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On the investment direction of a behavioral portfolio choice model

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ABSTRACT

The existing results show that for two-point distributions, the investment direction of a CPT-investor is determined by the actual (respectively, perceived) market opportunity when the investor is in a gain (respectively, loss) position. For general distributions this article shows that the result in the case of gain positions still holds when the CPT-investor is sufficiently loss-averse, but no longer holds in the case of loss positions by constructing counterexamples.

1. Introduction

It is well-known that under the framework of expected utility theory (EUT), in a market consisting of a risky asset (stock) and a risk-free asset, every investor with a strictly increasing and strictly concave utility function will optimally long (respectively, short) the stock if the stock has a positive (respectively, negative) expected excess return. In this article, we will investigate the investment direction of investors under the framework of Tversky and Kahneman's cumulative prospect theory (CPT) [3,6].

It is generally difficult to present explicit solutions for behavioral portfolio choice (BPC) optimization problems due to the non-concavity of CPT value functions (see Corollary 2 in [2]). In fact, except for the work [2], which derives the optimal investment strategy (OIS) in two special cases, most existing literature focuses on analyzing the properties of optimal solutions, see [1] and [5]. For tractability, the work [4] tries to identify the investment direction, i.e., to long or short the stock, for a single-period BPC model. Specifically, Lou et al. [4] show that for two-point distributions, the investment direction of a gain-position investor depends only on whether the mean of the excess return is positive or negative. This is consistent with the result in EUT. However, the investment direction of a loss-position investor no longer depends on the actual market opportunity, but the perceived market opportunity, which is jointly described by the investor's risk aversion coefficient and the market opportunity.

Identifying the investment direction of CPT-investors is important. For instance, for the multi-agent infinitely repeated BPC model proposed by Lou et al. [4], whether the actual and perceived market agree is crucial for the long-term dynamics of wealth growth and wealth gap (refer to Proposition 3 therein for more details). Furthermore, knowing the investment direction of CPT-investors is also important in real-world investment. For example, when the actual and perceived market disagree, there are potential investment opportunities for rational investors. Indeed, in this case CPT-investors short (respectively, long) an asset with a positive (respectively, negative) expected excess return, thereby decreasing (respectively, increasing) the price of the asset. A rational investor could potentially exploit this behavior. It is thus important for investment practitioners to be aware of the circumstances where such situations arise.

We continue to investigate whether it is still the actual (respectively, perceived) market opportunity that determines the investment direction for general distributions when the CPT-investor is in a gain (respectively, loss) position. To be specific, we show that the answer is in the affirmative when the investor is in a gain position, assuming that the investor is sufficiently loss-averse. However, the answer is in the negative when the investor is in a loss position, although in the affirmative for two-point distributions as shown by Lou et al. [4]. In fact, numerical examples for three-point distributions show that loss-position investors probably long (respectively, short) the stock in the presence of bad (respectively, good) perceived market opportunities. These observations add to the understanding of the market conditions that determine the investment direction of CPT-investors.

2. The behavioral portfolio choice model

We consider the (single-period) BPC model studied by Lou et al. [4]. The market consists of a risk-free asset and a risky

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asset (stock) with stochastic return $\tilde{R}$. It is assumed for simplicity that the risk-free asset does not generate interest. Let W_0 be an individual CPT-investor's initial wealth. Let θ be the amount invested in the stock and $W_0-\theta$ the amount invested in the risk-free asset by the investor. Consequently, the investor's total wealth at the end of the investment period is given by $W_0+(\tilde{R}-1)\theta$. Short-selling is allowed. The excess return $R:=\tilde{R}-1$ is a random variable with cumulative distribution function $F(\cdot)$. To exclude arbitrage opportunities, $0<\mathbb{P}(R<0)<1$ and $0<\mathbb{P}(R>0)<1$ are assumed.

Let B denote the reference point of the CPT-investor, which serves as a base point to separate gains from losses at the end of the investment period. This investor's S-shaped power utility function at wealth level x with respect to her reference point B is given by $(x-B)^\alpha$ if $x\geq B$, and $-K(B-x)^\alpha$ otherwise, where $K>1$ is the *loss aversion coefficient* and $0<\alpha<1$ is the *risk aversion coefficient*. This model does not consider probability weighting.

Let $\bar{B}=W_0-B$ denote the *relative wealth* of the CPT-investor. The investor is said to be in a *gain* (respectively, *loss*) position if $\bar{B}$ is positive (respectively, negative). The CPT preference value of a given investment strategy θ (i.e., the expected utility of the total wealth at the end of the investment period), denoted by $V(\theta)$, is expressed as

$$V(\theta)=\begin{cases}\int_{-\frac{\bar{B}}{\theta}}^{+\infty}(\theta t+\bar{B})^\alpha dF(t)-K\int_{-\infty}^{-\frac{\bar{B}}{\theta}}(-\theta t-\bar{B})^\alpha dF(t) & \text{for } \theta>0;\\ \int_{-\infty}^{-\frac{\bar{B}}{\theta}}(\theta t+\bar{B})^\alpha dF(t)-K\int_{-\frac{\bar{B}}{\theta}}^{+\infty}(-\theta t-\bar{B})^\alpha dF(t) & \text{for } \theta<0,\end{cases}$$

$V(0)=\bar{B}^\alpha$ when $\bar{B}\geq 0$, and $V(0)=-K|\bar{B}|^\alpha$ when $\bar{B}<0$. The investor maximizes her CPT preference value function

$$\max_{\theta\in\mathbb{R}} V(\theta). \tag{1}$$

We assume that the distribution of the excess return R is either discrete with finite values or absolutely continuous with a probability density function $f(t)=O(|t|^{-2-\epsilon})$, where $\epsilon>0$. This assumption guarantees that the optimization problem (1) is always finite-valued, i.e., $|V(\theta)|<+\infty$ for any $\theta\in\mathbb{R}$ (see Proposition 1 in [2]). We also assume that the well-posedness condition

$$K>\max\left\{\frac{\int_0^{+\infty}t^\alpha dF(t)}{\int_{-\infty}^0|t|^\alpha dF(t)},\frac{\int_{-\infty}^0|t|^\alpha dF(t)}{\int_0^{+\infty}t^\alpha dF(t)}\right\} \tag{2}$$

holds, which ensures that (1) has a finite OIS (see Theorem 2 in [2]).

Theorem 1 in [4] shows that the OIS of (1), denoted as θ^*, takes a piecewise linear form. Specifically, it shows that when $\bar{B}=0$, $\arg\max_{\theta\in\mathbb{R}}V(\theta)=0$, and when $\bar{B}\neq 0$, there exists γ^*, which depends only on the parameters α, K, $F(\cdot)$, and the *sign* of $\bar{B}$ (but *not* the *absolute* value of $\bar{B}$) such that the OIS θ^* takes the form of

$$\theta^*=\arg\max_{\theta\in\mathbb{R}}V(\theta)=\gamma^*\bar{B}. \tag{3}$$

In terms of the piecewise linear structure (3), when identifying the investment direction of (1) (i.e., to long or short the stock), it suffices to identify the investment direction in two special cases: $\bar{B}=1$ and $\bar{B}=-1$.

3. The investment direction for elliptical and two-point distributions

Here we introduce the existing results on the investment direction for elliptical and two-point distributions. The actual market is said to be good (respectively, bad) if the mean of the excess return R is positive (respectively, negative).

It is well-known that an EUT-investor with a strictly increasing and strictly concave utility function will optimally long (respectively, short) the stock in an actually good (respectively, bad) market. Hopefully it is expected that there also exists a simple market condition that can be used to identify a CPT-investor's investment direction. First, when the excess return follows an elliptical distribution, Corollary 1 in [4] shows that a CPT-investor will optimally long (respectively, short) the stock when the actual market is good (respectively, bad) no matter whether this investor is in a gain or loss position. This is consistent with the result in EUT.

Furthermore, for the special two-point distribution $(a_0,p_0;a_1,p_1)$ with $a_0<0$, $a_1>0$, $p_0>0$, $p_1>0$ and $p_0+p_1=1$, Lou et al. [4] show that

$$\arg\max_{\theta\in\mathbb{R}}V(\theta)\begin{cases}>0, & \text{if } \bar{B}<0 \text{ and } a_1^\alpha p_1-|a_0|^\alpha p_0>0;\\ <0, & \text{if } \bar{B}<0 \text{ and } a_1^\alpha p_1-|a_0|^\alpha p_0<0;\\ >0, & \text{if } \bar{B}>0 \text{ and } a_1p_1-|a_0|p_0>0;\\ <0, & \text{if } \bar{B}>0 \text{ and } a_1p_1-|a_0|p_0<0;\\ =0, & \text{if } \bar{B}>0 \text{ and } a_1p_1-|a_0|p_0=0;\\ =0, & \text{if } \bar{B}=0,\end{cases}$$

and there are two optimal solutions (where one is positive and the other is negative) when $\bar{B}<0$ and $a_1^\alpha p_1=|a_0|^\alpha p_0$. The above result reveals that when the investor is in a gain position, the investment direction is determined by the goodness and badness of the actual market. However, when the investor is in a loss position, the investment direction is no longer determined by the actual market opportunity, but the *perceived* market opportunity (i.e., $a_1^\alpha p_1>|a_0|^\alpha p_0$ or $a_1^\alpha p_1<|a_0|^\alpha p_0$). Note that in general there is no direct relationship between the signs of $a_1^\alpha p_1-|a_0|^\alpha p_0$ and $a_1p_1-|a_0|p_0$.

4. The investment direction for general distributions

In this section, we will discuss whether the identification condition for elliptical and two-point distributions (with an appropriate variant to capture the generality of distributions) can be applied to general distributions. To be specific, we will investigate whether it is the sign of

$$\int_0^{+\infty}tdF(t)-\int_{-\infty}^0|t|dF(t) \tag{4}$$

(i.e., the mean E[R] of the excess return R) that determines the investment direction of gain-position CPT-investors, and whether it is the sign of

$$\int_0^{+\infty}t^\alpha dF(t)-\int_{-\infty}^0|t|^\alpha dF(t) \tag{5}$$

that determines the investment direction of loss-position CPT-investors. In this article, to distinguish with the actual opportunity (4), (5) is defined as the perceived market opportunity. A positive (respectively, negative) value of (5) means a good (respectively, bad) perceived market opportunity.

Remark 1. When the excess return follows an elliptical distribution, (4) and (5) have the same sign and the sign is positive (respectively, negative) if the mean of the excess return is positive (respectively, negative). Furthermore, when the excess return follows a two-point distribution, it is clear that (4) and (5) reduce to $a_1p_1-|a_0|p_0$ and $a_1^\alpha p_1-|a_0|^\alpha p_0$, respectively. Thus, the answer to the above question is in the affirmative for elliptical and two-point distributions, and hence the two expressions (4) and (5) used to capture the generalization of distributions are reasonable.

4.1. Gain positions

When the reference point is much below the initial wealth, with high probability the final wealth is above the reference point and hence the investor is in the region of risk aversion. Consequently, the CPT model intuitively behaves like the classical EUT model to some extent and then the investment direction is determined by the actual market opportunity. The following theorem confirms this intuition.

Theorem 1. *Suppose the investor is in a gain position (i.e., the relative wealth $\bar{B} > 0$). Then when* $\mathrm{E}[R] > 0$ *(respectively,* $\mathrm{E}[R] < 0$*),* $\arg\max_{\theta\in\mathbb{R}} V(\theta) > 0$ *(respectively,* $\arg\max_{\theta\in\mathbb{R}} V(\theta) < 0$*) for sufficiently large K.*

Proof. Here we only show the case when $\mathrm{E}[R] > 0$ because the proof for the case when $\mathrm{E}[R] < 0$ is similar. We assume without loss of generality that $\bar{B} = 1$.

We first consider discrete distributions. Let $c_1 < c_2 < \cdots < c_m \le 0$ be all the non-positive values and $0 < d_1 < d_2 < \cdots < d_{n-1} < d_n$ be all the positive values that the excess return R takes. A simple computation shows that the CPT value function $V(\theta) = \sum_{i=1}^m (c_i\theta + 1)^\alpha p_i - K\sum_{i=1}^n (-d_i\theta - 1)^\alpha p_i$ for $\theta \le -d_1^{-1}$, $V(\theta) = \sum_{i=1}^m (c_i\theta + 1)^\alpha p_i + \sum_{i=1}^k (d_i\theta + 1)^\alpha p_i - K\sum_{i=k+1}^n (-d_i\theta - 1)^\alpha p_i$ for $-d_k^{-1} < \theta \le -d_{k+1}^{-1}$ and $k = 1, \ldots, n-1$, and $V(\theta) = \sum_{i=1}^m (c_i\theta + 1)^\alpha p_i + \sum_{i=1}^n (d_i\theta + 1)^\alpha p_i$ for $-d_n^{-1} < \theta < 0$. We conclude that the OIS of (1) must be positive based on the following arguments:

- Let $g(\theta) = \frac{\sum_{i=1}^m (c_i\theta+1)^\alpha p_i + \sum_{i=1}^{n-2} |d_i\theta+1|^\alpha p_i}{|d_n\theta+1|^\alpha p_n}$. Note that $\sup_{-\infty<\theta\le -d_{n-1}^{-1}} g(\theta) < \infty$ because g is continuous on $(-\infty, -d_{n-1}^{-1}]$ and $g(\theta) \to \frac{\sum_{i=1}^m c_i^\alpha p_i + \sum_{i=1}^{n-2} |d_i|^\alpha p_i}{d_n^\alpha p_n}$ as $\theta \to -\infty$. A simple derivation shows that when $K > \sup_{-\infty<\theta\le -d_{n-1}^{-1}} g(\theta)$, $V(\theta) \le \sum_{i=1}^m (c_i\theta + 1)^\alpha p_i + \sum_{i=1}^{n-2} |d_i\theta + 1|^\alpha p_i - K(-d_n\theta - 1)^\alpha p_n < 0$ for any $\theta \le -d_{n-1}^{-1}$. This implies that $V(\theta) < V(0) = 1$ for any $\theta \le -d_{n-1}^{-1}$.
- A sufficiently large K can be chosen such that $V'(\theta) = \alpha[\sum_{i=1}^m (c_i\theta + 1)^{\alpha-1} c_i p_i + \sum_{i=1}^{n-1} (d_i\theta + 1)^{\alpha-1} d_i p_i + K(-d_n\theta - 1)^{\alpha-1} d_n p_n] > 0$ for $-d_{n-1}^{-1} < \theta < -d_n^{-1}$.
- For $-d_n^{-1} < \theta < 0$, it holds that $V'(\theta) = \alpha[\sum_{i=1}^m (c_i\theta + 1)^{\alpha-1} c_i p_i + \sum_{i=1}^n (d_i\theta + 1)^{\alpha-1} d_i p_i]$ and $V''(\theta) = \alpha(\alpha - 1)[\sum_{i=1}^m (c_i\theta + 1)^{\alpha-2} c_i^2 p_i + \sum_{i=1}^n (d_i\theta + 1)^{\alpha-2} d_i^2 p_i] < 0$. Combining the previous analysis with the relation $\lim_{\theta\uparrow 0} V'(\theta) = \alpha\mathrm{E}[R] > 0$ implies that $V'(\theta) > 0$ for any $-d_n^{-1} < \theta < 0$.

We now consider absolutely continuous distributions. Theorem 2 in [2] shows that $\lim_{\theta\to-\infty} V(\theta) = -\infty$. So there exists a sufficiently large ϵ_1 such that $V(\theta) < V(0)$ for any $\theta \le -\epsilon_1$. We complete the proof by discussing the following two cases:

- The excess return R is unbounded from above, i.e., $\mathbb{P}(R > M) > 0$ for any $M > 0$. Proposition 3 in [2] informs us that the CPT value function $V(\cdot)$ is continuously differentiable on $(-\infty, 0)$ and $\lim_{\theta\uparrow 0} V'(\theta) = V'(0-) = \alpha\mathrm{E}[R] > 0$. Hence there exists a sufficiently small ϵ_2 with $0 < \epsilon_2 < \epsilon_1$ such that $V'(\theta) > 0$ for any $-\epsilon_2 \le \theta < 0$. We have that $V'(\theta) = \alpha[\int_{-\infty}^{-\frac{1}{\theta}} (\theta t + 1)^{\alpha-1} t dF(t) + K\int_{-\frac{1}{\theta}}^{+\infty} (-\theta t - 1)^{\alpha-1} t dF(t)]$ and $\int_{-\frac{1}{\theta}}^{+\infty} (-\theta t - 1)^{\alpha-1} t dF(t) \ge \frac{1}{\epsilon_2}\int_{\frac{1}{\epsilon_2}}^{+\infty} (-\theta t - 1)^{\alpha-1} dF(t) \ge \frac{1}{\epsilon_2}\int_{\frac{1}{\epsilon_2}}^{+\infty} (\epsilon_1 t - 1)^{\alpha-1} dF(t) > 0$ for $-\epsilon_1 \le \theta \le -\epsilon_2$, where the last inequality follows from the unboundedness of R. Therefore, $V'(\theta) > 0$ for any $-\epsilon_1 \le \theta \le -\epsilon_2$ when K is sufficiently large. Note that $V(\theta)$ (respectively, $V'(\theta)$) is a decreasing (respectively, increasing) function of the variable K for any fixed $\theta < 0$. The above analysis implies that the OIS of (1) is positive when K is sufficiently large.
- The excess return R is bounded from above, i.e., $R \le M$ almost surely for some $M > 0$. Without loss of generality, we assume that $M = \sup\{z | \mathbb{P}(R > z) > 0\} < \infty$. For $-\frac{1}{M} \le \theta < 0$, $V'(\theta) = \alpha(\int_{-\infty}^0 (\theta t + 1)^{\alpha-1} t dF(t) + \int_0^M (\theta t + 1)^{\alpha-1} t dF(t)) \ge \alpha\int_{-\infty}^M t dF(t) = \alpha\mathrm{E}[R] > 0$, where the inequality follows from the two relations $(\theta t + 1)^{\alpha-1} < 1$ for $t < 0$ and $(\theta t + 1)^{\alpha-1} > 1$ for $0 < t < M$. By the continuity of $V'(\cdot)$, we can then select a sufficiently small ϵ_3 such that $V'(\theta) > 0$ for any $-\frac{1}{M} - \epsilon_3 \le \theta < 0$. Furthermore, by similar arguments to the proof of the unbounded case, we can show that $V'(\theta) > 0$ for any $-\epsilon_1 \le \theta \le -\frac{1}{M} - \epsilon_3$ when K is sufficiently large. This implies that the OIS of (1) is positive when K is sufficiently large.

The proof is completed. □

Theorem 1 does not provide an answer to the investment direction when the CPT-investor is moderately loss-averse. In fact, when the loss aversion coefficient is not sufficiently large and only satisfies the well-posedness condition (2), possibly the BPC problem (1) has multiple positive and negative local maximum. In this case, it is generally difficult to identify which one is the global maximum and thus hard to identify the investment direction. However, the following example shows that for three-point distributions, it is indeed optimal for a moderately loss-averse gain-position investor to long (respectively, short) the stock in an actually good (respectively, bad) market. A three-point distribution $(b_1, p_1; b_2, p_2; b_3, p_3)$ means that it takes value b_i with positive probability p_i, where $b_1 < b_2 < b_3$, $b_1 < 0$, $b_3 > 0$, $p_1 + p_2 + p_3 = 1$.

Example 1. Here 100,000 three-point distributions $(b_1, p_1; b_2, p_2; b_3, p_3)$ with $b_1 < 0 < b_2 < b_3$ are randomly and independently generated in such a way that five random numbers are first generated independently of each other from the uniform distribution on (0, 1), denoted as ℓ_i, $i = 1, \ldots, 5$, and then we set $b_1 = -\ell_1$, $b_2 = \min\{\ell_2, \ell_3\}$, $b_3 = \max\{\ell_2, \ell_3\}$, $p_1 = \ell_4$, $p_2 = (1 - \ell_4)\ell_5$ and $p_3 = 1 - p_1 - p_2$. Take the relative wealth $\bar{B} = 1$, risk aversion coefficient $\alpha = 0.88$ and loss aversion coefficient $K = \max\{\frac{|b_1|^\alpha p_1}{b_2^\alpha p_2 + b_3^\alpha p_3}, \frac{b_2^\alpha p_2 + b_3^\alpha p_3}{|b_1|^\alpha p_1}\} + \epsilon$, where $\epsilon > 0$. For each value $\epsilon = 0.2, 0.5, 1, 2, 5$, the computations using MATLAB show that the optimal solution of (1) for each three-point distribution is positive (respectively, negative) if the mean of the excess return R is positive (respectively, negative).

4.2. Loss positions

The next example shows that a loss-position CPT-investor probably shorts the stock in the presence of good perceived market opportunities.

Example 2. Here we consider the three-point distribution $(-1, 0.45; 0.5, 0.2; 1, 0.35)$. Take $\bar{B} = -1$, $\alpha = 0.88$ and $K = K_0 + \epsilon$, where $K_0 = \max\{\frac{|b_1|^\alpha p_1}{b_2^\alpha p_2 + b_3^\alpha p_3}, \frac{b_2^\alpha p_2 + b_3^\alpha p_3}{|b_1|^\alpha p_1}\} = \max\{\frac{0.45}{0.5^{0.88} \times 0.2 + 0.35}, \frac{0.5^{0.88} \times 0.2 + 0.35}{0.45}\} = 1.0193$, and $\epsilon > 0$. For this three-point distribution, $b_2^\alpha p_2 + b_3^\alpha p_3 - |b_1|^\alpha p_1 \approx 0.0087$, which means that the perceived market opportunity is good. We summarize the approximate optimal solutions of (1) computed using MATLAB corresponding to different loss aversion coefficients in Table 1.

Table 1
Good perceived market opportunities but optimally short the stock.

ϵ	0.5	1	2	5	10	20	50
Optimal solution	−1.0590	−1.0054	−1.0002	−1.0000	−1.0000	−1.0000	−1.0000

For the three-point distribution $(b_1, p_1; b_2, p_2; b_3, p_3)$ in Example 2, we can construct one "asymmetric" three-point distribution $(-b_3, p_3; -b_2, p_2; -b_1, p_1)$ that can be used to show that a loss-position CPT-investor probably longs the stock in the presence of bad perceived market opportunities. Furthermore, for a three-point distribution $(b_1, p_1; 0, p_2; b_3, p_3)$ with $b_3^\alpha p_3 > |b_1|^\alpha p_1$, $K > \frac{b_3^\alpha p_3}{|b_1|^\alpha p_1}$ (respectively, $|b_1|^\alpha p_1 > b_3^\alpha p_3$, $K > \frac{|b_1|^\alpha p_1}{b_3^\alpha p_3}$) and a sufficiently small p_2, the result in Section 3 for two-point distributions tells us that possibly a loss-position CPT-investor optimally longs (respectively, shorts) the stock in a good (respectively, bad) perceived market.

The above discussions reveal that the result on investment directions in EUT also holds in CPT for gain-position investors when CPT-investors are sufficiently loss-averse. But not like in EUT, there is no simple and unified market condition that can determine the investment direction of loss-position CPT-investors for all distributions of the excess return. Whether the actual market opportunity can determine the investment direction of moderately loss-averse gain-position investors needs to be strictly proven or disproved by constructing counterexamples. These findings are interesting and provide insights into the characterization of market conditions that determine the investment direction of CPT-investors.

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