\phi^I + 2}{k\phi^I + 1} \right] = 0, \tag{55}$$

where

$$g_i(\phi^I;\lambda) = \frac{-b_i(\phi^I;\lambda) + \sqrt{b_i^2(\phi^I;\lambda) + 4k\phi^I \frac{k\phi^I+2}{k\phi^I+1}}}{2k}, \tag{56}$$

$$b_i(\phi^I;\lambda) = 2 - \frac{R_i^\top \eta_I}{\lambda \eta_I^\top R \eta_I}(k\phi^I + 2) - k\phi^I \frac{k\phi^I + 2}{k\phi^I + 1}. \tag{57}$$

It is convenient to think of λ taking integer values, $\{1, 2, \ldots\}$. We claim that $\lim_{\lambda \to \infty} \phi^I = \infty$. Suppose not. Then $\{\phi^I(\lambda)\}$ is a bounded sequence, which we can assume to be convergent without loss of generality (since we can always consider a convergent subsequence). From (13), (15) and (17),

$$\frac{k\phi^I + 2}{k\phi^I + 1} - \frac{g_i(\phi^I;\lambda)}{\phi^I} = \frac{g_i(\phi^I;\lambda)/\phi^I}{kg_i(\phi^I;\lambda) + 1} \left[1 - \frac{R_i^\top \eta_I}{\lambda \eta_I^\top R \eta_I}(k\phi^I + 2) \right].$$

Taking λ to be large enough so that the term in square brackets on the right-hand side is positive, we have

$$\frac{g_i(\phi^I;\lambda)}{\phi^I} = \frac{k\phi^I + 2}{k\phi^I + 1} \left(1 + \frac{1}{kg_i(\phi^I;\lambda) + 1} \left[1 - \frac{R_i^\top \eta_I}{\lambda \eta_I^\top R \eta_I}(k\phi^I + 2) \right] \right)^{-1}.$$

Since $\lim_{\lambda \to \infty} g_i(\phi^I;\lambda) \in [0, \infty)$, $\lim_{\lambda \to \infty} g_i(\phi^I;\lambda)/\phi^I < \lim_{\lambda \to \infty} (k\phi^I + 2)/(k\phi^I + 1)$. Therefore (55) cannot hold for sufficiently large λ (this is true even if L_U is empty). This is a contradiction. Hence we must have $\phi^I \to \infty$.

Since $\phi_i^U \geq \phi^I$ (Proposition 4.2), $\phi_i^U \to \infty$ as well, for all $i \in L_U$. From (12), $\alpha^I \to k^{-1} = \hat{\alpha}^I$. From (15),

$$\alpha_i^U = \frac{\phi_i^U}{k\phi_i^U + 1} \left[1 - \frac{R_i^\top \eta_I}{\lambda \eta_I^\top R \eta_I}(k\phi^I + 2) \right] = \frac{\phi_i^U}{k\phi_i^U + 1} \left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I} \left(\frac{k\phi^I}{\lambda} + \frac{2}{\lambda} \right) \right]. \tag{58}$$

From Lemma A3.1 in the Online Appendix, $k\phi^I/\lambda \to \gamma$, where γ is defined by (27). Therefore, α_i^U converges to $\hat{\alpha}_i^U$, given by (30). Similarly, from (16),

$$p = (k\phi^I + 2)^{-1}(\lambda\eta_I)^\top\theta = \left(\frac{k\phi^I}{\lambda} + \frac{2}{\lambda}\right)^{-1}\eta_I^\top\theta, \tag{59}$$

which converges to $\hat{p}$, given by (26).

Finally, we show that ϕ^I and ϕ^U are monotonic in λ. From (44), $\phi^I(\lambda)$ solves

$$f(\phi^I(\lambda);\lambda) := \frac{\phi^I}{k\phi^I + 1}\left[(k\phi^I + 2) - \lambda N^I\right] + \lambda\sum_{i\in L_U} N_i^U\left[g_i(\phi^I;\lambda) - \phi^I\frac{k\phi^I + 2}{k\phi^I + 1}\right] = 0, \tag{60}$$

where $g_i(\phi^I;\lambda)$ is given by (56), and $b_i(\phi^I;\lambda)$ by (57); we suppress the dependence of ϕ^I on λ to economize on notation. For given ϕ^I, $b_i(\phi^I;\lambda)$ is increasing in λ, and hence from (41), $g_i(\phi^I;\lambda) = \phi_i^U(\phi^I;\lambda)$ is decreasing in λ. We have

$$\begin{aligned}\frac{\partial f(\phi^I(\lambda);\lambda)}{\partial\lambda} &= -\frac{\phi^I}{k\phi^I + 1}N^I + \sum_{i\in L_U} N_i^U\left[g_i(\phi^I;\lambda) - \phi^I\frac{k\phi^I + 2}{k\phi^I + 1}\right] \\ &\quad + \lambda\sum_{i\in L_U} N_i^U\frac{\partial g_i(\phi^I,\lambda)}{\partial\lambda} \\ &= -\frac{\phi^I}{\lambda}\left[\frac{k\phi^I + 2}{k\phi^I + 1}\right] + \lambda\sum_{i\in L_U} N_i^U\frac{\partial g_i(\phi^I,\lambda)}{\partial\lambda},\end{aligned}$$

which is negative (the second equality follows from (60)). Hence, for any $\lambda \geq 1$, there exists $\epsilon > 0$ such that for all $\tilde{\lambda} \in (\lambda, \lambda + \epsilon)$, $f(\phi^I(\lambda), \tilde{\lambda}) < 0$. Since $\phi^I(\lambda)$ is defined as the highest solution to $f(\phi^I;\lambda) = 0$, and $\lim_{\phi^I\to\infty} f(\phi^I;\lambda) = \infty$ for given λ, we have $\phi^I(\tilde{\lambda}) > \phi^I(\lambda)$. It follows that ϕ^I is strictly increasing in λ.

Assuming that L_U is nonempty, we have (from (55)),

$$\phi^U(\phi^I(\lambda);\lambda) := (N^U)^{-1}\sum_{i\in L_U} N_i^U g_i(\phi^I;\lambda) = \frac{\lambda N^U - 1}{\lambda N^U}\phi^I\frac{k\phi^I + 2}{k\phi^I + 1} + \frac{N^I}{N^U}\frac{\phi^I}{k\phi^I + 1}.$$

Since $\phi^U(\cdot;\cdot)$ is strictly increasing in both arguments, and ϕ^I is strictly increasing in λ, it follows that ϕ^U is strictly increasing in λ. The result that price informativeness does not depend on λ is immediate from (22). □

Proof of Proposition 5.3. The proof is analogous to that of Proposition 5.2. See the Online Appendix, Section A3, for details. □

Proof of Proposition 5.4. *Proof of (i):* We need to check that the depth parameters are bounded, as well as bounded away from zero, as the number of uninformed agents in any group goes to infinity. From (44), ϕ^I satisfies

$$\frac{k\phi^I + 2}{k\phi^I + 1} - \frac{N^I}{k\phi^I + 1} + \sum_{i\in L_U} N_i^U\left[\frac{g_i(\phi^I)}{\phi^I} - \frac{k\phi^I + 2}{k\phi^I + 1}\right] = 0. \tag{61}$$

Suppose $N_i^U \to \infty$ for some i and consider the sequence $\{\phi^I(N_i^U)\}$. If $\phi^I \to \infty$, then from (47), and the assumption that $R_i^\top\eta_I > 0$, $\lim_{\phi^I\to\infty} g_i(\phi^I)/\phi^I > 1$, and hence (61) cannot be satisfied for large N_i^U. It follows that $\{\phi^I(N_i^U)\}$ is bounded. We assume without loss of generality that it is convergent (as otherwise we can choose a convergent subsequence). If $\phi^I(N_i^U) \to 0$, then using (45) and (46), and the assumptions that $N_j^I \geq 2$ and $R_j^\top \eta_I > 0$ for all j,

$$\lim_{\phi^I \to 0} \frac{g_i(\phi^I)}{\phi^I} = \left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}\right]^{-1} < \left[1 - \frac{1}{N_i^I}\right]^{-1} \leq 2. \tag{62}$$

Again, (61) cannot hold for large N_i^U, and consequently $\{\phi^I(N_i^U)\}$ must be bounded away from zero. In addition, for every $j \in L_U$, $\{\phi_j^U(N_i^U)\}$ is a bounded sequence due to (43), and bounded away from zero since $\phi_j^U \geq \phi^I$.

Proof of (ii): From (15) and (18), and recalling that $\phi_i^U = g_i(\phi^I)$, $\phi^I(N_i^U)$ solves

$$f(\phi^I(N_i^U); N_i^U) := \frac{\phi^I}{k\phi^I + 1}\left[(k\phi^I + 2) - N^I\right] - \sum_{j \in L_U} N_j^U \alpha_j^U(\phi^I) = 0, \tag{63}$$

where

$$\alpha_j^U(\phi^I) = \frac{g_j(\phi^I)}{kg_j(\phi^I) + 1}\left[1 - \frac{R_j^\top \eta_I}{\eta_I^\top R \eta_I}(k\phi^I + 2)\right], \tag{64}$$

and we suppress the dependence of ϕ^I on N_i^U. Consider N_i^U and $\check{N}_i^U$ satisfying $N_i^U > \check{N}_i^U \geq 1$. Three cases arise depending on the sign of $\alpha_i^U(\phi^I(\check{N}_i^U))$.

Case 1: $\alpha_i^U(\phi^I(\check{N}_i^U)) > 0$. From (63), $f(\phi^I(\check{N}_i^U); N_i^U) < f(\phi^I(\check{N}_i^U); \check{N}_i^U) = 0$. Since $\phi^I(N_i^U)$ is defined as the highest solution to $f(\cdot; N_i^U) = 0$, and $\lim_{\phi^I \to \infty} f(\phi^I; N_i^U) = \infty$ for given N_i^U, we have $\phi^I(N_i^U) > \phi^I(\check{N}_i^U)$.

Case 2: $\alpha_i^U(\phi^I(\check{N}_i^U)) < 0$. From (64), $\alpha_i^U(\phi^I) < 0$ for all $\phi^I \geq \phi^I(\check{N}_i^U)$. Hence, from (63), $f(\phi^I; N_i^U) > f(\phi^I; \check{N}_i^U) \geq f(\phi^I(\check{N}_i^U); \check{N}_i^U) = 0$ for all $\phi^I \geq \phi^I(\check{N}_i^U)$. Hence, $\phi^I(N_i^U) < \phi^I(\check{N}_i^U)$.

Case 3: $\alpha_i^U(\phi^I(\check{N}_i^U)) = 0$. From (63), $f(\phi^I(\check{N}_i^U); N_i^U) = f(\phi^I(\check{N}_i^U); \check{N}_i^U) = 0$. Moreover, for any $\phi^I > \phi^I(\check{N}_i^U)$, $\alpha_i^U(\phi^I) < 0$, so that $f(\phi^I; N_i^U) > f(\phi^I; \check{N}_i^U) > f(\phi^I(\check{N}_i^U); \check{N}_i^U) = 0$. Hence $\phi^I(\check{N}_i^U)$ is the highest zero of $f(\cdot; N_i^U)$, i.e. $\phi^I(N_i^U) = \phi^I(\check{N}_i^U)$.

Putting these three cases together, we see that $\phi^I(N_i^U) - \phi^I(\check{N}_i^U) \propto \alpha_i^U(\phi^I(\check{N}_i^U))$.

Proof of (iii): Since ϕ^I is bounded, so are Φ, α^I, and $\{\alpha_i^U\}_{i \in L_U}$, from Proposition 3.1. From (3), $\Phi = N^I\alpha^I + \sum_{j \in L_U} N_j^U \alpha_j^U$. Therefore, as N_i^U goes to infinity, $N_i^U \alpha_i^U$ remains bounded. This in turn implies that α_i^U converges to zero. □

Proof of Lemma 6.1. From (8), (10) and (12), we see that

$$\begin{aligned}\mathcal{U}_i^I &= \mathbb{E}\left[\mathbb{E}(W_i|\theta_i, p)\right] \\ &= \mathbb{E}\left[(\theta_i - p)q_i^I - \frac{k}{2}(q_i^I)^2\right]\end{aligned}$$

$$
\begin{aligned}
&= \mathbb{E}\left[\frac{k\phi^I + 2}{2\phi^I}(q_i^I)^2\right] \\
&= \frac{k\phi^I + 2}{2\phi^I}(\alpha^I)^2\sigma^2_{\theta_i - p} \qquad (65)\\
&= \frac{k\phi^I + 2}{2\phi^I}\left[\frac{\phi^I}{k\phi^I + 1}\right]^2 \sigma^2_{\theta_i - p}.
\end{aligned}
$$

Using (31) and (32), we obtain (33). For later use, we note that, from (16) and (31),

$$
G_i = 1 + \frac{\sigma_p^2}{\sigma_\theta^2} - 2\frac{\sigma_{\theta_i p}}{\sigma_\theta^2} = 1 + \frac{\eta_I^\top R\eta_I}{(k\phi^I + 2)^2} - 2\frac{R_i^\top \eta_I}{k\phi^I + 2}. \qquad (66)
$$

From (9), (11), (15) and (16),

$$
\begin{aligned}
\mathcal{U}_i^U &= \mathbb{E}\big[\mathbb{E}(W_i|p)\big] \\
&= \mathbb{E}\left[\big[\mathbb{E}(\theta_i|p) - p\big]q_i^U - \frac{k}{2}(q_i^U)^2\right] \\
&= \mathbb{E}\left[\frac{k\phi_i^U + 2}{2\phi_i^U}(q_i^U)^2\right] \\
&= \frac{k\phi_i^U + 2}{2\phi_i^U}(\alpha_i^U)^2\sigma_p^2 \qquad (67)\\
&= \frac{k\phi_i^U + 2}{2\phi_i^U}\left[\frac{\phi_i^U}{k\phi_i^U + 1}\right]^2\left[1 - \frac{R_i^\top \eta_I}{\eta_I^\top R\eta_I}(k\phi^I + 2)\right]^2 \frac{\sigma_\theta^2 \eta_I^\top R\eta_I}{(k\phi^I + 2)^2}.
\end{aligned}
$$

Using (32) and (66), we obtain (34). □

Proof of Proposition 6.2. We fix an equilibrium of a given economy (in particular, we fix $\phi^I, \alpha^I, \sigma_p^2$ and $\eta_I^\top R\eta_I$), and consider the utilities of agents in group i for different hypothetical values of $R_i^\top \eta_I$, and hence of β_i, given by (25). There is a one-to-one correspondence between β_i, $\mathcal{V}_i$, α_i^U and ϕ_i^U; a higher value of β_i is associated with a higher value of $\mathcal{V}_i$ and ϕ_i^U, and a lower value of α_i^U.

From (33) and (66), the utility of an informed agent can be written as

$$
\mathcal{U}_i^I = \frac{\sigma_\theta^2}{2}F(\phi^I)\left[1 + \frac{\eta_I^\top R\eta_I}{(k\phi^I + 2)^2}(1 - 2\beta_i)\right], \qquad (68)
$$

which is linear and strictly decreasing in β_i. From (13) and (67),

$$
\mathcal{U}_i^U = \frac{k\phi_i^U + 2}{2\phi_i^U}(\phi_i^U - \Phi)^2\sigma_p^2.
$$

Differentiating with respect to ϕ_i^U, we obtain

$$
\frac{\partial \mathcal{U}_i^U}{\partial \phi_i^U} = (\phi_i^U - \Phi)\left[k + (\phi_i^U)^{-1} + (\phi_i^U)^{-2}\Phi\right]\sigma_p^2, \qquad (69)
$$

$$\frac{\partial^2 \mathcal{U}_i^U}{(\partial \phi_i^U)^2} = \left[k + 2(\phi_i^U)^{-3}\Phi^2\right]\sigma_p^2. \tag{70}$$

From (41) and (42),

$$\frac{\partial \phi_i^U}{\partial \beta_i} = \frac{\phi_i^U}{2k\phi_i^U + b_i}, \tag{71}$$

$$\frac{\partial^2 \phi_i^U}{(\partial \beta_i)^2} = \frac{(2k\phi_i^U + b_i)\frac{\partial \phi_i^U}{\partial \beta_i} - \phi_i^U\left[2k\frac{\partial \phi_i^U}{\partial \beta_i} - 1\right]}{(2k\phi_i^U + b_i)^2} = \frac{2\phi_i^U(k\phi_i^U + b_i)}{(2k\phi_i^U + b_i)^3}. \tag{72}$$

From (17) and (41),

$$\phi_i^U(k\phi_i^U + b_i) = \Phi. \tag{73}$$

Differentiating $\mathcal{U}_i^U$ with respect to β_i, and noting that $2k\phi_i^U + b_i > 0$ (from (43)), we have

$$\frac{\partial \mathcal{U}_i^U}{\partial \beta_i} = \frac{\partial \mathcal{U}_i^U}{\partial \phi_i^U}\frac{\partial \phi_i^U}{\partial \beta_i} \propto \phi_i^U - \Phi. \tag{74}$$

Using (69)–(74),

$$\begin{aligned}
\frac{\partial^2 \mathcal{U}_i^U}{(\partial \beta_i)^2} &= \frac{\partial^2 \mathcal{U}_i^U}{(\partial \phi_i^U)^2}\left[\frac{\partial \phi_i^U}{\partial \beta_i}\right]^2 + \frac{\partial \mathcal{U}_i^U}{\partial \phi_i^U}\frac{\partial^2 \phi_i^U}{(\partial \beta_i)^2} \\
&\propto \left[k + 2(\phi_i^U)^{-3}\Phi^2\right]\frac{(\phi_i^U)^2}{(2k\phi_i^U + b_i)^2} \\
&\quad + (\phi_i^U - \Phi)\left[k + (\phi_i^U)^{-1} + (\phi_i^U)^{-2}\Phi\right]\frac{2\phi_i^U(k\phi_i^U + b_i)}{(2k\phi_i^U + b_i)^3} \\
&\propto \left[k(\phi_i^U)^3 + 2\Phi^2\right]\phi_i^U(2k\phi_i^U + b_i) \\
&\quad + 2(\phi_i^U - \Phi)\left[k(\phi_i^U)^2 + \phi_i^U + \Phi\right]\phi_i^U(k\phi_i^U + b_i) \\
&= \left[k(\phi_i^U)^3 + 2\Phi^2\right]\left[k(\phi_i^U)^2 + \Phi\right] + 2(\phi_i^U - \Phi)\left[k(\phi_i^U)^2 + \phi_i^U + \Phi\right]\Phi \\
&= k^2(\phi_i^U)^5 + 3k(\phi_i^U)^3\Phi + 2(\phi_i^U)^2\Phi,
\end{aligned}$$

which is positive. Hence, $\mathcal{U}_i^U$ is strictly convex in β_i, achieving its minimum value of 0 at $\beta_i = 1$, which corresponds to $\alpha_i^U = 0$ or $\phi_i^U = \Phi$.

Proof of (i): We will show that $\Delta\mathcal{U}_i := \mathcal{U}_i^I - \mathcal{U}_i^U$ is strictly decreasing in β_i. We have already established that $\mathcal{U}_i^I$ is linear in β_i, and $\mathcal{U}_i^U$ is strictly convex in β_i, so that $\Delta\mathcal{U}_i$ is strictly concave in β_i. Hence it suffices to show that $\partial(\Delta\mathcal{U}_i)/\partial\beta_i < 0$ at $\beta_i = 0$. From (68), (69), (71), and the relations $\sigma_p^2 = \sigma_\theta^2(k\phi^I + 2)^{-2}\eta_I^\top R\eta_I$, $\phi_i^U|_{\beta_i=0} = \phi^I$, and $(2k\phi_i^U + b_i)|_{\beta_i=0} = \sqrt{4 + \left(k\phi^I\frac{k\phi^I+2}{k\phi^I+1}\right)^2}$, we have

$$\begin{aligned}
\left.\frac{\partial(\Delta\mathcal{U}_i)}{\partial \beta_i}\right|_{\beta_i=0} &\propto \left.\left[\frac{\phi_i^U}{2k\phi_i^U + b_i}(\Phi - \phi_i^U)\left[k + (\phi_i^U)^{-1} + (\phi_i^U)^{-2}\Phi\right] - F(\phi^I)\right]\right|_{\beta_i=0} \\
&= \frac{\phi^I}{\sqrt{4 + \left(k\phi^I\frac{k\phi^I+2}{k\phi^I+1}\right)^2}}(\Phi - \phi^I)\left[k + (\phi^I)^{-1} + (\phi^I)^{-2}\Phi\right] - F(\phi^I)
\end{aligned}$$

$$
\begin{aligned}
&\propto (k\phi^I+1)^2+(k\phi^I+2)-(k\phi^I+2)\sqrt{4+\left(k\phi^I\frac{k\phi^I+2}{k\phi^I+1}\right)^2}\\
&\propto \left[(k\phi^I+1)^2+(k\phi^I+2)\right]^2(k\phi^I+1)^2\\
&\quad-(k\phi^I+2)^2\left[4(k\phi^I+1)^2+(k\phi^I)^2(k\phi^I+2)^2\right]\\
&=-2(k\phi^I)^3-8(k\phi^I)^2-12k\phi^I-7,
\end{aligned}
$$

which is negative.

Proof of (ii): We will show that $\Delta\mathcal{U}_i(\beta_i)>0$ if and only if $\beta_i\in[0,\beta^*)$, for some cutoff value $\beta^*>1$. This cutoff value corresponds to a slope parameter $\alpha^{U*}<0$ for uninformed agents, and price informativeness $\mathcal{V}^*$.

If $\beta_i=1$, then $\alpha_i^U=0$, and hence $\Delta\mathcal{U}_i=\mathcal{U}_i^I>0$. From part (i), $\Delta\mathcal{U}_i$ is strictly decreasing in β_i, implying that $\Delta\mathcal{U}_i(\beta_i)>0$ for all $\beta_i\in[0,1]$. If $\Delta\mathcal{U}_i(\beta_i)>0$ for all β_i, we can pick $\beta^*=\max_i\beta_i+\epsilon$, for some small $\epsilon>0$, and we are done. If not, $\Delta\mathcal{U}_i(\beta^*)=0$ for some $\beta^*>1$. Since $\Delta\mathcal{U}_i$ is strictly decreasing in β_i, it is positive for $\beta_i<\beta^*$ and negative for $\beta_i>\beta^*$.

Proof of (iii): Competitive equilibrium utilities can be calculated as in the imperfectly competitive case (see the proof of Lemma 6.1), but with zero price impact. From (28), (29), (65) and (67), we have

$$
\hat{\mathcal{U}}_i^I=\frac{k}{2}(\hat{\alpha}^I)^2\sigma^2_{\theta_i-\hat{p}}=\frac{1}{2k}\sigma^2_{\theta_i-\hat{p}}, \tag{75}
$$

$$
\hat{\mathcal{U}}_i^U=\frac{k}{2}(\hat{\alpha}_i^U)^2\sigma^2_{\hat{p}}=\frac{1}{2k}\left[1-\frac{\sigma_{\theta_i\hat{p}}}{\sigma^2_{\hat{p}}}\right]^2\sigma^2_{\hat{p}}. \tag{76}
$$

Therefore,

$$
\Delta\hat{\mathcal{U}}_i=\frac{1}{2k}\left[\sigma^2_\theta+\sigma^2_{\hat{p}}-2\sigma_{\theta_i\hat{p}}-\left(\sigma^2_{\hat{p}}+\frac{\sigma^2_{\theta_i\hat{p}}}{\sigma^2_{\hat{p}}}-2\sigma_{\theta_i\hat{p}}\right)\right]=\frac{\sigma^2_\theta}{2k}(1-\mathcal{V}_i), \tag{77}
$$

where the second equality follows from (22) and (26).

Using Lemma 6.1, (32) and (77), and recalling that $\phi_i^U\geq\phi^I$, and hence $F(\phi_i^U)\geq F(\phi^I)$, we have

$$
\Delta\mathcal{U}_i=\frac{\sigma^2_\theta}{2}\left[F(\phi_i^U)(1-\mathcal{V}_i)-\left[F(\phi_i^U)-F(\phi^I)\right]G_i\right]\leq\frac{\sigma^2_\theta}{2}F(\phi_i^U)(1-\mathcal{V}_i)<\frac{\sigma^2_\theta}{2k}(1-\mathcal{V}_i).
$$

The result follows from (77). □

Proof of Claim in Example 6.1. We first show that as σ_ν^2 goes to zero, $\phi^I(\sigma_\nu^2)$ converges to a strictly positive finite limit. From (44), $f(\phi^I;\sigma_\nu^2)$ is a smooth function of σ_ν^2, and hence any zero of $f(\cdot;\sigma_\nu^2)$ is almost everywhere continuous in σ_ν^2. It follows that the right-hand limit $\lim_{\sigma_\nu^2\to 0}\phi^I:=\lim_{\sigma_\nu^2\downarrow 0}\phi^I$ exists, possibly equal to ∞.

If $\phi^I(\sigma_\nu^2)\to\infty$, then from (47),

$$
\lim_{\sigma_\nu^2\to 0}\frac{g_i(\phi^I)}{\phi^I}=1+\lim_{\sigma_\nu^2\to 0}\frac{R_i^\top\eta_I}{\eta_I^\top R\eta_I},
$$

which is equal to $1+(N_1^I)^{-1}$ for $i=1$, and equal to 1 for $i=2,3$. Therefore, (61) cannot hold for σ_ν^2 close to zero, a contradiction.

If $\phi^I(\sigma_v^2) \to 0$, then using (45) and (46),

$$\lim_{\sigma_v^2 \to 0} \frac{g_i(\phi^I)}{\phi^I} = \left[1 - \lim_{\sigma_v^2 \to 0} \frac{R_i^\top \eta_I}{\eta_I^\top R \eta_I}\right]^{-1},$$

which is equal to $[1-(N_1^I)^{-1}]^{-1} \leq 2$ for $i=1$, and equal to 1 for $i=2,3$. Again, (61) cannot hold for σ_v^2 close to zero, a contradiction. Thus we have proved that $\lim_{\sigma_v^2 \to 0} \phi^I$ exists and lies in $(0,\infty)$.

Proof of (i): From (66),

$$\begin{aligned} G_1 &= 1 + \frac{(N_1^I)^2 + 2(N_2^I)^2(1-a^2)}{(k\phi^I+2)^2} - 2\frac{N_1^I}{k\phi^I+2} \\ &= \left[\frac{N_1^I}{k\phi^I+2} - 1\right]^2 + \frac{2(N_2^I)^2(1-a^2)}{(k\phi^I+2)^2}. \end{aligned}$$

If $\lim_{\sigma_v^2 \to 0} G_1 = 0$, then $\lim_{\sigma_v^2 \to 0}(k\phi^I + 2) = N_1^I$, implying from (15) that $\lim_{\sigma_v^2 \to 0} \alpha_i^U \geq 0$ for all i. Hence, from (18), $\lim_{\sigma_v^2 \to 0}(k\phi^I + 2) \geq N^I > N_1^I$, a contradiction. Therefore $\lim_{\sigma_v^2 \to 0} G_1 > 0$.

Proof of (ii): Since ϕ^I converges to a strictly positive finite limit as $\sigma_v^2 \to 0$, the same is true for $\phi_i^U = g_i(\phi^I)$. Moreover, since the limiting value of $\mathcal{V}_1$ is positive, it follows from (48) that $\lim_{\sigma_v^2 \to 0} \phi_1^U > \lim_{\sigma_v^2 \to 0} \phi^I$. □

Proof of Lemma 7.1. Using the same argument as in the proof of Proposition 5.4, we see that the sequences $\{\phi^I(N_1^U)\}$ and $\{\phi_1^U(N_1^U)\}$ are both bounded and bounded away from zero. Since we only require these results for $i=1$ rather than for all i, the conditions on $\{N_i^I, R_i^\top \eta_I\}_{i \in L}$ (given in Definition 7.1) are weaker than those in Proposition 5.4.

From Proposition 5.4 (iii), $\alpha_1^U = 0$. It follows from (11) that $p = \mathbb{E}(\theta_1|p) = \beta_1 p$, and hence $\beta_1 = 1$. Equation (36) follows from (25), and equations (16) and (36) together give us the right-hand side of (35). □

Proof of Proposition 7.2. Taking the limit of (26) as $N_1^U \to \infty$, we see that the price function of the competitive $\mathcal{F}_1$-economy coincides with that of the $\mathcal{F}_1$-economy, which is given by (35). Moreover, it is clear from (35) that this price function is invariant with respect to λ.

From (36), it is immediate that ϕ^I is strictly increasing in λ, and $\lim_{\lambda \to \infty} \phi^I = \infty$. From (12) and (28), α^I is also strictly increasing in λ and converges to $\hat{\alpha}^I$.

From (17), (36), (41) and (42), ϕ_i^U solves

$$k(\phi_i^U)^2 + b_i \phi_i^U - \Phi = 0, \tag{78}$$

where

$$b_i := 2 - \frac{R_i^\top \eta_I}{R_1^\top \eta_I} - k\Phi. \tag{79}$$

Note that

$$\frac{\partial \Phi}{\partial \phi^I} = 1 + \frac{1}{(k\phi^I + 1)^2} > 0. \tag{80}$$

From (78), the derivative of ϕ_i^U with respect to any variable x satisfies $2k\phi_i^U(\phi_i^U)' + b_i'\phi_i^U + b_i(\phi_i^U)' - \Phi' = 0$ (denoting derivatives with respect to x by a prime), which gives us

$$(\phi_i^U)' = \frac{\Phi' - b_i'\phi_i^U}{2k\phi_i^U + b_i}. \tag{81}$$

Note that, from (78), $2k\phi_i^U + b_i > k\phi_i^U + b_i = \Phi/\phi_i^U > 0$. Now taking x to be the variable λ, we have $\Phi' > 0$ due to (80), and $b_i' = -k\Phi' < 0$. Therefore, $(\phi_i^U)' > 0$. Moreover, since $\phi_i^U \geq \phi^I$, $\lim_{\lambda\to\infty}\phi_i^U = \infty$.

From (15) and (36),

$$\alpha_i^U = \frac{\phi_i^U}{k\phi_i^U + 1}\left[1 - \frac{R_i^\top \eta_I}{R_1^\top \eta_I}\right].$$

It is clear that if $\alpha_i^U = 0$, it is invariant with respect to λ, and if $\alpha_i^U \neq 0$, it depends on λ only through ϕ_i^U. Moreover, in the latter case, $|\alpha_i^U|' \propto (\phi_i^U)'$. Hence, the stated properties of $|\alpha_i^U|$ follow from the corresponding properties of ϕ_i^U.

From (33), $\mathcal{U}_i^I = (\sigma_\theta^2/2)F(\phi^I)G_i$. Since $p = \hat{p}$ for all λ, G_i is invariant with respect to λ. Since $F'(\phi^I) > 0$, and ϕ^I is strictly increasing in λ, $\mathcal{U}_i^I$ is strictly increasing in λ. It converges to $(2k)^{-1}\sigma_\theta^2 G_i$, which is equal to $\hat{\mathcal{U}}_i^I$, from (75). From (35), $\sigma_p^2 = \sigma_\theta^2\mathcal{V}_1$, which is invariant with respect to λ. Hence, the statements about $\mathcal{U}_i^U$ follow from (67) and (76).

For a proof of the result that $\mathcal{U}_i^I - \mathcal{U}_i^U$ is strictly increasing in λ, see the Online Appendix, Lemma A5.1. □

Proof of Proposition 7.3. *Proof of (i):* From (36), we obtain

$$\frac{\partial\phi^I}{\partial N_i^I} = k^{-1}\frac{2(R_1^\top\eta_I)(R_i^\top\eta_I) - \rho_{i1}\eta_I^\top R\eta_I}{(R_1^\top\eta_I)^2}. \tag{82}$$

Therefore,

$$\begin{aligned}\frac{\partial^2\phi^I}{(\partial N_i^I)^2} &\propto (R_1^\top\eta_I)^2 - 2\rho_{i1}(R_1^\top\eta_I)(R_i^\top\eta_I) + \rho_{i1}^2\eta_I^\top R\eta_I \\ &= (R_1^\top\eta_I - \rho_{i1}R_i^\top\eta_I)^2 + \rho_{i1}^2\eta_I^\top R\eta_I(1-\mathcal{V}_i),\end{aligned}$$

which is positive.

Proof of (ii): From (82),

$$\frac{\partial\phi^I}{\partial N_1^I} = k^{-1}(2 - \mathcal{V}_1^{-1}), \tag{83}$$

from which the result is immediate.

Proof of (iii): Suppose all pairwise correlations are equal to ρ. From (82), $\partial\phi^I/\partial N_i^I > 0$ if $\rho < 0$. For $\rho \geq 0$, using the condition that $N_1^I + N_i^I \geq N_j^I$ for all $j \neq 1, i$, we have

$$\frac{\partial\phi^I}{\partial N_i^I} \propto 2(R_1^\top\eta_I)(R_i^\top\eta_I) - \rho\eta_I^\top R\eta_I$$

$$
\begin{aligned}
&> (R_1^\top \eta_I)(R_i^\top \eta_I) - \rho \eta_I^\top R \eta_I \\
&= \Big[N_1^I + \rho N_i^I + \rho \sum_{j \neq 1,i} N_j^I\Big]\Big[N_i^I + \rho N_1^I + \rho \sum_{j \neq 1,i} N_j^I\Big] \\
&\quad - \rho(1-\rho)\Big[(N_1^I)^2 + (N_i^I)^2 + \sum_{j \neq 1,i} (N_j^I)^2\Big] - \rho^2\Big[N_1^I + N_i^I + \sum_{j \neq 1,i} N_j^I\Big]^2 \\
&= (1-\rho^2) N_1^I N_i^I + \rho(1-\rho)(N_1^I + N_i^I) \sum_{j \neq 1,i} N_j^I - \rho(1-\rho) \sum_{j \neq 1,i} (N_j^I)^2 \\
&\geq (1-\rho^2) N_1^I N_i^I,
\end{aligned}
$$

which is positive. □

Proof of Lemma 7.4. Using (66) and (36),

$$
G_i = 1 + \frac{(R_1^\top \eta_I)(R_1^\top \eta_I - 2R_i^\top \eta_I)}{\eta_I^\top R \eta_I} = 1 + \frac{(R_1^\top \eta_I - R_i^\top \eta_I)^2 - (R_i^\top \eta_I)^2}{\eta_I^\top R \eta_I}, \tag{84}
$$

which gives us the desired expression for G_i. □

Proof of Proposition 7.5. By the definition of an $\mathcal{F}_1$-economy, $\mathcal{U}_1^U = 0$ for all N_1^I. We will show that $\partial \mathcal{U}_2^I / \partial N_1^I < 0$, $\partial \mathcal{U}_2^U / \partial N_1^I < 0$, and $\partial \mathcal{U}_1^I / \partial N_1^I < 0$. Utilities are given by Lemma 6.1, where we recall that $F' > 0$. It is easy to check that $\partial \mathcal{V}_2 / \partial N_1^I < 0$. Hence it suffices to show that $\partial \phi^I / \partial N_1^I \leq 0$, $\partial \phi_2^U / \partial N_1^I < 0$, $\partial G_2 / \partial N_1^I < 0$, and $\partial G_1 / \partial N_1^I < 0$.

(i) Proof of $\partial \phi^I / \partial N_1^I \leq 0$: In view of (83), we can equivalently show that $\mathcal{V}_1 \leq 1/2$. It is easy to check that

$$
\mathcal{V}_1 = \frac{(\rho N_2^I + N_1^I)^2}{(N_2^I)^2 + (N_1^I)^2 + 2\rho N_2^I N_1^I} \leq \frac{1}{2}
$$

if and only if $N_1^I \leq \left(\sqrt{1-\rho^2} - \rho\right) N_2^I$. For $\rho \in (-1, 1/2]$, the function $\sqrt{1-\rho^2} - \rho$ is minimized at $\rho = 1/2$, and the minimum value is greater than 1/3. Hence, given our assumptions that $\rho \leq 1/2$ and $N_1^I \leq N_2^I/3$, we have $\mathcal{V}_1 \leq 1/2$.

(ii) Proof of $\partial \phi_2^U / \partial N_1^I < 0$: From (79) and (81),

$$
\frac{\partial \phi_2^U}{\partial N_1^I} \propto \frac{\partial \Phi}{\partial N_1^I} + \left[\frac{\partial \frac{R_2^\top \eta_I}{R_1^\top \eta_I}}{\partial N_1^I} + k \frac{\partial \Phi}{\partial N_1^I}\right] \phi_2^U. \tag{85}
$$

From (80), and the result in part (i) above,

$$
\frac{\partial \Phi}{\partial N_1^I} = \frac{\partial \Phi}{\partial \phi^I} \frac{\partial \phi^I}{\partial N_1^I} \leq 0. \tag{86}
$$

Also,

$$
\frac{\partial \frac{R_2^\top \eta_I}{R_1^\top \eta_I}}{\partial N_1^I} = \frac{\rho R_1^\top \eta_I - R_2^\top \eta_I}{(R_1^\top \eta_I)^2} = -\frac{(1-\rho^2) N_2^I}{(R_1^\top \eta_I)^2} < 0. \tag{87}
$$

Hence, from (85), $\partial\phi_2^U/\partial N_1^I < 0$.

(iii) Proof of $\partial G_2/\partial N_1^I < 0$*:* From (84),

$$\begin{aligned}\frac{\partial G_2}{\partial N_1^I} &= \frac{(\eta_I^\top R\eta_I)\big[R_1^\top\eta_I - 2R_2^\top\eta_I + (1-2\rho)R_1^\top\eta_I\big] - 2(R_1^\top\eta_I)(R_1^\top\eta_I - 2R_2^\top\eta_I)(R_1^\top\eta_I)}{(\eta_I^\top R\eta_I)^2}\\ &\propto (1-\mathcal{V}_1)(R_1^\top\eta_I - 2R_2^\top\eta_I) + R_2^\top\eta_I - \rho R_1^\top\eta_I\\ &= \frac{(1-\rho^2)(N_2^I)^2}{(N_2^I)^2 + (N_1^I)^2 + 2\rho N_2^I N_1^I}\big[(\rho-2)N_2^I + (1-2\rho)N_1^I\big] + (1-\rho^2)N_2^I\\ &\propto N_2^I\big[(\rho-2)N_2^I + (1-2\rho)N_1^I\big] + \big[(N_2^I)^2 + (N_1^I)^2 + 2\rho N_2^I N_1^I\big]\\ &= (N_1^I)^2 + N_2^I N_1^I - (1-\rho)(N_2^I)^2,\end{aligned}$$

which is negative if and only if $N_1^I < (\sqrt{5-4\rho}-1)N_2^I/2$. This condition is satisfied for $\rho \le 1/2$ and $N_1^I \le N_2^I/3$.

(iv) Proof of $\partial G_1/\partial N_1^I < 0$*:* This follows from (23) and (38). □

Proof of Proposition 8.2. *Proof of (i):* From (22), price informativeness for the two groups is given by

$$\mathcal{V}_1 = \frac{(\psi+\rho)^2}{\psi^2 + 2\rho\psi + 1}, \qquad \text{and} \qquad \mathcal{V}_2 = \frac{(\rho\psi+1)^2}{\psi^2 + 2\rho\psi + 1},$$

where $\psi := N_1^I/N_2^I$. Under the assumption that $\rho \ge 0$, it is easy to check that $\partial\mathcal{V}_1/\partial\psi > 0$ and $\partial\mathcal{V}_2/\partial\psi < 0$.

Proof of (ii): Using (80) and (82),

$$\begin{aligned}\frac{\partial\Phi}{\partial E} \propto \frac{\partial\phi^I}{\partial E} = \frac{\partial\phi^I}{\partial N_2^I} - \frac{\partial\phi^I}{\partial N_1^I}&\\ &\propto \big[2(R_1^\top\eta_I)(R_2^\top\eta_I) - \rho\eta_I^\top R\eta_I\big] - \big[2(R_1^\top\eta_I)^2 - \eta_I^\top R\eta_I\big]\\ &= 2(R_1^\top\eta_I)(R_2^\top\eta_I - R_1^\top\eta_I) + (1-\rho)\eta_I^\top R\eta_I\\ &= 2(N_1^I + \rho N_2^I)(1-\rho)(N_2^I - N_1^I) + (1-\rho)\big[(N_1^I)^2 + (N_2^I)^2 + 2\rho N_1^I N_2^I\big]\\ &\propto (1+2\rho)(N_2^I)^2 + 2N_1^I N_2^I - (N_1^I)^2,\end{aligned}$$

which is positive if and only if $N_1^I/N_2^I < 1 + \sqrt{2(1+\rho)}$. The result follows.

Proof of (iii): In an $\mathcal{F}_1$-economy, $\mathcal{U}_1^U = 0$ for all E. We will show that, under the conditions stated in part (iii), $\partial\mathcal{U}_1^I/\partial E$, $\partial\mathcal{U}_2^I/\partial E$ and $\partial\mathcal{U}_2^U/\partial E$ are all strictly positive. Utilities are given by Lemma 6.1. Using (38) and part (i) of this proposition, $\partial G_1/\partial E = -\partial\mathcal{V}_1/\partial E > 0$. From part (i), we also have $\partial\mathcal{V}_2/\partial E > 0$. Hence it suffices to show that $\partial\phi^I/\partial E > 0$, $\partial G_2/\partial E \ge 0$ and $\partial\phi_2^U/\partial E > 0$.

Proof of $\partial\phi^I/\partial E > 0$: The condition in part (iii) implies that $N_2^I > N_1^I$. Therefore, from part (ii), $\partial\phi^I/\partial E \propto \partial\Phi/\partial E > 0$.

Proof of $\partial G_2/\partial E \ge 0$: From (84),

$$G_2 = 1 + \frac{(1-2\rho)\psi^2 + 2(\rho-\rho^2-1)\psi + \rho(\rho-2)}{\psi^2 + 2\rho\psi + 1},$$

where $\psi := N_1^I / N_2^I$. Differentiating with respect to ψ, we see that

$$\begin{aligned}\frac{\partial G_2}{\partial \psi} &\propto (\psi^2 + 2\rho\psi + 1)\big[2(1-2\rho)\psi + 2(\rho - \rho^2 - 1)\big] \\ &\quad - (2\psi + 2\rho)\big[(1-2\rho)\psi^2 + 2(\rho - \rho^2 - 1)\psi + \rho(\rho - 2)\big] \\ &\propto \psi^2 + \psi - (1-\rho).\end{aligned}$$

Since $\partial\psi/\partial E < 0$, we have $\partial G_2/\partial E \geq 0$ if and only if $\psi^2 + \psi - (1-\rho) \leq 0$, or equivalently, $2\psi \leq -1 + \sqrt{1 + 4(1-\rho)}$.

Proof of $\partial\phi_2^U/\partial E > 0$: From (15), (25) and (36),

$$\Phi - \phi_2^U = \frac{\phi_2^U}{k\phi_2^U + 1}\left[1 - \frac{R_2^\top \eta_I}{\eta_I^\top R \eta_I}(k\phi^I + 2)\right] = \frac{\phi_2^U}{k\phi_2^U + 1}\left[1 - \frac{R_2^\top \eta_I}{R_1^\top \eta_I}\right].$$

Since $\partial\Phi/\partial E > 0$ and $\partial[R_2^\top \eta_I / R_1^\top \eta_I]/\partial E > 0$, we conclude that $\partial\phi_2^U/\partial E > 0$. $\square$

Appendix B. Online Appendix

Supplemental material related to this article can be found online at https://doi.org/10.1016/j.jet.2023.105756.

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