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The impact of a reference point determined by social comparison on wealth growth and inequality[☆,☆☆]

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We study the impact of a reference point determined by social comparison on wealth growth and inequality. The reference point of each individual investor contains both personal and social components. Whereas the personal component depends on the investor's own history of wealth, the social component is determined by the wealth level of other investors in her network. In the benchmark case without social interactions and under the assumption of homogeneous preferences, each investor's expected wealth grows at a common rate, the wealth gaps widens and the Gini coefficient remains constant. On the other hand, if the reference point is determined solely by social interactions, then it is possible that the network simultaneously experiences high wealth growth and a reduction in inequality. Finally, for the general case where the reference point incorporates both personal and social components and some extensions of the model, we numerically show that increasing the degree of social interactions is beneficial for both increasing wealth growth and reducing inequality.

1. Introduction

In financial markets, investors' decisions are often influenced not only by their individual experiences in the market and perceptions of the market opportunity, but also by other investors' behavior and performance. This motivates us to formulate a behavioral portfolio choice model where the reference point contains two components: a personal component

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☆☆ With deep sorrow, we have to inform you that our co-author, mentor, and friend Professor Duan Li passed away on November 16, 2020. Professor Duan Li initiated this project and many of his ideas are reflected throughout the paper. We are eternally grateful for to privilege to collaborate with him.

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depending on the investor's own wealth history and a social component depending on the wealth levels of other investors in her network. Our main objective is to study the impact of social interactions on wealth growth and inequality in such a network model.

Our model builds on key ingredients of the (cumulative) prospect theory of Kahneman and Tversky (1979) and Tversky and Kahneman (1992). Their pioneering work aims to capture investors' psychology in decision-making, which is ignored by the classical expected utility theory. One of the main elements of prospect theory is that people evaluate their assets on gains and losses with respect to a reference point.[1] We incorporate a reference-dependent and loss averse value function into a multi-investor, infinitely repeated portfolio choice model in an environment of social networks. All investors are assumed to be myopic[2] in the sense that, at the beginning of each period, every investor maximizes an S-shaped, reference dependent value function for the current time period in a pure-investment framework with exogenous asset prices based on her wealth level and her current reference point.[3] At the end of each time period, investors adjust their reference points based on their own wealth levels as well as the wealth levels of the neighbors in their social network and proceed forward to make the optimal investment decision for the next period. This process repeats infinitely many times.

The most prominent feature of our model is that investors' reference points contain a social component. The reference points impact each investor's investment behavior and the investment outcome under this behavior in turn impacts investors' reference point formation. The wealth dynamics and reference points hence evolve jointly and mutually depend on one another. We are interested in the impact of the reference point formation on the long run evolution of wealth growth and inequality as measured by the wealth gap and the Gini coefficient of the network.

Since our investors are myopic, we start with a single-period behavioral portfolio choice model for a single investor with an arbitrary reference point. Such a problem was investigated in greater generality for example in He and Zhou (2011a). We here do not take probability weighting into account in order to get more explicit results for a general position of the reference point. We find that the optimal investment strategy (OIS) is piecewise linear and increasing in the distance between current wealth and reference point. Besides obtaining the OIS, we are also able to determine conditions under which a loss averse investor with an S-shaped utility function would long, respectively short a risky asset. Whereas a classical expected utility maximizer would always take a long position in a stock with a positive expected excess return, this is not necessarily the case if the utility function is not concave.

We then extend the single-period setting for a single-investor with an arbitrary reference point to a multi-period model for a social network of multiple investors who form their respective reference points as a convex combination of both personal and social components. We consider two possibilities for the personal component: Either the reference point is updated recursively as a function of the investor's previous reference point and current wealth as was suggested in Arkes et al. (2008, 2010), or the reference point is updated non-recursively taking the whole history of wealth levels into account as was proposed by Baucells et al. (2011). These models have already been incorporated into dynamic behavioral portfolio selection models, for example, the recursive updating rule was employed in Shi et al. (2015b) and He and Yang (2019), and the non-recursive updating rule in Strub and Li (2020). A reference point containing a social component on the other hand has not yet been studied in the existing literature on portfolio selection to the best of our knowledge, although there have been some empirical work in other fields, for example, Viglia and Abrate (2014) study the effect of social comparison on reference point formation in a service context. We propose that such a reference point satisfies the two properties of "Upward-Looking" - each investor's reference point is never strictly lower than her own wealth - and "Moderation" - the difference between each investor's wealth and her reference point, is in some sense proportional to the wealth gap to the richest person within her network. These properties are motivated by empirical evidence, see, e.g., Collins (1996), Duesenberry (1949), or Ferrer-i-Carbonell (2005).

We first consider the case where the reference point is solely determined by the personal component and there are no social interactions. This case serves as a benchmark for our later analysis. We find that, in this case, each investor's wealth grows at a common rate and inequality as measured by the generalized Gini coefficient remains constant over time.

We then investigate in detail the case where the reference point is determined by social comparison alone, independent of each investor's history of wealth. We first show that a positive wealth gap is necessary and sufficient for a sustained wealth growth in this case. We then consider a specific reference point formation rule, where the reference point is a convex combination of the individual investor's wealth level and the maximum wealth level among all the members in her network. We call the weight given to the maximum wealth level in her network the coefficient of aspiration as it represents how strong the investors aspire to achieve high social status. We establish a threshold for the coefficient of aspiration which determines the long run behavior of the network and which can be computed in semi-closed form. If the coefficient of aspiration is above the threshold, each investor's expected wealth grows indefinitely with a common growth rate and the expected wealth gap diverges. We find that it is possible to have both wealth growth and a reduction of inequality in the

[1] The other three main elements of prospect theory are: (i) People are risk averse on gains and risk seeking on losses; (ii) People are loss averse, i.e., they are more sensitive to losses than gains; and (iii) People overweight small probabilities and underweight large probabilities. In this paper, we do take into account reference-dependence, loss aversion, and diminishing sensitivity to gains and losses. We ignore probability distortions for simplicity.

[2] Myopia of investors can be justified by empirical findings showing that investors often focus on their investment in the short term without taking into account the long-term returns (Kahneman and Lovallo, 1993; Thaler et al., 1997).

[3] We herein consider a social network consisting of a finite number of investors not representing the whole economy. In such a setting, the assumption of exogenous asset prices is reasonable.

presence of social interactions. On the other hand, if the coefficient of aspiration is below the threshold, each investor's expected wealth converges to a common finite level and the expected wealth gap disappears.

Finally, we consider a general reference point containing both personal and social components. We term the weight given to the social component the degree of social interactions and investigate numerically how it influences the long term dynamics of wealth growth and inequality. Our main finding is that, within our model, increasing the degree of social interactions typically is beneficial for achieving higher wealth growth and lower inequality simultaneously. In addition, we also consider several extensions of the model such as a no-bankruptcy constraint, heterogeneous risk attitude, and heterogeneous loss aversion, and find that the main finding is robust with regards to these extensions. The intuition behind this finding is as follows. Social interactions move the reference point upward from the benchmark case where the reference point does not include a social component. For an investor whose current wealth lies above the personal component of the reference point, taking social interactions into account thus typically leads to a reduction in the distance between wealth and reference point and, therefore, to smaller investments in the risky asset. These tend to be the richer investors of the network. Vice versa, for those investors whose wealth lies below their personal component of the reference point, taking the social component into account increases the distance between their wealth and the reference point and thus leads to higher investments in the risky asset. The social component thus tends to cause richer investors to reduce their appetite for risk because of a fear of loosing their position in the social network and falling below the reference point, while it simultaneously increases the risky investments of the poorer investors because they want to catch up with the richer ones. Because excess returns of the risky asset are positive on average, this leads to a reduction in inequality.

We remark that empirical evidence shows that richer investors typically invest more in stocks than poorer ones, and that this is a possible factor behind rising inequality, see, e.g., Fagereng et al. (2020). The reasons allowing richer investors to hold more stocks than poorer investors are manifold and integrating them into our model goes well beyond the scope of this paper. We stress that it is not our objective to build a model which replicates empirical wealth dynamics, but, rather, to study the effect of social interaction in isolation of other possible constraints on investors strategies.

1.1. Related literature

Behavioral Portfolio Choice. Behavioral portfolio choice problems under the framework of prospect theory have been widely studied under different market settings, see for example Barberis et al. 2001; Barberis and Xiong 2009; Berkelaar et al. 2004; Carassus and Rásonyi 2015; He and Zhou 2011a; 2011b; Jin and Zhou 2008; Shi et al. 2015a; 2015b; Strub and Li 2020.[4] For the single-period setting, Bernard and Ghossoub (2010) consider a no-shorting constraint and probability weighting, (He and Zhou, 2011a) study a setting with probability weighting and a general value function, and (Pirvu and Schulze, 2012) investigate a model where the excess return follows an elliptical distribution. Most of the existing literature focuses on the qualitative analysis of the properties of the OIS due to the difficulty of deriving an analytical optimal solution, although (He and Zhou, 2011a) are able to derive optimal solutions in analytical form for the two special cases where the value function is piecewise linear or the initial wealth equals the reference point. In this paper, we consider a market model where short-selling is allowed, the utility is isoelastic, there is no probability weighting and the excess return is allowed to follow a general distribution. While in some aspects less general than the existing literature, this specification allows us to obtain the OIS in semi-analytical form and to provide conditions on whether it is optimal to long or short the stock.

Reference Point Formation. The reference point plays a central role in prospect theory. In our model, it refers to a target or aspiration level that an individual investor aspires to attain. Several dynamic reference point adaptation rules have been proposed in the behavioral portfolio choice literature to model how investors adjust their reference points. With the notable exception of the endogenous reference point formation suggested by Kőszegi and Rabin (2006) and the models of mental reference point adjustment of He and Strub (2020), most of the literature considers a reference point determined solely by the history of wealth levels (see, e.g., Arkes et al. 2008; 2010; Baucells et al. 2011; Chiyachantana and Yang 2010; Gomes 2005; Shi et al. 2015b; Strub and Li 2020; Thaler and Johnson 1990).[5] However, in the context of social networks, people inevitably compare themselves to others. Our reference point adaptation rule differs from the existing ones as it depends not only on the investor's own wealth level, but also on wealth levels of the neighbors in her social network.

Wealth Growth and Inequality. The relation between wealth growth and inequality is generally complex and has received much attention in the literature (e.g. Bogliacino and Ortoleva 2013; Corneo and Jeanne 2001; Hopkins and Kornienko 2006; Knell 1999; Li and Zou 1998; Persson and Tabellini 1994; Rauscher 1997). Whereas most of the existing literature studies expected utility maximizers, investors in our model exhibit reference dependence and loss aversion. Most closely related to our work is the recent paper by Genicot and Ray (2017), which develops a theory of socially determined aspirations. In analogy to Genicot and Ray (2017), we also study a model where reference points, wealth growth and inequality evolve jointly and interact with one another. The most notable difference to the pioneering work of Genicot and Ray (2017) is that they consider a deterministic equilibrium framework with consumption, whereas our model is based on a pure investment economy with exogenously determined, stochastic asset prices. Consequently, the wealth trajectories and reference points of the investors in our model are not deterministic, but evolve as stochastic processes. In contrast to the findings of Genicot and

[4] A behavioral portfolio theory on the foundation of prospect theory was also developed in Shefrin and Statman (2000).

[5] There also have been some (empirical) work studying how the current reference price depends on the past sequence of prices in marketing science, e.g., see Ackerman and Perner (2004); Baucells and Hwang (2017); Dickson and Sawyer (1990); Kahneman et al. (1993); Nasiry and Popescu (2011).

Table 1
Model Parameters and Notations.

Symbol		Definition
θ		The amount invested in the risky asset
α		The parameter of risk attitude
β		The degree of social interactions
k		The coefficient of loss aversion
B		The reference point
$\bar{B}$		The relative wealth $W_0 - B$
R		The excess return of the risky asset in the single-investor, single-period model
$V(\cdot)$		The behavioral evaluation function
θ^*		The optimal investment strategy (OIS)
γ^*		The piecewise linearity coefficient in the OIS
γ_+ (γ_-)		The OIS (the negative value of the OIS) if $\bar{B}=1$ ($\bar{B}=-1$)
$\{R(t)\}_{t\geq 0}$		The i.i.d. excess returns of the risky asset
$F(\cdot)$		The cumulative distribution function (CDF) of the excess return
$f(\cdot)$		The probability density function (PDF) of the excess return
μ		The mean of the excess return
λ		The coefficient of aspiration
$\phi_\lambda(t)$	$\triangleq$	$R(t)\lvert\gamma_-\rvert\lambda$
ξ_λ	$\triangleq$	$\mathbb{E}[(1-\phi_\lambda(t))\mathbf{1}_{\{\phi_\lambda(t)\leq 1\}}]$, where $\mathbf{1}_{\{\cdot\}}$ is the indicator function.
η_λ	$\triangleq$	$\mathbb{E}[(\phi_\lambda(t)-1)\mathbf{1}_{\{\phi_\lambda(t)\geq 1\}}]$
n		Network size
$\mathcal{N}_i$		The neighbor set of investor i in the graph $\mathcal{G}$
$H(t)$ ($h(t)$)		The maximum (minimum) of investors' positions at time t
$E(\mu,\sigma,g)$		The elliptical distribution with mean μ and a strictly decreasing density generator g
$T(a_0,p_0;a_1,p_1)$		The two-point distribution taking two values a_0, a_1 with respective positive probabilities p_0, p_1, where $a_0<0<a_1$ and $p_0+p_1=1$.

Ray (2017), it is the coefficient of aspiration, not the degree of inequality of the initial wealth distribution, which determines the asymptotic behavior of wealth growth and inequality in our model.

The remainder of this paper is organized as follows. In Section 2 we present the single-period behavioral portfolio choice, determine the OIS and study under what conditions it is optimal to long the risky asset. We then extend the model to a multi-period setting of a social network of multiple investors and discuss plausible models for the reference point for this setting in Section 3. Our main results are in Section 4, where we study the impact of social interactions on wealth growth and inequality. Finally, we conclude this paper in Section 5. For a clear presentation of the main story line and the key results, we place all the proofs in the appendix of this paper. To help readers to follow our presentation with ease, we summarize the model parameters and notations in Table 1.

2. Single-period behavioral portfolio choice

We start our analysis with a single-investor behavioral portfolio choice model for a single investor. This problem was studied in greater generality in He and Zhou (2011a), who also consider probability weighting and are able to obtain explicit solutions for two special cases, namely when the reference point coincides with the risk-free return or when the value function is piecewise linear. Disregarding probability weighting on the other hand allows us to get more explicit results when taking diminishing sensitivity into account and allowing for a general position of the reference point. The latter point is particularly important for the aim of this paper as we later want to study the effect of social reference point formation on wealth growth and inequality. This section also contributes to the literature by showing conditions under which it is optimal for a loss averse investor with S-shaped utility function to long, respectively to short a risky asset.

The financial market under consideration consists of one risk-free account and one risky asset (stock) with stochastic return $\tilde{R}$. We assume that the risk-free asset does not generate interest for simplicity. Let W_0 be the initial wealth of an individual investor at the beginning of an investment period, and let θ denote the amount that she invests in the stock. The remaining wealth $W_0-\theta$ is invested in the risk-free account such that the strategy is self-financing. The investor's total wealth at the end of this period is then given by $W_0+(\tilde{R}-1)\theta$. We do not prohibit short selling. The excess return $R:=\tilde{R}-1$ is a random variable with cumulative distribution function (CDF) $F(\cdot)$. To exclude arbitrage opportunities we assume that $0<\mathbb{P}(R<0)<1$ and $0<\mathbb{P}(R>0)<1$.

The preferences of the investor are incorporating key ingredients of prospect theory: reference dependence, loss aversion, and an S-shaped utility function implying locally risk seeking behavior in the loss region and locally risk averse behavior in the gain region.[6] More specifically, if the reference point separating gains and losses is given by B, then the expected value

[6] When probability distortion is not considered, prospect theory reduces to expected utility maximization with an S-shaped utility function. The formulation of prospect theory of Kahneman and Tversky (1979) incorporates distortion of probabilities. We here do not consider probability weighting for

of a random variable X with CDF F_X is defined as

$$V(X) = \int_B^{+\infty} (x - B)^\alpha dF_X(x) - k \int_{-\infty}^{B} (B - x)^\alpha dF_X(x), \tag{1}$$

where $k > 1$ is the coefficient of loss aversion and $0 < \alpha < 1$ is the parameter of risk attitude[7].

Let $\bar{B} = W_0 - B$ denote the relative wealth of the investor. The expected utility evaluation of a given strategy θ, denoted as well by $V(\theta)$ by a slight abuse of notation, can then be expressed as

$$V(\theta) = \int_{-\frac{\bar{B}}{\theta}}^{+\infty} (\theta s + \bar{B})^\alpha dF(s) - k \int_{-\infty}^{-\frac{\bar{B}}{\theta}} (-\theta s - \bar{B})^\alpha dF(s)$$

when $\theta > 0$, as

$$V(\theta) = \int_{-\infty}^{-\frac{\bar{B}}{\theta}} (\theta s + \bar{B})^\alpha dF(s) - k \int_{-\frac{\bar{B}}{\theta}}^{+\infty} (-\theta s - \bar{B})^\alpha dF(s)$$

when $\theta < 0$, and as

$$V(0) = \begin{cases} \bar{B}^\alpha, & \bar{B} \geq 0; \\ -k|\bar{B}|^\alpha, & \bar{B} < 0. \end{cases}$$

The objective of the investor is to find the optimal investment strategy (OIS) θ^* such that

$$V(\theta^*) = \max_{\theta \in \mathbb{R}} V(\theta). \tag{2}$$

Since the behavioral evaluation function is not convex it might occur that there are multiple optimal investment strategies. In this case, we assume that the investor will choose the largest solution.[8] This assumption also applies to the infinitely repeated model we develop later.

Throughout this paper we make the following two assumptions.

Assumption 1. The CDF, $F(\cdot)$, of the excess return R is either a two-point distribution or absolutely continuous with the property that there exists an $\epsilon > 0$ such that the probability density function (PDF) $f(s) = O(|s|^{-2-\epsilon})$ for sufficiently large $|s|$.

Assumption 2. The following well-posedness condition holds,

$$k > \max\left\{ \frac{\int_0^{+\infty} s^\alpha dF(s)}{\int_{-\infty}^{0} |s|^\alpha dF(s)}, \frac{\int_{-\infty}^{0} |s|^\alpha dF(s)}{\int_0^{+\infty} s^\alpha dF(s)} \right\}, \tag{3}$$

where α is the parameter of risk attitude.

Assumption 1 is not essential for the analysis presented herein, and our results would in particular also hold in case of discrete distributions other than the two-point distribution. We restrict distributions to the two-point case mainly to explicitly illustrate and discuss the case where the investor takes a short position in a stock with a positive expected return in Appendix A. Assumption 2 on the other hand is essential and guarantees that the behavioral portfolio optimization model under consideration is well-posed and both the OIS θ^* and the optimal evaluation value $V(\theta^*)$ are finite, cf. He and Zhou (2011a). We refer to He and Zhou (2011a) for a discussion of Assumption 2.

It is clear that when the relative wealth $\bar{B} = W_0 - B = 0$, $\arg\max_{\theta \in \mathbb{R}} V(\theta) = 0$. The following theorem shows that the OIS is piecewise linear as a function of the relative wealth and provides conditions on whether it is optimal to long or short the stock.

Theorem 1. *Consider the behavioral portfolio choice problem (2) with $\bar{B} \neq 0$. Then there exists γ^* which depends only on α, k, F, and the sign of $\bar{B}$ (but not the absolute value of $\bar{B}$) such that the OIS θ^* takes the form of*

$$\theta^* = \arg\max_{\theta \in \mathbb{R}} V(\theta) = \gamma^* \bar{B}.$$

We emphasize that the piecewise linearity coefficient γ^* depends only on the parameter of risk attitude α, the coefficient of loss aversion k, and the market opportunity and in particular not on the relative wealth $\bar{B}$. The piecewise linear structure turns out to be very convenient, because it suffices to solve (2) only twice for the cases of $\bar{B} = 1$ and $\bar{B} = -1$, respectively.

simplicity. The main result of this section, namely the piecewise linear structure of the optimal policy, would still hold under probability weighting. However, it would be harder to identify whether the investor takes a long or short position in the stock.

[7] We remark that positive homogeneity of utility functions is sufficient for the optimal investment strategy of the single-period optimization problem (2) to be (piecewise) linear. The main theoretical results in Sections 4.1 and 4.2 therefore hold for any positive homogeneous utility function, and not just for power utility functions, provided that the market is good enough so that the investor always optimally longs the stock (i.e., Proposition 1 holds).

[8] Our results would also hold when taking another solution as long as the investor selects her strategy according to a predefined rule.

Denoting the resulting optimal coefficients by γ_+ and $-\gamma_-$ respectively (i.e., $\gamma^* = \gamma_+$ in the case of $\bar{B} = 1$ and $\gamma^* = \gamma_-$ in the case of $\bar{B} = -1$), the OIS must be $\gamma_+\bar{B}$ for any positive relative wealth $\bar{B}$ and $\gamma_-\bar{B}$ for any negative relative wealth $\bar{B}$. Similar results of piecewise linear structure can be found for example in Shi et al. (2015a), Shi et al. (2015b), or Strub and Li (2020).

For the upcoming analysis of the infinitely repeated behavioral portfolio choice model, we suppose that the coordination between the investor's preferences and the market is such that the stock offers a positive expected excess return and the investor takes a long position in the stock. We make the following assumption in the rest of the paper and refer to Appendix A for further insight on the investment direction and results on the wealth dynamics in a social network where Assumption 3 is violated.

Assumption 3. The expected excess return of the stock is positive and either of the following two conditions hold

(i) the CDF $F(\cdot)$ of the excess return is absolutely continuous and $F(s) + F(-s)$ is strictly increasing on $s > 0$;[9]
(ii) the CDF $F(\cdot)$ of the excess return is a two-point distribution $T(a_0, p_0; a_1, p_1)$ and $a_1^\alpha p_1 > |a_0|^\alpha p_0$.

Proposition 1. *The investor never takes a short position in the stock.*

3. Dynamic behavioral portfolio choice and reference point formation for a social network

We now extend the setting of the previous section to a multi-period framework for a social network. The social network consists of multiple investors described by a node set $\mathcal{V} = \{1, \ldots, n\}$ $(n \geq 2)$. Each investor holds a deterministic positive initial wealth $x_i(0)$ and invests in a financial market consisting of one risky asset and one risk-free asset. The key feature of our model is that investors not only consider their own current and past wealth levels but also observe the wealth levels of the investors in the network when forming their reference points. The links among investors can be characterized by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the arc set. The graph is undirected and connected, i.e., $(i, j) \in \mathcal{E}$ if and only if $(j, i) \in \mathcal{E}$ and there is a path between any pair of nodes in the graph. Let $\mathcal{N}_i = \{j | (j, i) \in \mathcal{E}\}$ denote the neighborhood set of investor i. Since each investor knows her own wealth level it is natural to assume that $(i, i) \in \mathcal{E}$, i.e., $i \in \mathcal{N}_i$ for all i.

We consider an infinite investment horizon and investment periods: [0,1), [1,2),...,$[t, t+1)$,.... The stochastic return and excess return of the t-th time period $[t, t+1)$ are denoted by $\tilde{R}(t)$ and $R(t) := \tilde{R}(t) - 1$, respectively. The excess returns are assumed to be i.i.d. over time with CDF $F(\cdot)$. All investors are myopic, i.e., at each time t they only consider the optimal portfolio choice problem for the time span $[t, t+1)$. We propose that investors' reference points are dynamic, adaptive over time and composed of two components: A personal component determined by each investor's individual history of wealth and a social component determined by comparison with the wealth levels of the other investors in the network. To be specific, at the beginning of each time period $[t, t+1)$, every investor i updates her reference point as a convex combination of her personal reference point $B_i^p(t)$ and the reference point determined by social comparison $B_i^s(t)$,

$$B_i(t) = (1 - \beta)B_i^p(t) + \beta B_i^s(t), \tag{4}$$

where $0 \leq \beta \leq 1$ represents the weight given to the social component of the reference point. We term β the *degree of social interactions*. She then solves the behavioral portfolio optimization problem (2) with respect to the reference point $B_i(t)$ and initial wealth $W_0 = x_i(t)$, to determine her OIS $\theta_i^*(t)$. At the end of this investment period, investor i receives returns from her investment and now owns $x_i(t+1) = x_i(t) + \theta_i^*(t)(\tilde{R}(t) - 1)$. This process repeats over all subsequent time periods. In the following we explicate in detail how the personal and social components of the reference point are formed respectively.

There are two competing rules for personal reference point updating based on past decisions and outcomes in a dynamic setting: The non-recursive updating rule of Baucells et al. (2011) and the recursive updating rule of Arkes et al. (2008, 2010). Both of those rules have been employed to study decision making in a financial context, the non-recursive rule in Strub and Li (2020) and the recursive rule in Shi et al. (2015b) and He and Yang (2019). Under the non-recursive updating rule, the personal reference point is determined by

$$B_i^p(t) = \rho x_i(t) + \sum_{r=0}^{t} \left(w\left(\frac{r+1}{t+1}\right) - w\left(\frac{r}{t+1}\right) \right) x_i(r), \quad t \geq 0. \tag{5}$$

The first term represents a built-in profit while the second term is a weighted sum of the investor's own past wealth levels. The weighting function w with $w(0) = 0$ and $w(1) = 1$ is typically inverse S-shaped, which leads to considerable weight being given to the current and the initial wealth levels and lower weights to the intermediate wealth levels.

On the other hand, under the recursive updating rule the personal reference point is formed as a function of the current wealth and previous reference point by

$$B_i^p(t) = \begin{cases} B_i^p(t-1) + \alpha_g\big(x_i(t) - B_i^p(t-1)\big), & \text{if } x_i(t) \geq B_i^p(t-1), \\ B_i^p(t-1) + \alpha_l\big(x_i(t) - B_i^p(t-1)\big), & \text{if } x_i(t) < B_i^p(t-1), \end{cases} \tag{6}$$

[9] We can show that when the excess return follows an elliptical distribution with a positive (negative) mean, $F(s) + F(-s)$ is strictly increasing (decreasing) on $s > 0$.

where $\alpha_g, \alpha_l \in [0, 1]$ are updating coefficients for prior gains and losses respectively. Note that $\alpha_g < 1$ and $\alpha_l < 1$ mean that the adaptation is partial. Arkes et al. (2008) have found that typically $\alpha_g > \alpha_l$, which implies that the reference point adapts faster to gains than to losses. This updating rule also requires to determine an initial reference point which we set to $B_i^p(0) = (1+\rho)x_i(0)$ such that the initial reference points under the recursive and non-recursive updating rules coincide.

A detailed comparison between the two updating rules can be found in Strub and Li (2020). While they find that the non-recursive updating rule leads to more realistic trading behavior, and the regression analysis in Baucells et al. (2011) also provides evidence showing that reference points are generally not recursive, we will consider both updating rules for the theoretical part of the paper and even focus on the recursive updating rule for the numerical analysis. The reason is that the evidence supporting the non-recursive updating rule is coming from shorter time horizons, up to 10 price sequences for a stock in Baucells et al. (2011) and a time horizon of one year in Strub and Li (2020) to be specific. For the current paper on the other hand we are interested in the long term wealth dynamics of the network. It is very plausible that, when an investor buys a stock and sells it a year later, then the purchasing price of the stock plays an important role in determining the reference point. However, it is not obvious at all that the wealth an investor owned at the age of twenty is an important component of the investor's reference point at the age of sixty.

For the social component B_i^s of the reference point in (4) we propose the following two properties:

- Upward-Looking: $x_i(t) \le B_i^s(t)$ for any $t \ge 0$;
- Moderation: There exists $0 < \iota < 1$ such that for any $t \ge 0$

$$\iota\left(\max_{j\in\mathcal{N}_i} x_j(t) - x_i(t)\right) \le B_i^s(t) - x_i(t) \le \max_{j\in\mathcal{N}_i} x_j(t) - x_i(t).$$

The "Upward-Looking" property states that each investor's reference point is never strictly lower than her own wealth. This property is in line with the argument in Collins (1996) that "... people often compare themselves with those whose abilities and attributes are better than their own" and the observation in Duesenberry (1949) and Ferrer-i-Carbonell (2005) that people generally look to those who are richer than themselves when they form their aspirations. Upward-looking aspiration/reference point formations have been studied for example in the supplement of Genicot and Ray (2017). Taking the personal component as a benchmark, the "Upward-Looking" property of the social component will shift the reference point upward. The effect this property is having on the risk taking of an individual investor thus depends on where the current wealth stands with regards to the personal component of the reference point. If the current wealth lies above the personal component, social interactions will reduce the amount invested in the stock since the applying reference point will move closer to the current wealth and the optimal strategy is piecewise linear in the difference between the two. On the other hand, if the current wealth lies below the personal component of the reference point, social interactions will increase the amount invested in the stock since they further increase the perception of being in a loss position of an individual investor.

The "Moderation" property asserts that the incentive, defined as the difference between the investor's reference point and her own wealth, is neither too small nor too large compared to the wealth gap to the richest person within her network. This property is consistent with the argument that "... goals that lie ahead–but not too far ahead–provide the best incentives" suggested by Genicot and Ray (2017) (Page 490). These two properties are maintained throughout this paper.

Remark 1. Genicot and Ray (2017) propose a general aspiration formation function which is formed by the whole population's wealth level and derive the implications of some reasonable properties of the formation function (regularity, scale-invariance, range-boundedness and social monotonicity) on economic growth. In fact, the "Upward-Looking" property and the upper bound assumption of the "Moderation" property coincide with the "Range-Boundedness" assumption of Genicot and Ray (2017) when one would limit the investor's "vision" or "aspiration window" to those neighbors who are not poorer than herself.

4. Wealth growth and inequality

This section contains the main results of our paper where we present the implications of social interactions on wealth growth and inequality in our model. We study two extreme cases: First, we consider the case where there are no social interactions and the reference point is determined by each investor's own history of wealth which serves as a benchmark. Second, we study the case where the reference point is formed by social comparison alone independent of the investor's own history of wealth. Finally, we will consider the general case where the reference point is determined by both the personal and social components. Let us start with some definitions.

We denote by

$$H(t) = \max_{1\le i\le n} x_i(t), \quad h(t) = \min_{1\le i\le n} x_i(t)$$

the maximum and minimum wealth levels in the network at the beginning of the t-th time period. The *wealth gap* of the network at the beginning of the t-th time period is then given by $H(t) - h(t)$. To avoid the trivial case, we assume that the wealth gap exists in the initial wealth distribution, i.e., $H(0) > h(0)$.

We refer to $\mathbb{E}[x_i(t)]$ as investor i's expected wealth at time t. One goal of this paper is to investigate the effect of the wealth gap and reference point on the wealth growth, i.e., how they influence the expected wealth dynamics $\{\mathbb{E}[x_i(t)]\}_{t\ge 0}$ over time. For the sake of concise terminology, we will use *wealth growth* to refer to growth of the expected wealth.

In addition to the wealth gap as a measure of absolute inequality introduced above, we are also interested in the *generalized Gini coefficient* as a measure of relative inequality. Let $(y_1, ..., y_n)$ denote $(x_1, ..., x_n)$ in ascending order and τ be the unique index satisfying that $\sum_{i=1}^{\tau} y_i \leq 0$ and $\sum_{i=1}^{\tau+1} y_i > 0$ if $y_1 < 0$, $\tau = 0$ otherwise. Then the *generalized Gini coefficient* (at time t) is defined as

$$GGC(t) \triangleq \begin{cases} NGC(x_1(t), ..., x_n(t)), & \text{if } \sum_{i=1}^{n} x_i(t) > 0; \\ NGC(-x_1(t), ..., -x_n(t)), & \text{if } \sum_{i=1}^{n} x_i(t) < 0, \end{cases}$$

where

$$NGC(x_1, ..., x_n) \triangleq \begin{cases} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j|}{2n\sum_{i=1}^{n} y_i}, & \text{if } \tau = 0; \\ \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j|}{2n\sum_{i=1}^{n} y_i} \frac{1}{1 + \frac{2}{n}\frac{\sum_{j=1}^{\tau} j y_j}{\sum_{i=1}^{n} y_i} + \frac{1}{n}\frac{\sum_{j=1}^{\tau} y_j}{\sum_{i=1}^{n} y_i}\left[\frac{\sum_{j=1}^{\tau} y_j}{y_{\tau+1}} - (1+2\tau)\right]}, & \text{otherwise,} \end{cases}$$

is the normalized Gini coefficient proposed in Berrebi and Silber (1985) for wealth distributions where some wealth levels are allowed to be negative, but with positive total wealth (i.e., $\sum_{i=1}^{n} x_i > 0$). The normalized Gini coefficient was first proposed in Chen et al. (1982) for a limited case of wealth distribution where there exists a t such that $\sum_{j=1}^{t} y_j = 0$. The adjustment factor is introduced in order to ensure that the value of the Gini coefficient is not greater than one.

The generalized Gini coefficient introduced above allows for more general wealth distributions where the total wealth is allowed to be negative. This is necessary for a model without restrictions on the investment strategies.

It is evident that the generalized Gini coefficient is proportional to the ratio of the total wealth gap within the network over the total wealth, with an adjustment factor due to the possibility of negative wealth levels. To a certain degree, the numerator in the generalized Gini coefficient plays a similar role as the wealth gap we have defined earlier. The generalized Gini coefficient thus combines the two measures of wealth gap and total wealth in a single number. It lies between zero and one, where one represents complete inequality and zero perfect equality.

4.1. The case without social interactions

We first consider the case where there are no social interactions, which serves as a benchmark. The reference point of investor i is updated recursively, either by means of the non-recursive updating rule in (5) or the recursive updating rule in (6).

Proposition 2. *Suppose that each investor employs the same updating rule for the personal component of the reference point whose evolution is specified either in (5) or (6) and there are no social interactions, i.e., $\beta = 0$. Then the optimal terminal wealth of investor i is given by*

$$x_i(t) = S(t)x_i(0), \ \ i = 1, \dots, n,$$

where $S(t)$ is an $\mathcal{F}_t$-measurable random variable which does not depend on the specific investor, with $\mathcal{F}_t$ being the σ-algebra generated by the risky returns $R(s), s \leq t-1$.

Remark 2. We can easily see that Proposition 2 still holds when there is an additional constraint that the amount invested by each investor at each time period is not greater than her current wealth level, and the return of the risky asset is nonnegative.

From Proposition 2 we immediately obtain the following corollary on the wealth gap, wealth growth and generalized Gini coefficient.

Corollary 1. *Suppose that there are no social interactions, i.e., $\beta = 0$, and each investor employs the same updating rule for her personal component of the reference point as in Proposition 2. Then*

(i) *The expected wealth gap of the network evolves as $\mathbb{E}[H(t) - h(t)] = (H(0) - h(0))\mathbb{E}[S(t)]$;*
(ii) *The expected wealth growth of each investor is given by $\mathbb{E}[x_i(t)] = x_i(0)\mathbb{E}[S(t)] > \mathbb{E}[x_i(t-1)]$;*
(iii) *The generalized Gini coefficient of the network remains constant, $GGC(t) = GGC(0)$.*

In plain language, Corollary 1 tells us that if there are no social interactions, then each investor's wealth, and consequently the wealth gap within the network, grows at a linear rate. In particular, relative inequality as measured by the generalized Gini coefficient remains constant over time.

4.2. The case without a personal component

In the case where each investor's reference point is determined by social interactions alone, investor i's reference point at time t is equal to $B_i^s(t)$. We refer to the difference between investors' reference points and their current wealth levels as the *incentives* to invest. High reference points generally induce large incentives for investors to invest in the risky asset.

Note that for any i and t, $B_i^s(t) \geq x_i(t)$ by the "Upward-Looking" property. So by Theorem 1, investor i's wealth ($i = 1, \ldots, n$) evolves in the following way:

$$\begin{aligned} x_i(t+1) &= x_i(t) + R(t)\gamma_-[x_i(t) - B_i^s(t)] \\ &= x_i(t) + R(t)|\gamma_-|[B_i^s(t) - x_i(t)]. \end{aligned} \tag{7}$$

The following proposition establishes a relationship between the expected cumulative wealth of the network and the expected wealth gap.

Proposition 3. *Suppose that there are only social interactions in the reference point updating, i.e., $\beta = 1$. Then each investor's expected wealth will increase over time. Furthermore, it is necessary and sufficient for a sustained wealth growth without upper bound, i.e., for $\mathbb{E}[\sum_{i=1}^n x_i(t)]$ to converge to positive infinity as t goes to infinity, that the series of the sum of the expected wealth gaps is divergent. Specifically, we have that for all t,*

$$\iota\mu|\gamma_-|\sum_{r=0}^{t}\mathbb{E}[H(r) - h(r)] \leq \mathbb{E}\Big[\sum_{i=1}^{n} x_i(t+1)\Big] - \sum_{i=1}^{n} x_i(0) \leq n\mu|\gamma_-|\sum_{r=0}^{t}\mathbb{E}[H(r) - h(r)],$$

where ι is defined in the definition of "Moderation" and n is the network size. Consequently, the following are equivalent:

(a) $\lim_{t\to\infty}\mathbb{E}[H(t)] < +\infty$;
(b) $\lim_{t\to\infty}\mathbb{E}[x_i(t)] < +\infty$, $i = 1, \ldots, n$;
(c) $\sum_{t=0}^{\infty}\mathbb{E}[H(t) - h(t)] < +\infty$.

Furthermore, if $\lim_{t\to\infty}\mathbb{E}[H(t)] =: H^* < +\infty$, then $\lim_{t\to\infty}\mathbb{E}[x_i(t)] = H^*$ for all i.

The above proposition states that, if the incentive to invest is small in the sense that the series of the sum of the expected wealth gaps is bounded, investors' expected wealth will grow to a common finite level. On the other hand, if the incentive is large enough for the series of the sum of the expected wealth gaps to diverge, investors' expected wealth will grow without upper bound. This implies that a large wealth gap is beneficial for wealth growth.

In the above proposition, we investigated the impact of absolute wealth inequality on wealth growth. Absolute wealth inequality affects investors' incentives to invest and the investment outcomes in turn determine wealth growth and affect investors' reference point formation. Wealth inequality, wealth growth and the reference points evolve jointly and in mutual dependence. We proceed in the following to explore the evolution of wealth inequality and wealth growth for complete graphs under a specific choice for the reference point formation rule. We impose the following assumption for the rest of this paper. The network graph $\mathcal{G}$ is said to be complete if $\mathcal{N}_i = \{1, \ldots, n\}$ for any i, or in other words, every investor compares her own wealth to the wealth of every other investor in the network.

Assumption 4. The network graph is complete and the reference point of investor i at time period t is given by

$$B_i^s(t) = \lambda H(t) + (1-\lambda)x_i(t) \tag{8}$$

for some $\lambda \in [0, 1]$.

The parameter λ in (8) is referred to as the *coefficient of aspiration* of the investors and represents the strength of investors' concern for their relative status/position compared with the richest member of the network. The closer the parameter λ is to one, the more the investor strives to stand at the top. This specific rule satisfies the properties of "Upward-Looking" and "Moderation" as well as all the properties specified for the aspiration formation function in Genicot and Ray (2017). The updating rule captures the empirical evidence that an individual's aspiration level increases with her past income and depends positively on the incomes of others in her reference group. Put differently, her utility is negatively affected by the incomes of others in her reference group (see Clark et al. 2008; Ferrer-i-Carbonell 2005; Luttmer 2005; Senik 2009; Stutzer 2004).

By (7) and the specific reference point formation rule (8), the investor i's wealth ($i = 1, \ldots, n$) evolves in the following way:

$$x_i(t+1) = x_i(t) + \phi_\lambda(t)(H(t) - x_i(t)), \tag{9}$$

where $\phi_\lambda(t) := R(t)|\gamma_-|\lambda$.

We will investigate the impact of the coefficient of aspiration λ on the wealth gap, wealth growth and wealth inequality measured by generalized Gini coefficient under the specific reference point formation rule (8) in the following three subsections.

Remark 3. While we believe that the wealth of the richest member of the social network is the most salient and Assumption 4 is thus intuitively plausible, it does lead to the arguably unrealistic prediction that poorer investors invest more in the stock than richer ones if there is no personal component and the reference point is determined by social interactions alone. Empirical evidence on the other hand shows that poorer investors typically invest less in stocks than richer ones, see, e.g., Fagereng et al. (2020). This suggests that the reference point should contain both a personal and social component, or that other models for the formation of social reference points might also be plausible. In order to replicate empirical wealth

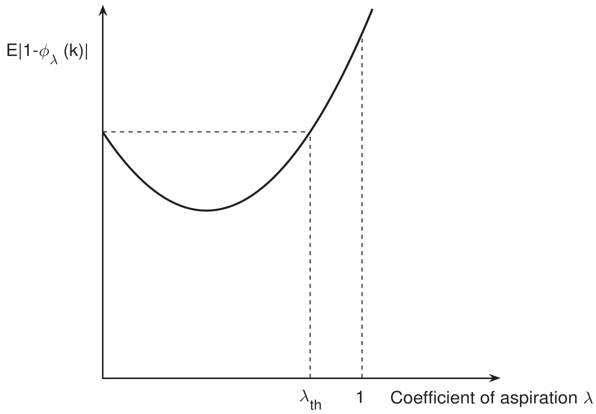


Fig. 1. The wealth gap measure and the coefficient of aspiration. *Notes.* $0 < \lambda_{th} \le 1$ is the threshold such that the wealth gap measure $\mathbb{E}|1-\phi_{\lambda_{th}}(t)| = 1$. When $0 < \lambda < \lambda_{th}$, the expected wealth gap disappears and each investor's expected wealth converges to a common finite level. On the other hand, when $\lambda_{th} < \lambda \le 1$, the expected wealth gap diverges and each investor's expected wealth grows indefinitely with a common growth rate.

dynamics, one would likely have to integrate other factors driving investment decisions into our model. This goes beyond the scope of this paper, which aims to study the impact of a reference point determined by social comparison on wealth growth and inequality in isolation of other factors driving wealth dynamics.

4.2.1. Wealth gap

The main reason why we start the investigation with the wealth gap $H(t) - h(t)$, instead of the equivalent $\sum_{i=1}^n \sum_{j=1}^n |x_i(t) - x_j(t)|$,[10] is that the analysis of the dynamics $(H(t))_{t=0}^\infty$ and $(h(t))_{t=0}^\infty$ can help us to analyze the wealth growth rate.

The following lemma shows that the wealth gap dynamics can be described by a linear iterative equation, where the piecewise linearity coefficient depends only on the market condition and the coefficient of aspiration, but not on investors' current wealth levels.[11]

Lemma 1. *Suppose that there are only social interactions in the reference point updating, i.e., $\beta = 1$. Then it holds that $H(t+1) - h(t+1) = |1-\phi_\lambda(t)|(H(t) - h(t))$ for any $t \ge 0$.*

According to the above lemma and the time-dependence of the excess returns, we have

$$\mathbb{E}[H(t+1) - h(t+1)] = \mathbb{E}[|1-\phi_\lambda(t)|]\mathbb{E}[H(t) - h(t)].$$

It becomes evident that the expected wealth gap will asymptotically disappear if $\mathbb{E}[|1-\phi_\lambda(t)|] < 1$, and widen when $\mathbb{E}[|1-\phi_\lambda(t)|] > 1$. We next present this threshold result.

Proposition 4. *Suppose that there are only social interactions in the reference point updating, i.e., $\beta = 1$. Then there exists a threshold $0 < \lambda_{th} \le 1$, such that if $\lambda < \lambda_{th}$, the expected wealth gap will disappear, and if $\lambda > \lambda_{th}$, the expected wealth gap will widen.*

Proposition 4 shows that a moderate coefficient aspiration incentives the poorer members of the network to catch up with the richest and reduces inequality, while an excessive coefficient of aspiration leads to divergence. The intuition behind this result is as follows. Under a moderate coefficient of aspiration, the poorer members of the network typically have a reference point slightly, but not disproportionately, above their current wealth and thus take positions in the stock which lead them to approach, but not exceed, the wealth of the richest members of the network under a typical sample path. If the aspirations of the investors become too large on the other hand, then the poorer members are so ambitious that they take excessive positions in the stock, often by taking leverage, such that their resulting wealth considerably exceeds those of the richest members after a favorable outcome of the stock market, or drops even further below after an adverse outcome.

The next proposition provides a semi closed-form expression for the computation of the threshold.

[10] In terms of the relation $H(t) - h(t) \le \sum_{i=1}^n \sum_{j=1}^n |x_i(t) - x_j(t)| \le n(n-1)(H(t) - h(t))$, the equivalence is in the sense that the two measures will either converge to a finite number or diverge to infinity with the same rate.

[11] We can show that the maximum and minimum wealth levels can be specified by means of two linear iterative equations with non-state-dependent linearity coefficients for complete graphs, cf. the proof of Proposition 6. This considerably facilitates the analysis of wealth growth. However, this description would fail for general network graphs. The analysis of the wealth gap and wealth growth is rather challenging for general network graphs and will be studied in our future work.

Proposition 5. *Suppose that there are only social interactions in the reference point updating, i.e., $\beta = 1$. Then the following hold.*

(i) *The threshold λ_{th} is given by $\lambda_{th} = \min\left\{\frac{1}{|\gamma_-|s^*}, 1\right\}$, where s^* is the unique root of the following equation*

$$\int_s^{+\infty} r dF(r) + s(F(s) - 1) = \frac{\mu}{2} \text{ on } (0, +\infty). \tag{10}$$

In particular,

(a) *Suppose $R(t)$ follows an elliptical distribution $E(\mu, \sigma, g)$ with $\mu > 0$. Then*

$$\lambda_{th} = \min\left\{\frac{1}{(\mu/\sigma + \bar{s})|\gamma_-|\sigma}, 1\right\},$$

where $\bar{s}$ is the unique root of the following equation

$$\int_s^{+\infty} r f_\Phi(r) dr + s(F_\Phi(s) - 1) = \frac{\mu}{2\sigma} \text{ on } \left(-\frac{\mu}{\sigma}, +\infty\right), \tag{11}$$

F_Φ and f_Φ are the CDF and PDF of the standard elliptical distribution, respectively.

(b) *Suppose $R(t)$ follows a two-point distribution $T(a_0, p_0; a_1, p_1)$ with $a_1^\alpha p_1 > |a_0|^\alpha p_0$. Then*

$$\lambda_{th} = \min\left\{\frac{2p_1}{|\gamma_-|(a_1 p_1 + |a_0| p_0)}, 1\right\}.$$

(ii) *The threshold $\lambda_{th} = 1$ if $\mathbb{E}[|1 - \phi_1(t)|] \le 1$ and $\lambda_{th} < 1$ if $\mathbb{E}[|1 - \phi_1(t)|] > 1$. Consequently, the threshold $\lambda_{th} = 1$ if and only if*

$$\mu \ge 2\left[\int_{|\gamma_-|^{-1}}^{+\infty} s dF(s) + |\gamma_-|^{-1}(F(|\gamma_-|^{-1}) - 1)\right],$$

and $\lambda_{th} < 1$ for sufficiently small μ if $\lim_{0<\mu\to 0} \gamma_-(\mu) < 0$. For symmetric two-point distribution $T(-a_1, p_0; a_1, p_1)$ with $p_1 > p_0$, $\lambda_{th} = 1$ if and only if $k \ge \frac{p_1}{p_0}\left(\frac{2p_1+1}{2p_1-1}\right)^{1-\alpha}$.

The above result reveals that for elliptical distributions, the threshold λ_{th} depends on μ and σ only through the Sharpe (1994) ratio μ/σ. For the following five specific distributions, we can get more explicit formulas for the threshold by, for the first four distributions, calculating the integrals in (10) and (11), and, for the last distribution, substituting the expression for γ_- in (B.1) into the threshold.

- **Normal distribution.**$\bar{s}$ is the unique root of the equation $s(F_\Phi^{NOR}(s) - 1) + \frac{1}{\sqrt{2\pi}} e^{-\frac{s^2}{2}} = \frac{\mu}{2\sigma}$ on $(-\frac{\mu}{\sigma}, +\infty)$, where the superscript “NOR” stands for the normal distribution.
- **Student's** t distribution with $\nu \ge 2$.$\bar{s}$ is the unique root of the equation $s(F_\Phi^{ST}(s) - 1) + \frac{\nu}{\nu-1}\frac{\Gamma(\frac{1+\nu}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})}(1 + \frac{s^2}{\nu})^{\frac{1-\nu}{2}} = \frac{\mu}{2\sigma}$ on $(-\frac{\mu}{\sigma}, +\infty)$, where $\Gamma(\cdot)$ is the gamma function.
- **Logistic distribution.**$\bar{s}$ is the unique root of the equation $s(F_\Phi^{LOG}(s) - 1) + \frac{s}{1+e^s} - s + \ln(1 + e^s) = \frac{\mu}{2\sigma}$ on $(-\frac{\mu}{\sigma}, +\infty)$, where the superscript “LOG” stands for the logistic distribution.
- **Lognormal distribution.**s^* is the unique root of the equation $e^{\rho+\frac{\sigma^2}{2}}\left[F_\Phi^{NOR}\left(\frac{\ln s - \rho - \sigma^2}{\sigma}\right) - \frac{1}{2}\right] + s\left[1 - F_\Phi^{NOR}\left(\frac{\ln s - \rho}{\sigma}\right)\right] = \frac{1}{2}$ on $(0, +\infty)$.
- **Symmetric two-point distribution.** Suppose that $R(t) \sim T(-a_1, p_0; a_1, p_1)$ with $p_1 > p_0$. In this case, $\lambda_{th}(p_1) = \min\left\{2p_1 \frac{\left(\frac{kp_0}{p_1}\right)^{\frac{1}{1-\alpha}} - 1}{\left(\frac{kp_0}{p_1}\right)^{\frac{1}{1-\alpha}} + 1}, 1\right\}$ and consequently, $\lim_{\frac{1}{2}<p_1\to\frac{1}{2}} \lambda_{th}(p_1) = \frac{k^{\frac{1}{1-\alpha}} - 1}{k^{\frac{1}{1-\alpha}} + 1}$.

4.2.2. Wealth growth

We next show how wealth growth depends on the coefficient of aspiration. Recall that we defined $\mathbb{E}[x_i(t)]$ as investor i's expected wealth at time t.

Proposition 6. *Suppose that there are only social interactions in the reference point updating, i.e., $\beta = 1$. Then if $\lambda < \lambda_{th}$, every investor's expected wealth grows to a common finite level with the exponential rate $\mathbb{E}[|1 - \phi_\lambda(t)|] < 1$, and if $\lambda > \lambda_{th}$, every investor's expected wealth increases without bound with the exponential rate $\mathbb{E}[|1 - \phi_\lambda(t)|] > 1$.*

Proposition 6 shows that the threshold for bounded/unbounded wealth growth is exactly the same as the threshold for a diminishing/widening wealth gap established in Proposition 4. It reveals an interesting choice: One can either have a narrow wealth gap but with bounded wealth growth, or a wide wealth gap but with unbounded wealth growth. In other words, there is a trade-off between sustained growth and a widening wealth gap, that is, we cannot have both economic growth as well as an absolutely equal society.

The proof of Proposition 6 explicitly specifies the common exponential growth rate of wealth. Specifically, when $\lambda_{th} < \lambda < 1$,

$$\lim_{t\to\infty}(\mathbb{E}[x_i(t)])^{\frac{1}{t}} = \mathbb{E}[|1-\phi_\lambda(t)|] > 1,\ i = 1,\ldots,n.$$

From the proof of Proposition 4 we know that the wealth growth rate $\mathbb{E}[|1-\phi_\lambda(t)|]$ is strictly increasing on $(\lambda_{th}, 1]$. One result of the constant elasticity growth model in Genicot and Ray (2017) shows that when the initial wealth distribution has a high degree of equality, all investors' wealth will grow at a common rate. We get a similar result of a common growth rate, however in our model it is the coefficient of aspiration, and not the initial wealth distribution, that determines the common growth rate.

4.2.3. Wealth inequality

Here we consider the impact of the coefficient of aspiration on the dynamics of the generalized Gini coefficient. Let us consider the following measure of wealth inequality in terms of investors' wealth level distribution:

$$W(t) \triangleq \frac{\sum_{i=1}^n \sum_{j=1}^n |x_i(t) - x_j(t)|}{2n\sum_{i=1}^n x_i(t)}.$$

This is the original definition of the Gini coefficient for a nonnegative wealth distribution in Sen (1973). It follows from (9) that the following relation between the difference of the wealth levels of two investors at two consecutive times holds:

$$|x_i(t+1) - x_j(t+1)| = |1-\phi_\lambda(t)||x_i(t) - x_j(t)| \tag{12}$$

for all $1 \le i, j \le n$. Consequently, we have the following dynamics for the inverse of the Gini coefficient:

$$\begin{aligned}\frac{1}{W(t+1)} &= \frac{2n\sum_{i=1}^n x_i(t) + 2n\phi_\lambda(t)\sum_{i=1}^n (H(t) - x_i(t))}{|1-\phi_\lambda(t)|\sum_{i=1}^n\sum_{j=1}^n |x_i(t)-x_j(t)|}\\ &= \frac{1}{|1-\phi_\lambda(t)|}\frac{1}{W(t)} + \frac{2n\phi_\lambda(t)\sum_{i=1}^n (H(t)-x_i(t))}{|1-\phi_\lambda(t)|\sum_{i=1}^n\sum_{j=1}^n |x_i(t)-x_j(t)|}.\end{aligned} \tag{13}$$

We have the following result.

Proposition 7. *Suppose* $\mathbb{E}\Big[\frac{\phi_\lambda(0)}{|1-\phi_\lambda(0)|}\Big] > 0$,[12] *and that there are only social interactions in the reference point updating, i.e.,* $\beta = 1$. *Then the followings hold.*

(i) $\mathbb{E}[\frac{1}{W(t)}]$ *increases in t without bound if* $\mathbb{E}\Big[\frac{1}{|1-\phi_\lambda(t)|}\Big] > 1$, *and* $\{\mathbb{E}[\frac{1}{W(t)}]\}_{t\ge 0}$ *is bounded if* $\mathbb{E}\Big[\frac{1}{|1-\phi_\lambda(t)|}\Big] < 1$.[13]

(ii) *Suppose* $\lambda_{th} < 1$. *Then there exists* $\lambda_{th} < \lambda^* \le 1$ *such that when* $0 < \lambda < \lambda^*$, $\mathbb{E}\Big[\frac{1}{|1-\phi_\lambda(t)|}\Big] > 1$.

(iii) *There exists* $\lambda_{th} < \lambda^* \le 1$ *such that when* $0 < \lambda < \lambda^*$, $\mathbb{E}[\frac{1}{GGC(t)}]$ *increases in t without bound.*

Proposition 7 is interesting because it suggests that there is a region for the coefficient of aspiration where it is possible to achieve wealth growth and relative equality simultaneously even in the absence of a personal component for the reference point. We emphasize that, within our model, this is not possible in the absence of social interactions.

4.3. The case where the reference point contains both social and personal components

Comparing the theoretical results of the previous two subsections suggests that social interactions can have a positive effect on inequality. In this subsection we investigate this further by considering the general case $0 < \beta < 1$, where the reference point is composed of both social and personal components.

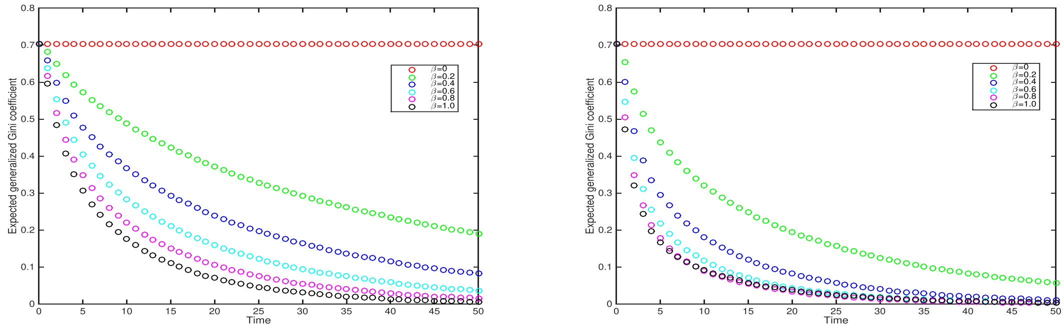
Since it is difficult to obtain analytical results we investigate the effect of social interactions numerically by means of simulation. We work with the following assumptions and values for the parameters of our model: Suppose that the random return $\tilde{R}(t)$ follows a *lognormal* distribution $LN(0.02, 0.3)$. We consider the following parameters specifying the preferences of the investor $\alpha = 0.88$ and $k = 2.25$ corresponding to the values estimated in Tversky and Kahneman (1992). We can then numerically compute the threshold value for the coefficient of aspiration, $\lambda_{th} \approx 0.4440$, as well as the piecewise linearity coefficients for the investment strategy, $\gamma_- \approx -5.3838$ and $\gamma_+ \approx 2.6119$. We here consider the recursive updating rule with updating coefficients $\alpha_g = \alpha_l = 0.6$ and suppose that the built-in profit determining the initial reference point is given by $\rho = 0.1$. We consider a stylized complete network consisting of five investors, where the initial wealth levels are set as follows: $x_i(0) \in \{1, 5, 10, 50, 300\}$ which corresponds to an initial Gini of approximately $GGC(0) \approx 0.7$. This value is typical for wealth inequality, see, e.g., Davies et al. (2008).

Fig. 2 shows the evolution of the expected[14] generalized Gini coefficient of the network over time for different degrees of social interactions dictated by the parameter $\beta \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ and two coefficients of aspirations, namely $\lambda = 0.2$

[12] This is true when the excess return follows an elliptical distribution with a positive mean.

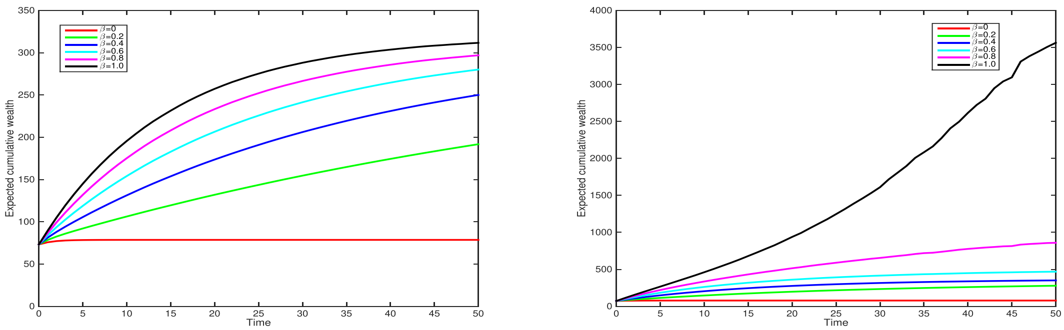
[13] We can see from equation (13) that for the special case of $n = 2$, $\lim_{t\to\infty}\mathbb{E}[\frac{1}{W(t)}] = \frac{2\mathbb{E}[\frac{\phi_\lambda(0)}{|1-\phi_\lambda(0)|}]}{1-\mathbb{E}[\frac{1}{|1-\phi_\lambda(0)|}]}$ because $\sum_{i=1}^2\sum_{j=1}^2 |x_i(t)-x_j(t)| = 2\sum_{i=1}^2 (H(t)-x_i(t)) = 2(H(t)-h(t))$.

[14] The numerical results presented in this paper would be similar if one considers the median instead of the expected generalized Gini coefficient.



(a) $\lambda = 0.2$ (b) $\lambda = 0.5$

Fig. 2. Expected generalized Gini coefficient *Notes.* The figure shows the evolution of the expected generalized Gini coefficient over time for different degrees of social interactions. Subfigure (a) shows the expected Gini for a case where the coefficient of aspiration is below the threshold and Subfigure (b) shows the expected Gini for a case where the coefficient of aspiration is above the threshold. The simulation is based on 100,000 scenarios.



(a) $\lambda = 0.2$ (b) $\lambda = 0.5$

Fig. 3. Expected cumulative wealth of the network *Notes.* The figure shows the evolution of the expected and median cumulative wealth of all investors in the network for different degrees of social interactions. Subfigure (a) shows the expected cumulative wealth for a case where the coefficient of aspiration is below the threshold and Subfigure (b) shows the expected cumulative wealth for a case where the coefficient of aspiration is above the threshold. The simulation is based on 1,000,000 scenarios.

below and $\lambda = 0.5$ above the threshold. We first note that the Gini coefficient is constant over time when there are no social interactions. This was to be expected from Corollary 1. We also observe that increasing the degree of social interactions β typically decreases the expected and median Gini coefficient. This indicates that social interactions can potentially reduce inequality. Increasing the coefficient of aspiration also has a positive effect on reducing inequality. In other words, a larger weight is given to the wealth level of the richest member of the network when forming the social component of their reference point, the faster inequality is reduced on average.

Fig. 3 shows the evolution of the expected cumulative wealth of the network, again for different degrees of social interactions $\beta \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ and two coefficient of aspirations, namely $\lambda = 0.2$ below and $\lambda = 0.5$ above the threshold. We note that increasing the degree of social interactions has a positive effect of wealth growth. In particular, social interactions can simultaneously achieve higher wealth growth and lower inequality. Intuitively, the social component moves the reference point towards the wealth level of the richest member of the network and, compared with the case where there is no social component, increases the reference point for every member of the social network. For those investors whose current wealth lies below the personal component of the reference, this further increases the distance between wealth and reference point and thus causes those investors to increase their position in the risky asset. These are typically the poorer members of the social network and, moreover, the increase due to the social component is larger for the poorer investors because of the difference between their and the richest investor's wealth. On the other hand, for those investors whose current wealth lies above their personal component of the reference point, the increase in the reference point due to social interactions decreases the distance between current wealth and reference point and thus leads to lower investments in the stock. These tend to be the richer investors of the network. To summarize, a social component in the reference point incentivizes poorer investors to catch up with the richest in the network and thus invest more in the risky asset and simultaneously causes the richer investors to become more concerned about their wealth falling below the reference point and thus to make a more cautious investment. This drives the reduction in inequality.

Remark 4. We note that wealth quickly converges to a constant level when there are no social interactions and the reference point is updated recursively. This replicates a finding of Strub and Li (2020) in a different setting. The reason for this phenomenon is that the current wealth typically converges to the current reference point after a few rounds of trading when the latter is recursively updated.

Remark 5. One would obtain qualitatively similar results if the reference point is updated according to the non-recursive updating rule of Baucells et al. (2011) with one exception: For some parameter values, a lower degree of social interactions achieves a higher expected wealth growth at long time horizons than a higher degree of social interactions. The reason for this is that the initial wealth constitutes an important part of the reference point when the reference point is updated non-recursively. When t is large, then the current wealth is typically much larger than the initial wealth. Thus, the difference between reference point and current wealth is typically large which leads to a large investment in the risky asset. On average, the current wealth and thus also the difference between current wealth and reference point further increases. To summarize, the large weight which the non-recursive updating rule gives to the initial wealth level at later times leads to ever more investment in the risky asset in a self-enforcing manner and consequently to large expected wealth levels.

In the following we will show that our main findings that increasing social interactions can reduce inequality while simultaneously increase wealth growth are robust with regards to several extensions of the model. We will first consider the case where there is a no-bankruptcy constraint preventing investors from taking leverage. We will then consider the cases where investors are heterogeneous, first in terms of their individual risk attitude and second in terms of their individual loss aversion.

4.3.1. No-bankruptcy constraint

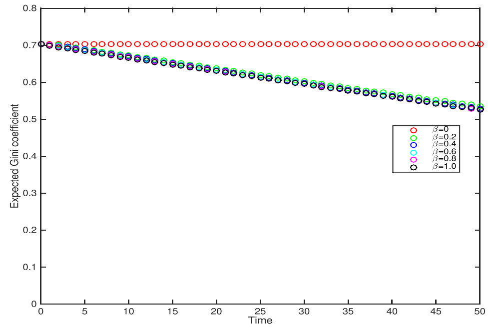
One potential objection to our result is that investors with a socially determined reference point take a lot of leverage. This is indeed the case since the optimal investment policy is linear in the difference between current wealth and reference point and the difference between an investor's wealth and the richest member within her network can be very large. To robustify our findings we thus consider a setting where there is a no-bankruptcy constraint, i.e., no investor can invest more in the risky asset than her own current wealth. Figs. 4 and 5 replicate our previous analysis in this setting with the same market and preference parameters as before. Although the quantitative results are different and in particular the expected wealth growth is much smaller since the investors can not leverage a stock with positive excess return, the qualitative interpretation remains similar: Social interactions have a positive effect on both wealth growth and inequality. Whereas the positive effect on the expected wealth growth is clear from Fig. 5, the analysis of the effect on social interactions is more delicate, cf. Figure 4 which displays the effect for varying degrees of social interactions $\beta \in \{0, 0.2, 0.4, 0.6, 0.8, 1.0\}$ for Subfigures (a) and (b) and $\beta \in \{0, 0.01, 0.02, 0.03, 0.04, 0.05\}$ for Subfiugres (c) and (d). Looking first at Subfigures (a) and (b) where the values of β are as in Figure 2, it appears that the degree of social interactions does not have a significant effect on inequality in the constraint setting as long as there are social interactions at all. The main reason why the no-bankruptcy constraint dampens the positive effect of social interactions on inequality is that it mostly limits the poorer members of the network while it is often not binding for the richer members. This is especially the case when the wealth levels of the poorer members are close to zero and the no-bankruptcy constraint is thus truly constraining them from catching up with the richer members of the network. Indeed, a thorough analysis of the model under a large variety of model parameters showed that the effect would be larger if one considers a more equal initial wealth distribution such that this constraining effect applies to all members of the network, a longer time horizon allowing poorer members to move their wealth sufficiently above the constraining bankruptcy bound, or smaller values for the degree of social interactions β. The latter case is displayed in Subfigures (c) and (d) showing that, in this range, increasing social interactions typically leads to a strictly lower expected Gini coefficient.

4.3.2. Heterogeneous risk attitude

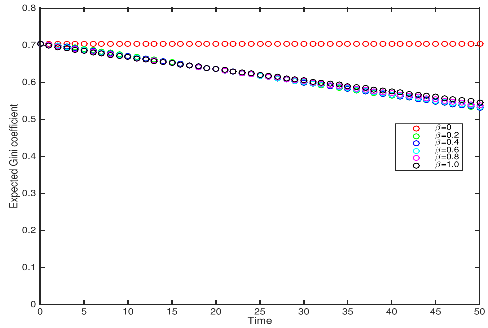
We consider the situation where investors not only differ in their initial wealth and reference points, but also in their risk attitude. Specifically, we consider the following values for the parameter of risk attitude: $\alpha \in \{0.75, 0.8, 0.85, 0.9, 0.95\}$ around the benchmark value $\alpha = 0.88$ estimated in Tversky and Kahneman (1992). The corresponding piecewise linearity coefficients are shown in Table 2.

We observe that the absolute value of γ_- is decreasing in α whereas γ_+ is increasing in α. This conforms with the theoretical finding in Lou (2020) for the case of two-point distributions. Consequently, agents with a lower parameter of risk attitude invest less in the risky asset when they are in a gain position, but more when they are in a loss position. This is intuitive given the S-shape of the utility function.

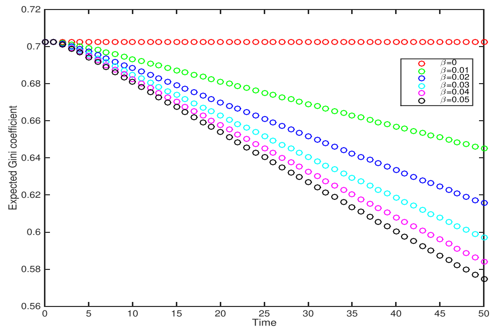
We consider two scenarios, one where the initially richer investors have a higher parameter of risk attitude, i.e., the investor with initial wealth $x_1(0) = 1$ has parameter of risk attitude $\alpha_1 = 0.75$, the investor with initial wealth $x_2(0) = 5$ has parameter of risk attitude $\alpha_2 = 0.8$ and so on, and another where the order is reversed and the initially poorer investors have a higher parameter of risk attitude, i.e., the investor with initial wealth $x_1(0) = 1$ has parameter of risk attitude $\alpha_1 = 0.95$, the investor with initial wealth $x_2(0) = 5$ has parameter of risk attitude $\alpha_2 = 0.9$ and so on. We then repeat the simulation



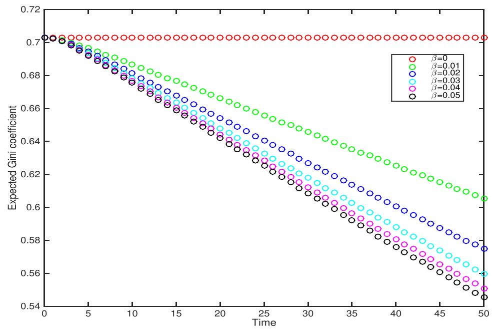
(a) $\lambda = 0.2$



(b) $\lambda = 0.5$



(c) $\lambda = 0.2$



(d) $\lambda = 0.5$

Fig. 4. Expected Gini coefficient with bankruptcy prohibition *Notes.* Here investors' wealth levels are nonnegative and so generalized Gini coefficient reduces to the classical Gini coefficient. The figure shows the evolution of the expected and median Gini coefficient over time when wealth is required to remain nonnegative for different degrees of social interactions, $\beta \in \{0, 0.2, 0.4, 0.6, 0.8, 1.0\}$ for Subfigures (a) and (b) and $\beta \in \{0, 0.01, 0.02, 0.03, 0.04, 0.05\}$ for Subfiugres (c) and (d). Subfigures (a) and (c) show a case where the coefficient of aspiration is below the threshold, while Subfigures (b) and (d) show a case where the coefficient of aspiration is above the threshold. The simulation is based on 100,000 scenarios.

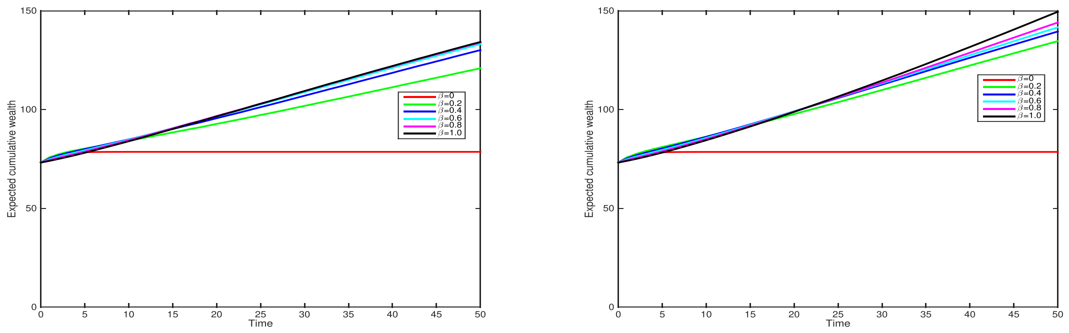
Table 2
Piecewise linearity coefficients for varying risk attitude.

α	γ_-	γ_+
0.75	−7.8488	1.9759
0.8	−6.8445	2.1651
0.85	−5.9117	2.4170
0.9	−5.0460	2.7657
0.95	−4.2562	3.2662

Notes. Piecewise linearity coefficients when the loss aversion is $k = 2.25$ and returns are log-normal $LN(0.02, 0.3)$.

of Section 4.3 and show the evolution of the expected Gini coefficient in Fig. 6 and the evolution of the expected cumulative wealth in Fig. 7. The scenario where the initially poorer investors are more risk averse is represented in Subfigures (a) and (b) whereas the scenario where the initially richer investors are more risk averse is represented in Subfigures (c) and (d). In short, the main findings of our previous analysis still hold when investors have heterogeneous risk attitude: Increasing the degree of social interactions tends to a reduction in inequality and higher expected cumulative wealth of the network. This shows that it is the position of the reference point, and not individual differences in risk attitudes, that drives these main long term results.

We remark that the reduction in expected inequality is not monotone anymore when the coefficient of aspiration is large and risk attitudes are heterogeneous. Note that the region for the coefficient of aspiration where both wealth growth and



(a) $\lambda = 0.2$ (b) $\lambda = 0.5$

Fig. 5. Expected cumulative wealth of the network with bankruptcy prohibition *Notes.* The figure shows the evolution of the expected cumulative wealth of all investors in the network for different degrees of social interactions when wealth is required to remain nonnegative. Subfigure (a) shows a case where the coefficient of aspiration is below the threshold and Subfigure (b) shows a case where the coefficient of aspiration is above the threshold. The simulation is based on 1,000,000 scenarios.

Table 3
Piecewise linearity coefficients for varying loss aversion.

k	γ_-	γ_+
2.25	−5.3838	2.6119
2.5	−4.5202	2.4995
2.75	−4.0318	2.4192
3	−3.7172	2.3565
3.25	−3.4972	2.3059

Notes. Piecewise linearity coefficients when the parameter of risk attitude is $\alpha = 0.88$ and returns are lognormal $LN(0.02, 0.3)$.

relative equality are possible we identified in Proposition 7 will change under heterogeneous risk attitudes. A small weight given to the personal component thus has a positive moderating effect on expected inequality in this setting.

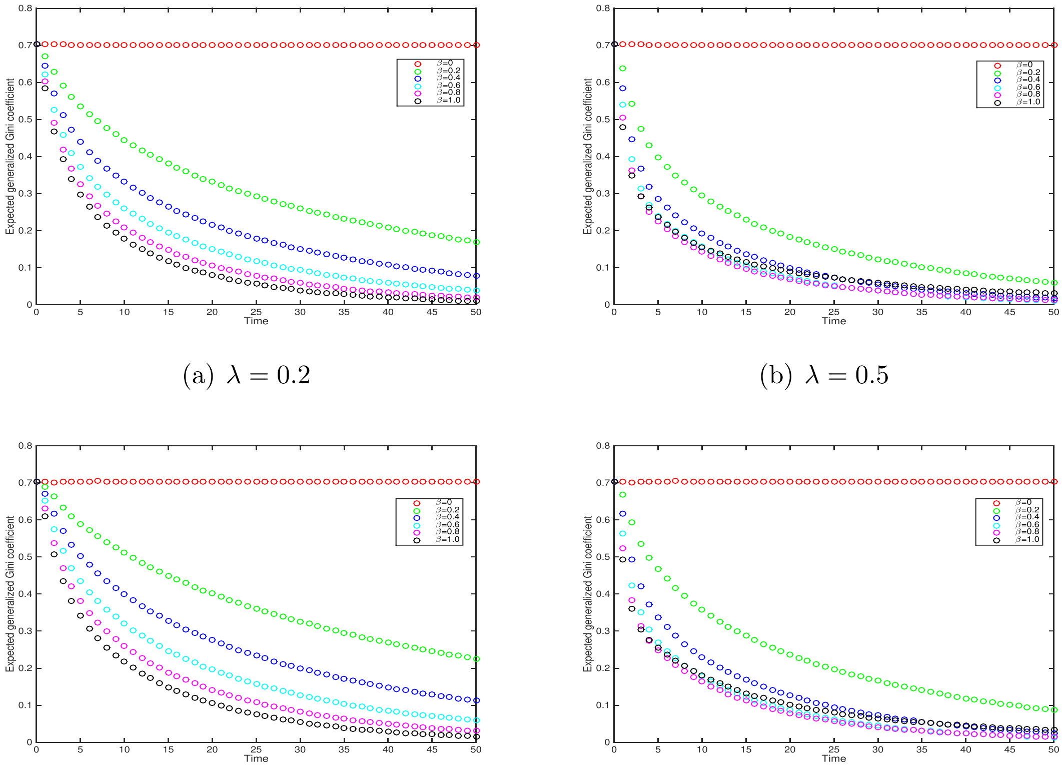
Remark 6. From Fig. 6, it appears that the expected Gini coefficient remains constant when there are no social interactions even under heterogeneous risk attitudes of the investors. This is not the case, and a careful analysis shows that the expected Gini coefficient slightly increases when the initially poorer members of the network have a larger parameter of risk attitude than the initially richer members and slightly decreases when the poorer members have a smaller parameter of risk attitude than the initially richer members. However, the differences are miniscule and hard to identify on the plot.

4.3.3. Heterogenous loss aversion

We further consider the situation where investors have heterogeneous loss aversion parameters, i.e., investors not only differ in their initial wealth and reference points, but also in their loss aversion. We consider the following values for the loss aversion parameter $k \in \{2.25, 2.5, 2.75, 3, 3.25\}$ increasing from the benchmark value $k = 2.25$ estimated in Tversky and Kahneman (1992). We show the corresponding piecewise linearity coefficients in Table 3.

We observe that the absolute value of γ_- and γ_+ are both decreasing in k conforming with the theoretical finding in Lou (2020). Consequently, agents with a higher loss aversion take smaller positions in the risky asset as one would expect.

We consider two scenarios, one where the initially richer investors have a higher loss aversion coefficient, i.e., the investor with initial wealth $x_1(0) = 1$ has loss aversion coefficient $k_1 = 2.25$, the investor with initial wealth $x_2(0) = 5$ has loss aversion coefficient $k_2 = 2.5$ and so on, and another where the order is reversed and the initially poorer investors are more loss averse than initially richer investors, i.e., the investor with initial wealth $x_1(0) = 1$ has loss aversion coefficient $k_1 = 3.25$, the investor with initial wealth $x_2(0) = 5$ has loss aversion coefficient $k_2 = 3$ and so on. We then repeat the simulation of Section 4.3 and show the evolution of the expected Gini coefficient in Fig. 8 and the evolution of the expected cumulative wealth in Fig. 9. The scenario where the initially richer investors are more loss averse is represented in Subfigures (a) and (b) whereas the scenario where the initially poorer investors are more loss averse is represented in Subfigures (c) and (d). As in the case of heterogeneous risk attitude, the main findings of our previous analysis are replicated when investors have heterogeneous loss aversion: Increasing the degree of social interactions leads to a reduction in inequality



(a) $\lambda = 0.2$ (b) $\lambda = 0.5$

(c) $\lambda = 0.2$ (d) $\lambda = 0.5$

Fig. 6. Expected generalized Gini coefficient where investors have heterogeneous risk attitude *Notes.* The setting is the same as that in Fig. 2 except that here investors have heterogeneous risk attitude. The parameters of risk attitude of the five investors in Subfigures (a) and (b) are respectively 0.75, 0.8, 0.85, 0.9 and 0.95 in an increasing order, while those in Subfigures (c) and (d) are respectively 0.95, 0.9, 0.85, 0.8 and 0.75 in a decreasing order. The simulation is based on 100,000 scenarios.

and higher expected cumulative wealth of the network. We can thus again conclude that it is the position of the reference point, and not individual differences in loss aversion, that drives these main long term results.

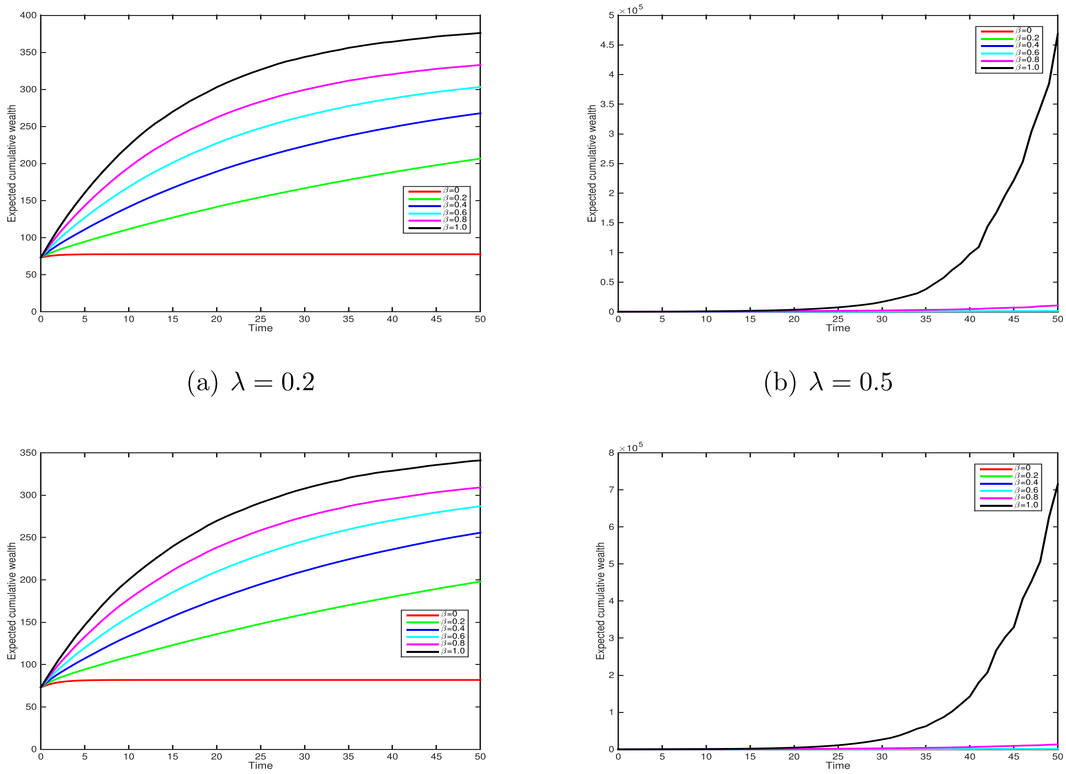
Remark 7. Similar to Remark 6, the expected Gini coefficient displayed in Fig. 8 when there are no social interactions is not constant, but slightly increases when the poorer investors are more loss averse than the richer investors and slightly decreases when the poorer investors are less loss averse than than the richer investors.

5. Conclusions

We investigate the long run behavior of wealth growth and inequality in a dynamic network of multiple behavioral investors. Investors' preferences are myopic, incorporate key ingredients of prospect theory, and are homogeneous with the exception of the reference point which is the focus of our study. For the setting of dynamic social networks, we propose a reference point consisting of two components: A personal component incorporating each investor's own history of wealth and a social component which is determined by comparison with the current wealth levels of other investors in the network. We term the weight given to the social component the degree of social interactions and investigate its influence on wealth growth and inequality of the network.

To this end, we first consider a benchmark case where the degree of social interactions is zero and the reference point is thus solely determined by each investor's individual history of wealth. In this case, each investor's expected wealth grows at a common rate, the wealth gap widens and the Gini coefficient remains constant over time.

Next, we study in detail the case where the reference point is solely determined by social comparison for a specific reference point formation rule prescribing that the reference point is a convex combination of the individual investor's wealth and the maximum wealth level of all members in her network. This formation rule is parametrized by the coefficient of aspiration and we establish a threshold for this parameter, below which investors' expected wealth grows to a common finite level, the expected wealth gap vanishes and the generalized Gini coefficient keeps at a relatively low level, and above which



(a) $\lambda = 0.2$ (b) $\lambda = 0.5$

(c) $\lambda = 0.2$ (d) $\lambda = 0.5$

Fig. 7. Expected cumulative wealth of the network where investors have heterogeneous risk attitude *Notes.* The setting is the same as that in Fig. 3 except that here investors have heterogeneous risk attitude. The parameters of risk attitude of the five investors in Subfigures (a) and (b) are respectively 0.75, 0.8, 0.85, 0.9 and 0.95 in an increasing order, while those in Subfigures (c) and (d) are respectively 0.95, 0.9, 0.85, 0.8 and 0.75 in a decreasing order. The simulation is based on 1,000,000 scenarios.

investors' expected wealth grows without bound at a common rate and the expected wealth gap widens. This latter case indicates that it is possible to have both wealth growth and a reduction of inequality in the presence of social interactions.

Finally, we consider the general case where the reference point contains both the personal and social components. Our numerical analysis highlights the importance and positive impact of social interactions potentially have on wealth growth and inequality. Indeed, we find that increasing the degree of social interactions can lead to a Pareto improvement in that it leads simultaneously to an increase in wealth growth and a decrease in inequality as measured by the Gini coefficient. From the perspective of a social planer or policy maker, it would thus be desirable to increase connectivity among the population and transparency about the current distribution of wealth within a society.

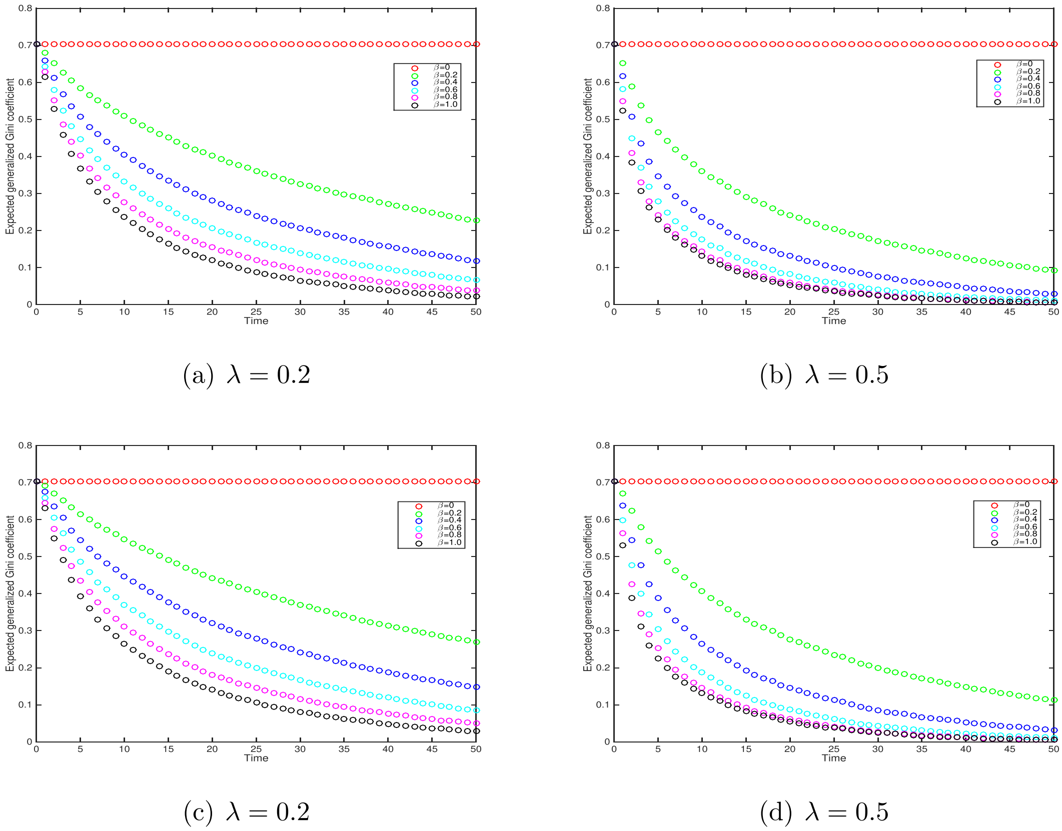
We close with the remark that our conclusions are so far only suggestive. They depend on the assumptions we impose on the social component of the reference point, in particular that the social component is upward looking and driven by the wealth of the richest member of the network. Furthermore, the general case where the reference point contains both a personal and social component was studied numerically since it was difficult to derive theoretical results even under those assumptions. It is in particular an interesting direction for future research to further investigate how reference points are formed in the context of social interactions, and how those mechanisms affect the wealth dynamics within social networks.

Appendix A. The actual market and the market perceived by the investor

In this appendix, we provide some further results about the actual market and perceived market, and the proofs of all the statements in this paper.

The following definition specifies what we mean by a good or bad actual market, respectively by the market perceived by the investor being good or bad.

Definition 1. (i) We say that the *actual* market is good if the mean μ of the excess return R is positive, and bad if it is negative.



(a) $\lambda = 0.2$ (b) $\lambda = 0.5$

(c) $\lambda = 0.2$ (d) $\lambda = 0.5$

Fig. 8. Expected generalized Gini coefficient where investors are heterogeneously loss averse *Notes.* The setting is the same as that in Fig. 2 except that here investors are heterogeneously loss averse. The loss aversion coefficients of the five investors in Subfigures (a) and (b) are respectively 2.25, 2.5, 2.75, 3 and 3.25 in an increasingly loss aversion order, while those in Subfigures (c) and (d) are respectively 3.25, 3, 2.75, 2.5 and 2.25 in a decreasingly loss aversion order. The simulation is based on 100,000 scenarios.

Table 4
The four cases for the market condition with a negative relative wealth.

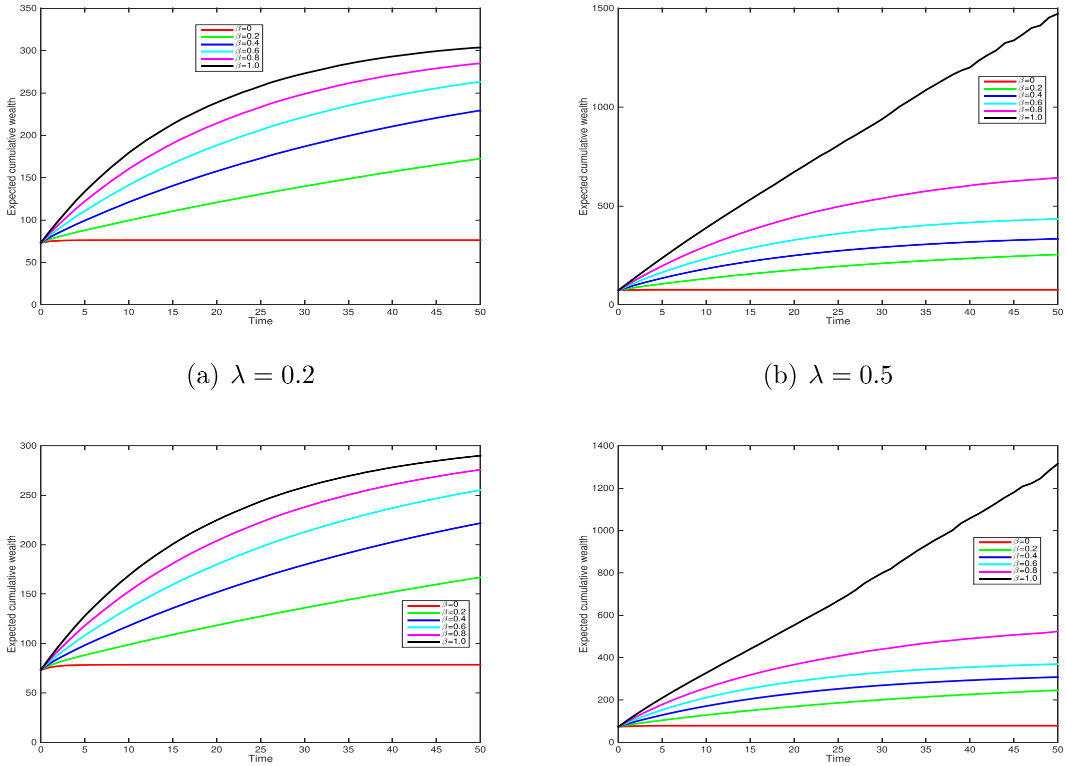
	$\mu > 0$	$\mu < 0$
$\gamma_- < 0$	**Case 1**	**Case 3**
$\gamma_- > 0$	**Case 2**	**Case 4**

Notes. The market perceived by the investor matches (mismatches) the actual market in the two cases of **Case 1** and **Case 4** (respectively **Case 2** and **Case 3**).

(ii) We say that the market *perceived* by the investor (which is subjective in accordance to the view of the investor) is good, if the optimal solution θ^* of the behavioral portfolio choice problem (2) is positive, and bad if it is negative.

Proposition 9 (i) in Appendix B and Footnote[9] tell us that for elliptical distributions, the investor always longs the stock when the actual market is good and always shorts the stock when the actual market is bad. In the case of a two-point distribution $T(a_0, p_0; a_1, p_1)$, Proposition 9 (ii) shows that this result still holds, but need not be the case when the investor is in a loss position. It can thus happen that the investor shorts an asset with a positive expected excess return, a phenomenon which never appears under expected utility theory.

The “Upward-Looking” property of the reference point formation implies that each investor’s relative wealth is always nonpositive and we thus only need to consider the loss region. Table 4 illustrates four possible relations between the actual market and the market perceived by the investor.



(a) $\lambda = 0.2$ (b) $\lambda = 0.5$

(c) $\lambda = 0.2$ (d) $\lambda = 0.5$

Fig. 9. Expected cumulative wealth of the network where investors are heterogeneously loss averse *Notes.* The setting is the same as that in Fig. 3 except that here investors are heterogeneously loss averse. The loss aversion coefficients of the five investors in Subfigures (a) and (b) are respectively 2.25, 2.5, 2.75, 3 and 3.25 in an increasingly loss aversion order, while those in Subfigures (c) and (d) are respectively 3.25, 3, 2.75, 2.5 and 2.25 in a decreasingly loss aversion order. The simulation is based on 1,000,000 scenarios.

We will say that the market perceived by the investor *matches* the actual market, if the optimal solution of the optimization problem (2) with a negative relative wealth is positive (i.e., $\gamma_- < 0$) in an actually good market, i.e., Case 1 in Table 4 occurs, or the optimal solution with a negative relative wealth is negative (i.e., $\gamma_- > 0$) in an actually bad market, i.e., Case 4 occurs. If Case 2 or Case 3 occurs we say that the market perceived by the investor *mismatches* the actual market. Note that because the investors in our model have the same (power) value function, whether the market perceived by the investor matches the actual market or not does not depend on the individual investor.

The following proposition shows that it is impossible for the wealth to grow or the wealth inequality to be reduced if the market perceived by the investor mismatches the actual market for any (connected) social network and reference point formation rule satisfying the two properties of "Upward-Looking" and "Moderation".

Proposition 8 (Impossibility Theorem)**.** *Suppose that the market perceived by the investor mismatches the actual market (i.e., Case 2 and Case 3 in Table 4). Then the expected total wealth of the network converges to negative infinity and the expected wealth gap increases without upper bound.*

Appendix B. Proofs of all statements

Proof of Theorem 1.. Proposition 1 and Theorem 2 in He and Zhou (2011a) show that under Assumptions 1 and 2, the expected utility evaluation $V(\theta)$ is finite for any $\theta \in \mathbb{R}$ and the optimization problem under consideration is well-posed, i.e., the OIS θ^* is finite. The first part about the piecewise linear OIS can be shown by similar arguments as given in Theorem 4.2 of Shi et al. (2015a). $\square$

Proof of Proposition 1.. We show the following more general proposition which specifies further conditions on whether the investor takes a long or short position in the stock and, in the case of a two-point distribution, computes the investment strategy explicitly. $\square$

Proposition 9. *When $\bar{B} \neq 0$, we have the following results on the sign of the investment policy (where the positive and negative signs correspond to long and short positions, respectively).*[15]

(i) *Suppose that CDF $F(\cdot)$ is absolutely continuous. Then*

$$\arg\max_{\theta\in\mathbb{R}} V(\theta) \begin{cases} > 0, & \text{if } F(s)+F(-s) \text{ is strictly increasing on } s>0; \\ < 0, & \text{if } F(s)+F(-s) \text{ is strictly decreasing on } s>0. \end{cases}$$

(ii) *Suppose that CDF $F(\cdot)$ is a two-point distribution $T(a_0, p_0; a_1, p_1)$. Then we have the following explicit formula for the optimal investment policy,*[16]

$$\gamma^* = \begin{cases} \dfrac{1 - \dfrac{\frac{a_1}{|a_0|}+1}{\left(\frac{a_1 p_1}{|a_0| p_0}\right)^{\frac{1}{1-\alpha}} + \frac{a_1}{|a_0|}}}{|a_0|}, & \text{if } \bar{B} > 0; \\ \dfrac{\dfrac{\frac{|a_0|}{a_1}+1}{\left(k\frac{|a_0| p_0}{a_1 p_1}\right)^{\frac{1}{1-\alpha}} - \frac{|a_0|}{a_1}}+1}{-a_1}, & \text{if } \bar{B} < 0 \text{ and } a_1^\alpha p_1 > |a_0|^\alpha p_0; \\ \dfrac{\dfrac{\frac{a_1}{|a_0|}+1}{\left(k\frac{a_1 p_1}{|a_0| p_0}\right)^{\frac{1}{1-\alpha}} - \frac{a_1}{|a_0|}}+1}{|a_0|}, & \text{if } \bar{B} < 0 \text{ and } a_1^\alpha p_1 < |a_0|^\alpha p_0. \end{cases} \tag{B.1}$$

In particular,

$$\arg\max_{\theta\in\mathbb{R}} V(\theta) \begin{cases} > 0, & \text{if } \bar{B} < 0 \text{ and } a_1^\alpha p_1 > |a_0|^\alpha p_0; \\ < 0, & \text{if } \bar{B} < 0 \text{ and } a_1^\alpha p_1 < |a_0|^\alpha p_0; \\ > 0, & \text{if } \bar{B} > 0 \text{ and } a_1 p_1 > |a_0| p_0; \\ < 0, & \text{if } \bar{B} > 0 \text{ and } a_1 p_1 < |a_0| p_0; \\ = 0, & \text{if } \bar{B} > 0 \text{ and } a_1 p_1 = |a_0| p_0. \end{cases}$$

We start with the proof of (i) and only show the monotonically increasing case since the proof for the monotonically decreasing case is similar. First, it follows from Proposition 3 in He and Zhou (2011a) that $V(\cdot)$ is continuously differentiable on $(0, +\infty)$ and $(-\infty, 0)$. Note that here it suffices to only consider the two cases of $\bar{B} = 1$ and $\bar{B} = -1$ based on the result in the first part. We first show that in the case of $\bar{B} = 1$, $V'(\theta) > 0$ for any $\theta < 0$. In fact, for $\theta < 0$, we have

$$\begin{aligned} \frac{V'(\theta)}{\alpha} &= \int_{-\infty}^{-\frac{1}{\theta}} (\theta s + 1)^{\alpha-1} s dF(s) + k \int_{-\frac{1}{\theta}}^{+\infty} (-\theta s - 1)^{\alpha-1} s dF(s) \\ &\geq \int_{-\infty}^{-\frac{1}{\theta}} (\theta s + 1)^{\alpha-1} s dF(s) + \int_{-\frac{1}{\theta}}^{+\infty} (-\theta s - 1)^{\alpha-1} s dF(s) \\ &= \int_{-\infty}^{0} (\theta s + 1)^{\alpha-1} s dF(s) + \int_{0}^{+\infty} |\theta s + 1|^{\alpha-1} s dF(s), \end{aligned} \tag{B.2}$$

where the inequality follows from $k > 1$, which comes from the well-posedness condition (3), and the relation $\int_{-\frac{1}{\theta}}^{+\infty} (-\theta s - 1)^{\alpha-1} s dF(s) \geq 0$. Then by doing a change of variables in the integral of the first term in (B.2), we get

$$\frac{V'(\theta)}{\alpha} \geq \int_0^{+\infty} |\theta s + 1|^{\alpha-1} s dF(s) - \int_0^{+\infty} (-\theta s + 1)^{\alpha-1} s d[-F(-s)].$$

Using the above inequality, the relation $|\theta s + 1|^{\alpha-1} > (-\theta s + 1)^{\alpha-1}$ for $\theta < 0$, $s > 0$, and the strict increasingness of $F(s) + F(-s)$ on $s > 0$, we get that $V'(\theta) > 0$ for $\theta < 0$.

We now show that in the case of $\bar{B} = -1$, $d[V(\theta) - V(-\theta)]/d\theta > 0$ for any $\theta > 0$. It is easy to see

$$\begin{aligned} \frac{d[V(\theta) - V(-\theta)]}{d\theta} = \alpha\Bigg(&\int_{\frac{1}{\theta}}^{+\infty} (\theta s - 1)^{\alpha-1} s dF(s) + k \int_{-\infty}^{\frac{1}{\theta}} (-\theta s + 1)^{\alpha-1} s dF(s) \\ &+ \int_{-\infty}^{-\frac{1}{\theta}} |\theta s + 1|^{\alpha-1} s dF(s) + k \int_{-\frac{1}{\theta}}^{+\infty} (\theta s + 1)^{\alpha-1} s dF(s) \Bigg). \end{aligned} \tag{B.3}$$

Because $F(s) + F(-s)$ is strictly increasing on $s > 0$, it follows that

$$\int_{\frac{1}{\theta}}^{+\infty} (\theta s - 1)^{\alpha-1} s dF(s) + \int_{-\infty}^{-\frac{1}{\theta}} |\theta s + 1|^{\alpha-1} s dF(s) = \int_{\frac{1}{\theta}}^{+\infty} (\theta s - 1)^{\alpha-1} s d[F(s) + F(-s)] > 0,$$

[15] If there is a no-shorting constraint, the optimal policy is still piecewise linear. We also conclude that $\arg\max_{\theta\geq 0} V(\theta) > 0$ if the actual market is good, and $\arg\max_{\theta\geq 0} V(\theta) = 0$ if the actual market is bad based on the fact that $V'(0+) = \alpha\mu$ for $\bar{B} = 1$ and $V'(0+) = k\alpha\mu$ for $\bar{B} = -1$.

[16] From the proof of this theorem we find that for the case of $\bar{B} < 0$ and $a_1^\alpha p_1 = |a_0|^\alpha p_0$, the behavioral portfolio choice problem (2) has two optimal solutions: $\frac{1}{a_1} + \frac{\frac{|a_0|}{a_1}+1}{\left(k^{\frac{1}{1-\alpha}} - 1\right)|a_0|}$ and $-\left[\frac{1}{|a_0|} + \frac{\frac{a_1}{|a_0|}+1}{\left(k^{\frac{1}{1-\alpha}} - 1\right)a_1}\right]$.

and

$$\int_{-\infty}^{\frac{1}{\theta}}(-\theta s+1)^{\alpha-1}sdF(s)+\int_{-\frac{1}{\theta}}^{+\infty}(\theta s+1)^{\alpha-1}sdF(s)=\int_{-\infty}^{\frac{1}{\theta}}(-\theta s+1)^{\alpha-1}sd[F(s)+F(-s)]>0.$$

The preceding two inequalities together with (B.3) imply that $d[V(\theta)-V(-\theta)]/d\theta>0$ for $\theta>0$.

Moreover, by Proposition 3 in He and Zhou (2011a), $V'(0+)=\alpha\mu$ when $\bar{B}=1$, and $V'(0+)=k\alpha\mu$ when $\bar{B}=-1$. Note that when $s\mapsto F(s)+F(-s)$ is strictly increasing on $s>0$, the mean $\mu>0$ and we thus conclude that the optimal solution of (2) must be positive for the monotonically increasing case.

(ii). We first consider the case of $\bar{B}=-1$. It is easy to see that

$$V(\theta)=\begin{cases}(\theta a_0-1)^\alpha p_0-k(-\theta a_1+1)^\alpha p_1, & -\infty<\theta<\frac{1}{a_0};\\ -k\big((-\theta a_1+1)^\alpha p_1+(-\theta a_0+1)^\alpha p_0\big), & \frac{1}{a_0}<\theta<\frac{1}{a_1};\\ (\theta a_1-1)^\alpha p_1-k(-\theta a_0+1)^\alpha p_0, & \frac{1}{a_1}<\theta<+\infty,\end{cases}$$

as well as

$$V'(\theta)=\begin{cases}\alpha(\theta a_0-1)^{\alpha-1}a_0p_0+k\alpha(-\theta a_1+1)^{\alpha-1}a_1p_1, & -\infty<\theta<\frac{1}{a_0};\\ k\alpha\big((-\theta a_1+1)^{\alpha-1}a_1p_1+(-\theta a_0+1)^{\alpha-1}a_0p_0\big), & \frac{1}{a_0}<\theta<\frac{1}{a_1};\\ \alpha(\theta a_1-1)^{\alpha-1}a_1p_1+k\alpha(-\theta a_0+1)^{\alpha-1}a_0p_0, & \frac{1}{a_1}<\theta<+\infty.\end{cases}$$

It thus follows that $\lim_{\theta\to\pm\infty}V'(\theta)=0$, $\lim_{\theta\to1/a_0}V'(\theta)=-\infty$, $\lim_{\theta\to1/a_1}V'(\theta)=+\infty$, $\lim_{\theta\to0}V'(\theta)=k\alpha(a_1p_1+a_0p_0)$. This together with $V''(\theta)=k\alpha(1-\alpha)[(-\theta a_1+1)^{\alpha-2}a_1^2p_1+(-\theta a_0+1)^{\alpha-2}a_0^2p_0]>0$ on $(1/a_0,1/a_1)$ implies that the maximum point of $V(\cdot)$ must be in $(-\infty,1/a_0)\cup(1/a_1,+\infty)$ no matter whether $a_1p_1+a_0p_0>0$, $a_1p_1+a_0p_0=0$ or $a_1p_1+a_0p_0<0$. Then letting $V'(\theta)=0$ gives that $V(\cdot)$ has two candidates for extreme points $\theta_-\in(-\infty,1/a_0)$ and $\theta_+\in(1/a_1,+\infty)$:

$$\theta_-=\frac{\frac{\frac{a_1}{|a_0|}+1}{\left(k\frac{a_1p_1}{|a_0|p_0}\right)^{\frac{1}{1-\alpha}}-\frac{a_1}{|a_0|}}+1}{a_0},\quad \theta_+=\frac{\frac{\frac{|a_0|}{a_1}+1}{\left(k\frac{|a_0|p_0}{a_1p_1}\right)^{\frac{1}{1-\alpha}}-\frac{|a_0|}{a_1}}+1}{a_1}.$$

The maximum point of $V(\cdot)$ is then given by either θ_- or θ_+. Note that from the well-posedness condition (3), it holds that $[(ka_1p_1)/(|a_0|p_0)]^{\frac{1}{1-\alpha}}-a_1/|a_0|>0$ and

$[(k|a_0|p_0)/(a_1p_1)]^{\frac{1}{1-\alpha}}-|a_0|/a_1>0$, implying $\theta_-<1/a_0$ and $\theta_+>1/a_1$.

Let $\varrho=(a_1^\alpha p_1)/(|a_0|^\alpha p_0)$ and substitute $p_1=(|a_0|^\alpha p_0\varrho)/a_1^\alpha$ into $V(\theta_+)$ and $V(\theta_-)$, respectively. We then get that $V(\theta_+)=-(1+|a_0|/a_1)^\alpha p_0[(k/\varrho)^{\frac{1}{1-\alpha}}-1]^{1-\alpha}\varrho$ and $V(\theta_-)=-(1+|a_0|/a_1)^\alpha p_0[(k\varrho)^{\frac{1}{1-\alpha}}-1]^{1-\alpha}$. Note that $k\varrho>1$ and $k/\varrho>1$ from the well-posedness condition. It is easy to verify that $V(\theta_+)>V(\theta_-)$ if $\varrho>1$, and $V(\theta_+)<V(\theta_-)$ if $\varrho<1$. Thus, the optimal solution of (2) for the case of $\bar{B}=-1$ is positive if $a_1^\alpha p_1>|a_0|^\alpha p_0$ and negative if $a_1^\alpha p_1<|a_0|^\alpha p_0$.

We now consider the case of $\bar{B}=1$. Similar to the case of $\bar{B}=-1$, some simple calculations give $\lim_{\theta\to\pm\infty}V'(\theta)=0$, $\lim_{\theta\to1/|a_0|}V'(\theta)=-\infty$, $\lim_{\theta\to-1/a_1}V'(\theta)=+\infty$, $\lim_{\theta\to0}V'(\theta)=\alpha(a_1p_1+a_0p_0)$, and $V''(\theta)<0$ on $(-1/a_1,0)\cup(0,1/|a_0|)$. As a consequence, there is a unique local maximum point that locates in $(0,1/|a_0|)$ if $\mu=a_1p_1+a_0p_0>0$ and in $(-1/a_1,0)$ if $\mu<0$, and is equal to zero if $\mu=0$. We next show that $V'(\theta)\neq0$ for any $\theta\in(1/|a_0|,+\infty)$. Suppose, to the contrary, that $V'(\theta)=0$ for some $\bar{\theta}\in(1/|a_0|,+\infty)$. Some simple calculations give that

$$\bar{\theta}=\frac{1+\frac{\frac{a_1}{|a_0|}+1}{\left(\frac{a_1p_1}{k|a_0|p_0}\right)^{\frac{1}{1-\alpha}}-\frac{a_1}{|a_0|}}}{|a_0|}.$$

It follows from the well-posedness condition that $\left(\frac{a_1p_1}{k|a_0|p_0}\right)^{\frac{1}{1-\alpha}}-\frac{a_1}{|a_0|}<0$ and hence $\bar{\theta}<1/|a_0|$, which yields a contradiction. Similarly, $V'(\theta)\neq0$ for any $\theta\in(-\infty,-1/a_1)$. Thus, in the case of $\bar{B}=1$, the optimal solution of (2) is positive when $\mu>0$, negative when $\mu<0$ and zero when $\mu=0$. The expression in (B.1) can be obtained by letting the derivative of $V(\theta)$ being zero.

Proof of Proposition 2.. We only show the case where the reference point is updated according to the non-recursive updating rule (5). The recursive case is analogous. We prove the statement by induction over t. The case $t=0$ is trivial by taking $S(0)=1$. Suppose that the statement holds up to time t. Then

$$\begin{aligned}B_i^s(t)&=\rho x_i(t)+\sum_{r=0}^{t}\left[w\Big(\frac{r+1}{t+1}\Big)-w\Big(\frac{r}{t+1}\Big)\right]x_i(r)\\&=x_i(0)\Big(\rho S(t)+\sum_{r=0}^{t}\left[w\Big(\frac{r+1}{t+1}\Big)-w\Big(\frac{r}{t+1}\Big)\right]S(r)\Big).\end{aligned}$$

We can see that whether the current wealth lies over or below the current reference point does not depend on the investor. Specifically, the expression $x_i(t) \geq B_i^s(t)$ is equivalent to

$$S(t) \geq \rho S(t) + \sum_{r=0}^{t} \left[w\left(\frac{r+1}{t+1}\right) - w\left(\frac{r}{t+1}\right) \right] S(r).$$

Thus, the piecewise linearity coefficient depends only on $\mathcal{F}_t$, but not the investor $i = 1, \ldots, n$. Hence,

$$\begin{aligned} x_i(t+1) &= x_i(t) + \gamma^*(t)(x_i(t) - B_i^s(t))R(t) \\ &= x_i(0)\left(S(t) + \gamma^*(t)\left((1-\rho)S(t) - \sum_{r=0}^{t}\left[w\left(\frac{r+1}{t+1}\right) - w\left(\frac{r}{t+1}\right)\right]S(r)\right)R(t)\right) \end{aligned}$$

is of the form $x_i(t+1) = S(t+1)x_i(0)$ by taking

$$S(t+1) = S(t) + \gamma^*(t)\left((1-\rho)S(t) - \sum_{r=0}^{t}\left[w\left(\frac{r+1}{t+1}\right) - w\left(\frac{r}{t+1}\right)\right]S(r)\right)R(t), \tag{B.4}$$

which is independent of the investor. Note that here $\gamma^*(t)$ is defined by

$$\gamma^*(t) = \begin{cases} \gamma_+, & \text{if } (1-\rho)S(t) - \sum_{r=0}^{t}\left[w\left(\frac{r+1}{t+1}\right) - w\left(\frac{r}{t+1}\right)\right]S(r) > 0; \\ \gamma_-, & \text{otherwise,} \end{cases}$$

and is thus $\mathcal{F}_t$-measurable. □

Proof of Proposition 3.. Clearly, it follows from wealth dynamics (7) that $\{H(t)\}_{t\geq 0}$ is nondecreasing with probability one, hence either the limit $\lim_{t\to\infty} \mathbb{E}[H(t)]$ exists or $\lim_{t\to\infty} \mathbb{E}[H(t)] = +\infty$.

Note that under Assumption 3, $\mu > 0$ and $\gamma_- < 0$. Thus, from (7) we have $\mathbb{E}[x_i(t+1)] = \mathbb{E}[x_i(t)] + \mu|\gamma_-|\mathbb{E}[B_i^s(t) - x_i(t)] \geq \mathbb{E}[x_i(t)]$. The first conclusion follows. From (7) we also have $\mathbb{E}\left[\sum_{i=1}^n x_i(t+1)\right] \geq \mathbb{E}\left[\sum_{i=1}^n x_i(t)\right] + \iota\mu|\gamma_-|\mathbb{E}[H(t) - h(t)]$, where we use the moderation property of the reference point formation rule implying that $\sum_{i=1}^n (B_i^s(t) - x_i(t)) \geq \iota\sum_{i=1}^n (\max_{j\in\mathcal{N}_i} x_j(t) \ -x_i(t)) \geq \iota(H(t) - h(t))$. Similarly, $\mathbb{E}[\sum_{i=1}^n x_i(t+1)] \leq \mathbb{E}[\sum_{i=1}^n x_i(t)] + n\mu|\gamma_-|\mathbb{E}[H(t) - h(t)]$ and thus the inequalities in this proposition follow.

We now show the equivalence of (a), (b) and (c). (a)$\Longrightarrow$(b) is obvious, and (b)$\Longrightarrow$(c) follows from the first inequality of the relation in this proposition. We next show that (c)$\Longrightarrow$(a). It follows from the dynamics (7) that $h(t) \leq H(t-1)$ and then $H(t) - H(t-1) \leq H(t) - h(t)$. Therefore, $\sum_{t=1}^\infty \mathbb{E}[H(t) - H(t-1)] \leq \sum_{t=1}^\infty \mathbb{E}[H(t) - h(t)] < +\infty$, which implies $\lim_{t\to\infty} \mathbb{E}[H(t)] < +\infty$. Thus, (c)$\Longrightarrow$(a) follows.

We now show the second part. First it follows from (c) that $\lim_{t\to\infty} \mathbb{E}[H(t) - h(t)] = 0$. As a result, $\lim_{t\to\infty} \mathbb{E}[x_i(t)] = H^*$ for all i because $\mathbb{E}[h(t)] \leq \mathbb{E}[x_i(t)] \leq \mathbb{E}[H(t)]$. □

Proof of Lemma 1.. Let i_0, j_0 be two arbitrary nodes with $x_{i_0}(t) = h(t)$ and $x_{j_0}(t) = H(t)$. We have $x_i(t+1) = (1 - \phi_\lambda(t))x_i(t) + \phi_\lambda(t)H(t)$, $i = 1, \ldots, n$. Then $x_{j_0}(t+1) = x_{j_0}(t) = H(t)$. We complete the proof by considering the following two cases.

- $1 - \phi_\lambda(t) \geq 0$. In this case, it is easy to verify that $x_{i_1}(t+1) \leq x_{i_2}(t+1)$ for any i_1, i_2 with $x_{i_1}(t) \leq x_{i_2}(t)$. This implies $h(t+1) = x_{i_0}(t+1) = (1-\phi_\lambda(t))h(t) + \phi_\lambda(t)H(t)$ and $H(t+1) = x_{j_0}(t+1) = H(t)$. Therefore,
$$H(t+1) - h(t+1) = x_{j_0}(t+1) - x_{i_0}(t+1) = (1-\phi_\lambda(t))(H(t) - h(t)).$$
- $1 - \phi_\lambda(t) < 0$. In this case, $x_{i_1}(t+1) \geq x_{i_2}(t+1)$ for any i_1, i_2 with $x_{i_1}(t) \leq x_{i_2}(t)$, which implies that $H(t+1) = x_{i_0}(t+1)$ and $h(t+1) = x_{j_0}(t+1)$. Therefore,
$$H(t+1) - h(t+1) = x_{i_0}(t+1) - x_{j_0}(t+1) = (\phi_\lambda(t) - 1)(H(t) - h(t)).$$

The proof is completed. □

Proof of Proposition 4.. Denote

$$\zeta(\lambda) := \mathbb{E}|1 - R(t)|\gamma_-|\lambda| = \int_{-\infty}^{\frac{1}{|\gamma_-|\lambda}} (1 - |\gamma_-|\lambda s)dF(s) + \int_{\frac{1}{|\gamma_-|\lambda}}^{+\infty} (|\gamma_-|\lambda s - 1)dF(s),$$

which is a convex function on [0,1]. We first consider the absolutely continuous case. It is easy to verify that

$$\zeta'(\lambda) = |\gamma_-|\left(\int_{\frac{1}{|\gamma_-|\lambda}}^{+\infty} sdF(s) - \int_{-\infty}^{\frac{1}{|\gamma_-|\lambda}} sdF(s)\right) = |\gamma_-|\left(2\int_{\frac{1}{|\gamma_-|\lambda}}^{+\infty} sdF(s) - \mu\right),$$

and hence $\lim_{0<\lambda\to 0} \zeta'(\lambda) = -|\gamma_-|\mu < 0$. We complete the proof of (i) by considering the following two cases.

- $\zeta(1) > 1$. It can be seen that the root of $\zeta(\cdot) = 1$ on (0,1) is unique, denoted as λ_{th}. Then it follows that $\zeta(\lambda) < 1$ for $0 < \lambda < \lambda_{th}$, and $\zeta(\lambda) > 1$ for $\lambda_{th} < \lambda \leq 1$.

• $\zeta(1) \le 1$. In this case, $\zeta(\lambda) < 1$ for any $\lambda \in (0,1)$.

We now consider the discrete distribution case. We observe that $\zeta(\cdot)$ is piecewise linear, continuous and differentiable except at points in set $\Delta := \{1/(|\gamma_-|a_i), |\gamma_-|a_i > 1\}$, and $\lim_{0<\lambda\to 0, \lambda\notin\Delta} \zeta'(\lambda) = -|\gamma_-|\mu$. The conclusion for discrete distribution case follows from the preceding observations. □

Proof of Proposition 5.. (i) Here we prove only the first part for the case of an absolutely continuous distribution since the proof for the case of a discrete distribution is similar. We can see that the function

$$\hbar(s) := \int_s^{+\infty} s dF(s) + s(F(s) - 1)$$

is nonincreasing on $(0,+\infty)$ and $\lim_{s\to+\infty} \hbar(s) = 0$ by noting that $\hbar'(s) = F(s) - 1 \le 0$ and $\lim_{s\to+\infty}(F(s)-1)/(1/s) = -\lim_{s\to+\infty} s^2 f(s) = 0$ from Assumption 1. Because $\hbar(0) > \mu/2$, we claim that the equation $\hbar(s) = \mu/2$ has a unique root, denoted as s^*, on $(0,+\infty)$. If $\hbar'(s) < 0$ for all $s > 0$, the claim is obvious. Otherwise let $\hat{s}$ be the smallest number such that $\hbar'(\hat{s}) = 0$ (i.e., $F(\hat{s}) = 1$). Therefore, $\hbar(s) = 0$ for $s \ge \hat{s}$, which implies that $\hbar(\cdot)$ has a unique root on $(0,\hat{s})$ and then the claim follows.

Recall the notation $\zeta(\lambda) = \mathbb{E}[|1 - R(t)|\gamma_-|\lambda|]$ given in last proof. By some simple algebraic computations we obtain that

$$\zeta(\lambda) = 1 + 2|\gamma_-|\lambda\left[\hbar\left(\frac{1}{|\gamma_-|\lambda}\right) - \frac{\mu}{2}\right].$$

If $|\gamma_-|s^* \le 1$, then $\hbar(1/(|\gamma_-|\lambda)) \le \hbar(1/|\gamma_-|) \le \hbar(s^*) = \frac{\mu}{2}$, implying that $\zeta(\lambda) \le 1$ for any $0 < \lambda \le 1$ and thus $\lambda_{th} = 1$. If $|\gamma_-|s^* > 1$, then $\zeta(1/(|\gamma_-|s^*)) = 1$ and thus $\lambda_{th} = 1/(|\gamma_-|s^*)$. This completes the proof of the first part of (i). The formula for elliptical distributions can be obtained by a change of the variable $z = (s-\mu)/\sigma$ and the formula for two-point distributions can be obtained by solving directly (10).

(ii) Here we only show the conclusion $\lambda_{th} < 1$ for sufficiently small μ if $\lim_{0<\mu\to 0} \gamma_-(\mu) < 0$ since all other conclusions can be obtained directly from the arguments given in (i). We can see that it suffices to show that when $\mu = 0$, $\int_\tau^{+\infty} s dF(s) + \tau(F(\tau) - 1) > 0$, where $0 < \tau = \lim_{0<\mu\to 0} 1/|\gamma_-(\mu)| < +\infty$. This is true since otherwise $F(\tau) = 1$, which contradicts the following Lemma 2. Therefore, the desired conclusion follows. □

Lemma 2. *Suppose that the excess return of $R(t)$ is bounded from above: $R(t) \le M$ for some $M > 0$ and $\int_{-\infty}^M s dF(s) > 0$. Then $|\gamma_-|M > 1$.*

Proof. To prove this, it suffices to show that when $\bar{B} = -1$, $V'(\theta) > 0$ for $0 < \theta \le 1/M$. In fact, when $0 < \theta \le 1/M$, $V(\theta) = -k\int_{-\infty}^M (-\theta s + 1)^\alpha dF(s)$ and

$$\begin{aligned}
V'(\theta) &= k\alpha \int_{-\infty}^M (-\theta s + 1)^{\alpha-1} s dF(s) \\
&= k\alpha\left(\int_{-\infty}^0 (-\theta s + 1)^{\alpha-1} s dF(s) + \int_0^M (-\theta s + 1)^{\alpha-1} s dF(s)\right) \\
&\ge k\alpha \int_{-\infty}^M s dF(s) > 0,
\end{aligned}$$

where the first inequality follows from $(-\theta s + 1)^{\alpha-1} < 1$, $s < 0$ and $(-\theta s + 1)^{\alpha-1} > 1$, $0 < s < M$. □

Proof of Proposition 6.. Recall that $\phi_\lambda(t) = R(t)|\gamma_-|\lambda$. In this proof we abbreviate $\xi_\lambda, \eta_\lambda$ as ξ, η for simplicity. From the proof of Lemma 1, we have

$$h(t+1) = \begin{cases} H(t), & \text{if } \phi_\lambda(t) > 1; \\ h(t) + \phi_\lambda(t)(H(t) - h(t)), & \text{if } \phi_\lambda(t) \le 1, \end{cases}$$
$$H(t+1) = \begin{cases} h(t) + \phi_\lambda(t)(H(t) - h(t)), & \text{if } \phi_\lambda(t) > 1; \\ H(t), & \text{if } \phi_\lambda(t) \le 1. \end{cases}$$

As a result,

$$\begin{pmatrix} \mathbb{E}[h(t+1)] \\ \mathbb{E}[H(t+1)] \end{pmatrix} = \begin{pmatrix} \xi & 1-\xi \\ -\eta & 1+\eta \end{pmatrix} \begin{pmatrix} \mathbb{E}[h(t)] \\ \mathbb{E}[H(t)] \end{pmatrix}.$$

It is easy to verify that the two eigenvalues of the system matrix

$$A := \begin{pmatrix} \xi & 1-\xi \\ -\eta & 1+\eta \end{pmatrix}$$

are 1 and $\xi+\eta$, and when $\xi+\eta\neq 1$, the eigenvectors corresponding to eigenvalues 1, $\xi+\eta$ are $(1,1)^T$ and $(1-\xi,\eta)^T$ (T denotes the transpose of a vector), respectively. Consequently,

$$\begin{aligned} A^t &= \begin{pmatrix} 1 & 1-\xi \\ 1 & \eta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & (\xi+\eta)^t \end{pmatrix}\begin{pmatrix} 1 & 1-\xi \\ 1 & \eta \end{pmatrix}^{-1} \\ &= \tfrac{1}{1-\xi-\eta}\begin{pmatrix} (1-\xi)(\xi+\eta)^t-\eta & 1-\xi-(1-\xi)(\xi+\eta)^t \\ \eta(\xi+\eta)^t-\eta & 1-\xi-\eta(\xi+\eta)^t \end{pmatrix}. \end{aligned}$$

With this result we ge

$$\begin{aligned} \mathbb{E}[h(t)] &= \tfrac{(1-\xi)(\xi+\eta)^t(H(0)-h(0))-(1-\xi)H(0)+\eta h(0)}{\xi+\eta-1}, \\ \mathbb{E}[H(t)] &= \tfrac{\eta(\xi+\eta)^t(H(0)-h(0))-(1-\xi)H(0)+\eta h(0)}{\xi+\eta-1}. \end{aligned} \tag{B.5}$$

Note that $\xi_\lambda+\eta_\lambda=\mathbb{E}[|1-\phi_\lambda(t)|]$. From the proof of Proposition 4, we have $\xi_\lambda+\eta_\lambda<1$ for $0<\lambda<\lambda_{th}$, and $\xi_\lambda+\eta_\lambda>1$ for $\lambda>\lambda_{th}$. We claim that $\eta_\lambda>0$ for any $\lambda\geq\lambda_{th}$ since otherwise $\mathbb{P}\big(R(t)>1/(|\gamma_-|\lambda_0)\big)=0$ for some $\lambda_0\geq\lambda_{th}$ and hence $\xi_{\lambda_0}=1-\mu|\gamma_-|\lambda_0<1$. This contradicts $\xi_\lambda+\eta_\lambda\geq 1$ for $\lambda\geq\lambda_{th}$, so $\eta_\lambda>0$ for $\lambda\geq\lambda_{th}$. Then it follows from (B.5) that

$$\begin{aligned} \frac{(1-\xi)(\xi+\eta)^t(H(0)-h(0))}{\xi+\eta-1} &= \mathbb{E}[h(t)]-\varpi \\ &\leq \mathbb{E}[x_i(t)]-\varpi \\ &\leq \mathbb{E}[H(t)]-\varpi = \frac{\eta(\xi+\eta)^t(H(0)-h(0))}{\xi+\eta-1}, \end{aligned}$$

where $\varpi:=[(1-\xi)H(0)-\eta h(0)]/[1-\xi-\eta]$. Consequently, when $\lambda<\lambda_{th}$, each investor's expected wealth converges to the common finite level ϖ with the exponential rate $\xi_\lambda+\eta_\lambda<1$. The first conclusion follows noting that each investor's expected wealth is increasing over time as shown in Proposition 3.

We now show the second conclusion. First, it follows from (9) that

$$\mathbb{E}[x_i(t+1)]=(1-\mu|\gamma_-|\lambda)\mathbb{E}[x_i(t)]+\mu|\gamma_-|\lambda\mathbb{E}[H(t)],$$

which implies that

- when $\mu|\gamma_-|\lambda\geq 1$, $\mathbb{E}[x_i(t+1)]\geq\mathbb{E}[H(t)]$;
- when $\mu|\gamma_-|\lambda<1$, $\mathbb{E}[x_i(t+1)]\geq(1-\mu|\gamma_-|\lambda)x_i(0)+\mu|\gamma_-|\lambda\mathbb{E}[H(t)]$ noting the nondecreasingness of $\{\mathbb{E}[x_i(t)]\}_{t\geq 0}$.

In summary, $\min\{1,\mu|\gamma_-|\lambda\}\mathbb{E}[H(t)]\leq\mathbb{E}[x_i(t+1)]\leq\mathbb{E}[H(t+1)]$. This combined with (B.5) implies that when $\lambda>\lambda_{th}$, $\lim_{t\to\infty}(\mathbb{E}[x_i(t)])^{1/t}=\xi_\lambda+\eta_\lambda>1$, which is the implication of the exponential growth rate in the second expression of the theorem. □

Proof of Proposition 7.. From (13) we have

$$\mathbb{E}\left[\frac{1}{W(t+1)}\right]=\mathbb{E}\left[\frac{1}{|1-\phi_\lambda(t)|}\right]\mathbb{E}\left[\frac{1}{W(t)}\right]+2n\mathbb{E}\left[\frac{\phi_\lambda(t)}{|1-\phi_\lambda(t)|}\right]\mathbb{E}\left[\frac{\sum_{i=1}^n(H(t)-x_i(t))}{\sum_{i=1}^n\sum_{j=1}^n|x_i(t)-x_j(t)|}\right].$$

Therefore, the conclusion (i) follows from the above equality and the boundedness

$$\begin{aligned} \frac{\sum_{i=1}^n(H(t)-x_i(t))}{\sum_{i=1}^n\sum_{j=1}^n|x_i(t)-x_j(t)|} &\leq \frac{n(H(t)-h(t))}{\sum_{i=1}^n\sum_{j=1}^n|x_i(t)-x_j(t)|} \\ &= \frac{n(H(0)-h(0))}{\sum_{i=1}^n\sum_{j=1}^n|x_i(0)-x_j(0)|}, \end{aligned}$$

where the equality follows from Lemma 1 and the relation (12).

When $0<\lambda<\lambda_{th}$, $\mathbb{E}[|1-\phi_\lambda(t)|]<1$ by Proposition 4. The conclusion (ii) follows from the strict inequality $\mathbb{E}[1/|1-\phi_\lambda(t)|]>1/\mathbb{E}[|1-\phi_\lambda(t)|]$ and the continuity of function $\lambda\mapsto\mathbb{E}[1/|1-\phi_\lambda(t)|]$.

We observe that $\mathbb{E}[1/GGC(t)]\geq\mathbb{E}[1/W(t)]$ since a wealth distribution with a negative total wealth contributes to a positive value of $1/GGC(t)$ but a negative value of $1/W(t)$, and because for a wealth distribution with positive total wealth, the adjustment factor in the definition of the generalized Gini coefficient is no greater than one. Thus, (iii) follows from the results in (i), (ii) and the preceding observation. □

Proof of Proposition 8.. Here we only show Case 2 in Table 4 because the proof for Case 3 is similar. Note that $\gamma_->0$ in Case 2. From the "Upward-Looking" property in the reference point formation rule and Theorem 1 we have $x_i(t+1)=x_i(t)-R(t)\gamma_-(B_i^s(t)-x_i(t))$ and consequently, from the "Moderation" property,

$$\mathbb{E}\left[\sum_{i=1}^n x_i(t+1)\right]\leq\mathbb{E}\left[\sum_{i=1}^n x_i(t)\right]-\iota\mu\gamma_-\mathbb{E}[H(t)-h(t)]. \tag{B.6}$$

We first show that $\sum_{t=0}^{\infty}\mathbb{E}[H(t)-h(t)]=+\infty$. Suppose, to the contrary, that $\sum_{t=0}^{\infty}\mathbb{E}[H(t)-h(t)]<+\infty$. Consequently, $\lim_{t\to\infty}\mathbb{E}[H(t)-h(t)]=0$ and thus $\lim_{t\to\infty}\mathbb{E}[H(t)-x_i(t)]=0$ for all i because $h(t)\le x_i(t)$. However, a contradiction arises since $\left\{\mathbb{E}[x_i(t)]\right\}_{t\ge 0}$ is nonincreasing, $\{\mathbb{E}[H(t)]\}_{t\ge 0}$ is nondecreasing and there exists some investor i such that $x_i(0)<H(0)$.

From (B.6) we have $\mathbb{E}[\sum_{i=1}^{n}x_i(t+1)]\le\sum_{i=1}^{n}x_i(0)-\iota\mu\gamma_-\sum_{s=0}^{t}\mathbb{E}[H(s)-h(s)]$. Together with $\sum_{t=0}^{\infty}\mathbb{E}[H(t)-h(t)]=+\infty$, that leads to $\lim_{t\to\infty}\mathbb{E}[\sum_{i=1}^{n}x_i(t)]=-\infty$. Hence, $\lim_{t\to\infty}\mathbb{E}[h(t)]=-\infty$, and $\lim_{t\to\infty}\mathbb{E}[H(t)-h(t)]=+\infty$ follows noting that $\{\mathbb{E}[H(t)]\}_{t\ge 0}$ is nondecreasing. □

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