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The impact of relative wealth concerns on wealth gap and welfare in a noisy rational expectations economy[☆]

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ABSTRACT

We employ a noisy rational expectations equilibrium model to investigate the influence of relative wealth concerns on wealth gap and welfare. Our analysis reveals that the impact is sensitive to the exogeneity or endogeneity of information. When information is exogenous, the average wealth gap between high-precision and low-precision investors is either decreasing or, initially decreases and eventually increases in the degree of relative wealth concerns. Moreover, we identify two or three potential patterns regarding the monotonicity pattern of the welfare of low-precision and high-precision investors. However, when information becomes endogenous, the average wealth gap and welfare decrease.

1. Introduction

Relative wealth concerns (RWC) refers to the fact that investors care not only about their own wealth, but also about how their wealth compares to others', akin to the notion of "Keeping up with Joneses" preferences. This gives rise to several interesting questions: Does an individual's happiness increase with the degree of RWC? Does the wealth gap between individuals expand as RWC intensifies? Can the wealth gap be reduced and overall welfare be improved by adjusting the degree of RWC? This paper aims to address these questions within the context of an exchange economy.

We investigate the influence of RWC on the wealth gap and welfare using a rational expectations paradigm (Hellwig, 1980; Grossman and Stiglitz, 1980; Verrecchia, 1982). Specifically, we adopt the noisy rational expectations equilibrium model proposed by García and Strobl (2011), which involves two assets – one risky asset and one riskless asset – and a continuum of risk-averse investors. Investors evaluate the conditional utility of their relative wealth, defined as the difference between their own wealth and the average wealth of all investors in the economy, weighted by a coefficient of RWC. A higher coefficient indicates a greater degree of concern about the wealth of others. The economy consists of two types of rational investors: high-precision investors, who can obtain high-precision private signals at a high cost, and low-precision investors, who can acquire low-precision signals at a low cost. Investors' information includes their private signals and publicly observable price information. To prevent prices from fully revealing, the economy incorporates a random supply. Investors optimize their demands to maximize their conditional utility, and market-clearing conditions endogenously determine the equilibrium price.

The main contribution of the paper is that it is the first attempt to analyze the impact of RWC on wealth gap and welfare within a framework of rational expectations equilibrium. Our findings reveal that the effects of the degree of RWC on the wealth gap and welfare are contingent upon whether information is exogenous or endogenous. In the case of exogenous information, we demonstrate that in informationally inefficient markets, the average wealth gap between high-precision and low-precision investors decreases as the degree of RWC increases. However, in sufficiently informationally efficient markets, the average wealth gap initially decreases and eventually increases with the degree of RWC. Moreover, depending on model parameters, two or three potential patterns exist for the monotonicity of low-precision/high-precision investors' welfare as the degree of RWC varies. Additionally, in the case of endogenous information, we establish that for any model parameter, both the average wealth gap and the welfare of each investor decrease as the coefficient of RWC increases.

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Our results demonstrate that in three scenarios – endogenous information, exogenous information in an informationally inefficient market, and exogenous information in an informationally efficient market but the degree of RWC is confined to the range of [0,1] – a trade-off between reducing the wealth gap and improving welfare is always present. In other words, a social planner cannot simultaneously narrow the wealth gap and enhance investors' welfare by manipulating the degree of RWC. From the standpoint of social planners or policymakers, it is therefore desirable to appropriately adjust the extent of social comparison among the population and enhance transparency regarding wealth distribution within society, aiming to achieve an optimal balance between the wealth gap and welfare.

It is worth noting that studying the wealth gap in financial markets plays an important role in comprehending the multifaceted dimensions of economic inequality. As we know, there are two types of household income: labor income and capital income which includes interest, dividends and other realized investment returns. Thomas Piketty claims in his popular book *Capital in the Twenty-First Century* that capital income is more unequally distributed than labor income, and a transfer from labor income to capital income will increase inequality (Piketty, 2014). Moreover, Bengtsson and Waldenstrom (2018) empirically find that labor income has been declining as a share of total income earned in the U.S. since the early 1980s, and there appears to be a fairly strong positive relationship between higher capital shares and income inequality over the long run. Similarly, Jacobson and Occhino (2012) find that for the U.S., a 1 percent increase in the capital share tended to raise the Gini by between 0.15 percent and 0.33 percent.

Our paper is closely related to the vast literature on the impact of RWC on market outcomes (Admati and Pfleiderer, 1997; Qiu, 2017; García and Strobl, 2011; Breugem and Buss, 2019). In the context of delegated portfolio management, Qiu (2017) investigates the remuneration structure for fund managers, considering both absolute and relative performance metrics compared to their peer group instead of assuming "Keeping up with the Joneses" preferences. Qiu (2017) centers on the informativeness of asset prices under different information structures. On a related note, Admati and Pfleiderer (1997) and Breugem and Buss (2019) examine investor behavior with regard to their performance relative to a benchmark, such as passive benchmarks, in addition to their own absolute performance. These studies shed light on the interplay between relative performance and market dynamics. García and Strobl (2011) offer a particularly relevant contribution to our research as it focuses on the endogenous acquisition of information by agents. In contrast, our study investigates the impact of RWC on the wealth gap and welfare in both cases of exogenous and endogenous information. By extending the existing literature, our analysis provides valuable insights into the relationship between RWC and market outcomes.

2. Model

In this paper, we adopt the model introduced by García and Strobl (2011) to examine the impact of RWC. Our analysis focuses on a perfectly competitive market with multiple dates, specifically $t = 0, 1, 2, 3$. The market consists of a continuum of investors, indexed by $i \in [0, 1]$, and offers two trading assets: a riskless asset with a normalized price and payoff of 1 and a risky asset with a random final payoff $\theta \sim \mathcal{N}(\mu_\theta, \tau_\theta^{-1})$ at $t = 3$, where $\tau_\theta > 0$.

Trading occurs at $t = 2$, and the preferences of investor $i \in [0, 1]$ are captured by the Constant Absolute Risk Aversion (CARA) utility function. The CARA utility function is defined as:

$$u\left(W_i - \gamma \overline{W}\right) = -\exp\left(-\rho\left(W_i - \gamma \overline{W}\right)\right),$$

where $W_i = x_i(\theta - p)$ represents the terminal wealth of investor i, x_i denotes the number of shares of the risky asset bought by investor i, p is the publicly observable price of the risky asset, $\overline{W}$ is the average wealth of all investors in the economy, and $\rho > 0$ and $\gamma > 0$ represent the risk aversion parameter and coefficient of RWC, respectively. We assume zero initial wealth for investors due to the CARA assumption. The random supply of the risky asset equals $Z \sim \mathcal{N}(0, \tau_z^{-1})$, where $\tau_z > 0$. The randomness of the supply prevents equilibrium prices from fully revealing the asset payoff.

Signal precisions exhibit heterogeneity among investors. To maintain simplicity, we assume the presence of only two types of signals in this market. Investors can obtain either high-precision or low-precision signals. Without loss of generality, we designate investors who receive high-precision signals as type $H = [0, \lambda]$, while those who receive low-precision signals as type $L = (\lambda, 1]$. Specifically, at time $t = 1$, type-H investor $i \in H$ observes a private signal $y_i = \theta + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \tau_H^{-1})$. On the other hand, type-L investor $j \in L$ observes a private signal $y_j = \theta + \epsilon_j$, where $\epsilon_j \sim \mathcal{N}(0, \tau_L^{-1})$ and $\tau_L < \tau_H$. The random variables $\{\epsilon_i\}_{i\in[0,1]}$, θ, and Z are mutually independent. When $\tau_L = 0$, our model reduces to the one proposed by García and Strobl (2011). In Section 3, we endow investors with signals, and the parameter λ is exogenously given. Fig. 1 illustrates the timeline of the economy with exogenous information, representing the periods $t = 1, 2, 3$. In Section 4, we consider the scenario where investors incur costs to acquire signals, necessitating a trade-off between the informativeness of signals and the cost of information acquisition at time $t = 0$. In this case, the parameter λ is endogenously determined. The timeline of the economy with endogenous information is depicted in Fig. 1, encompassing the periods $t = 0, 1, 2, 3$.

A *noisy rational expectations equilibrium* (with exogenous information) is characterized by a set of trading strategies $\{x_i^*\}_{i\in[0,1]}$ and a price function p satisfying the following conditions:

(i) Each investor i selects her trading strategy x_i^* to maximize her expected utility conditional on her information. Formally, x_i^* solves the optimization problem:

$$\max_{x_i} \mathbb{E}\left[-\exp\left(-\rho\left(W_i - \gamma \overline{W}\right)\right)\Big| y_i, p\right], \ i \in [0, 1].$$

(ii) The market clears, i.e.,

$$\int_0^1 x_i^* di = Z.$$

As is standard in this literature, we confine our analysis to linear equilibria. Thus, we assume that the equilibrium price is a linear function of the average signal of all investors and the supply, denoted as:

$$p = a + b\theta - sZ, \tag{1}$$

where a, b, and s are three constants to be determined later. Furthermore, we propose that in a linear equilibrium, the linear trading strategy of an investor in H (respectively L) takes the form $x_i = \xi_H + \beta_H y_i - \kappa_H p$ (respectively $x_i = \xi_L + \beta_L y_i - \kappa_L p$), where ξ_H, β_H, κ_H, ξ_L, β_L, and κ_L are endogenously determined constants. For the sake of convenience, let us denote $\beta_\lambda = \frac{\lambda\tau_H + (1-\lambda)\tau_L}{\rho}$ as the risk-adjusted average signal precision in the economy.

Suppose $\{\{x_i^*\}_{i\in[0,1]}, p\}$ represents a noisy rational expectations equilibrium, the *average wealth gap* between L and H is defined as:

$$\mathbb{E}\left(\frac{1}{\lambda}\int_0^\lambda x_i^*(\theta - p)di - \frac{1}{1-\lambda}\int_\lambda^1 x_i^*(\theta - p)di\right).$$

This measure captures the difference in wealth between high-precision and low-precision investors. Moreover, the *welfare* (expected utility) of investor i can be expressed as:

$$\mathbb{E}\left[u\left(W_i - \gamma\overline{W}\right)\right] = \mathbb{E}\left[u\left(x_i^*(\theta - p) - \gamma\int_0^1 x_j^*(\theta - p)dj\right)\right].$$

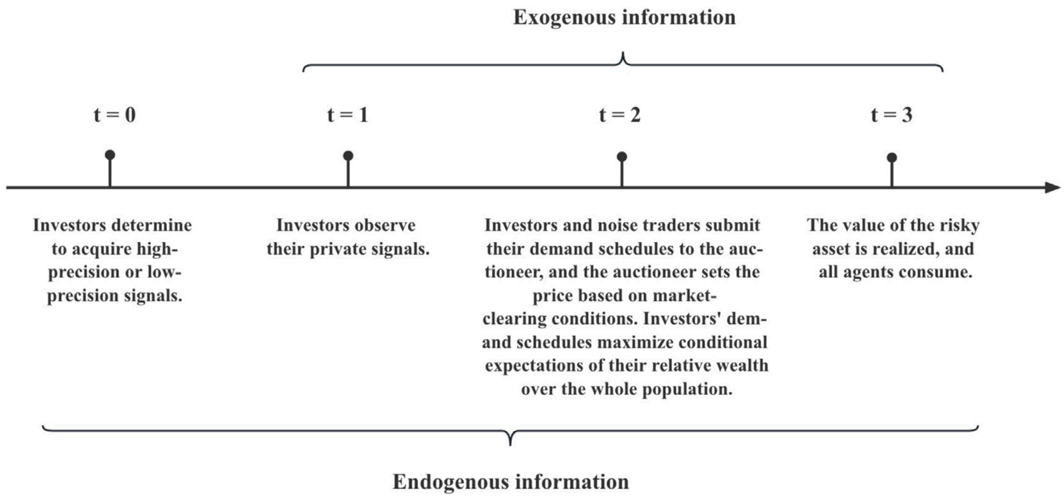

Fig. 1. Timeline of the model.

3. Exogenous information

In this section, we examine the case where investors are endowed with private signals and the parameter λ is exogenously given.

Proposition 1. *Suppose information is exogenous with an exogenously given fraction λ of high-precision investors, $\gamma < \frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda}$, and $\gamma \neq 1+\beta_\lambda\tau_z\rho^{-1}$. Then, a unique linear noisy rational expectations equilibrium exists, where the coefficients in the linear optimal strategy x_i^* by investor i are as follows:*

$$\beta_H = \frac{\tau_H}{\rho},$$
$$\beta_L = \frac{\tau_L}{\rho},$$
$$\kappa_H = \frac{\tau_\theta}{\rho(1-\gamma)+\beta_\lambda\tau_z} + \frac{\tau_H}{\rho},$$
$$\kappa_L = \frac{\tau_\theta}{\rho(1-\gamma)+\beta_\lambda\tau_z} + \frac{\tau_L}{\rho},$$
$$\xi_H = \xi_L = \frac{\tau_\theta\mu_\theta}{\rho(1-\gamma)+\beta_\lambda\tau_z},$$

and the coefficients in the equilibrium price $p = a + b\theta - sZ$ are as follows:

$$a = \frac{\tau_\theta\mu_\theta}{\tau_\theta+\beta_\lambda\rho(1-\gamma)+\beta_\lambda^2\tau_z}, \quad (2)$$
$$b = 1 - \frac{\tau_\theta}{\tau_\theta+\beta_\lambda\rho(1-\gamma)+\beta_\lambda^2\tau_z}, \quad (3)$$
$$s = \frac{\rho(1-\gamma)+\beta_\lambda\tau_z}{\tau_\theta+\beta_\lambda\rho(1-\gamma)+\beta_\lambda^2\tau_z}. \quad (4)$$

Next, we investigate the influence of RWC on the average wealth gap and investor welfare. Define

$$x_H := \frac{1}{\lambda}\int_0^\lambda x_i^* di = \xi_H + \beta_H\theta - \kappa_H p,$$
$$x_L := \frac{1}{1-\lambda}\int_\lambda^1 x_i^* di = \xi_L + \beta_L\theta - \kappa_L p$$

as the average number of shares of the risky asset bought by investors in the high-precision (H) and low-precision (L) regimes, respectively. Accordingly, the average wealth of investors in the high-precision regime (denoted by W_H) and low-precision regime (denoted by W_L) is given by

$$W_H := \frac{1}{\lambda}\int_0^\lambda x_i^*(\theta-p)di = x_H(\theta-p),$$
$$W_L := \frac{1}{1-\lambda}\int_\lambda^1 x_i^*(\theta-p)di = x_L(\theta-p).$$

Consequently, the average wealth gap can be expressed as $\mathbb{E}(W_H - W_L)$.

In the subsequent sections of this paper, we make the implicit assumption that the condition $\gamma \neq 1+\beta_\lambda\tau_z\rho^{-1}$ is satisfied to ensure the existence of equilibrium when discussing the monotonicity of the average wealth gap and investors' welfare. The following proposition, which assumes exogenous information, presents the results:

Proposition 2. *Suppose information is exogenous. We establish the following results:*

(i) If $\frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda} \leq 1$, then the average wealth gap between L and H and the welfare of each investor are both strictly decreasing in $\gamma \in \left(0, \frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda}\right)$.

(ii) If $\frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda} > 1$, then the average wealth gap between L and H is initially strictly decreasing in $\gamma \in (0,1)$ and eventually strictly increasing in $\gamma \in \left(1, \frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda}\right)$. Moreover, there exists $1 < \gamma_k^* < \frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda}$ such that the welfare of each type-k investor is strictly decreasing in $\gamma \in (0,\gamma_k^*)$, where $k \in \{H, L\}$. However, the monotonicity of low-precision investors' welfare on the interval $\left(\gamma_k^*, \frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda}\right)$ can follow two patterns, while high-precision investors' welfare can follow three patterns.

The average wealth gap $\mathbb{E}(W_H - W_L)$ can be expressed as (see equations (S20) and (S23) in the Online Appendix):

$$\frac{\tau_H - \tau_L}{\rho}\mathrm{Var}(\theta - p),$$

which positively correlates with the return volatility $\mathrm{Var}(\theta-p)$. Intuitively, as the return on the risky asset becomes more volatile, high-precision investors have more market opportunities to take advantageous positions and generate higher profits, resulting in a larger wealth gap between high-precision and low-precision investors.

Since $\mathrm{Var}^{-1}[\theta|y_i,p] = \tau_\theta+\tau_L+\beta_\lambda^2\tau_z$ for $i \in L$, the condition $\frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda} \leq 1$ can be interpreted as an indication of informationally inefficient markets, while $\frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda} > 1$ can be seen as a sign of sufficiently informationally efficient markets. From Eqs. (1), (3), and (4), we can decompose the return volatility $\mathrm{Var}(\theta-p)$ into a fundamental-driven component $(1-b)^2\tau_\theta^{-1}$ and a random-supply-driven component $s^2\tau_z^{-1}$. As the coefficient of RWC γ increases from zero, s decreases and $(1-b)$ increases. However, the random-supply-driven component dominates the fundamental-driven component, resulting in a decrease in return volatility with γ. Moreover, when γ increases from one (assuming the market is sufficiently informationally efficient), the decrease in s becomes small, and the increase in $(1-b)$ becomes significant since $(1-b)$ is a convex function and s is a concave function of γ.[1] Consequently, the fundamental-driven component dominates the random-supply-driven component, leading to an increase in return volatility with γ.

The welfare of investor i can be represented as (see equation (S25) in the Online Appendix):

$$-\left(\frac{\Psi_i}{\Sigma_i}\,\mathrm{Var}(\theta-p)\right)^{-\frac{1}{2}},$$

which consists of three components. Firstly, $\Sigma_i^{-1} = \mathrm{Var}^{-1}[\theta|y_i,p] = \tau_\theta + \tau_i + \beta_\lambda^2\tau_z$ measures price informativeness and remains independent of γ. Secondly, $\Psi_i = 1-2\rho\gamma\beta_\lambda\Sigma_i$ decreases as γ increases. The term $2\rho\gamma\beta_\lambda\Sigma_i$ can be interpreted as the welfare loss due to RWC. A larger value of γ leads to a greater welfare loss. Thirdly, the return volatility $\mathrm{Var}(\theta-p)$, which is influenced by γ, plays a role in determining welfare. A low value of $\mathrm{Var}(\theta-p)$ implies low investment risk faced by investors. However, low risk also leads to low expected returns, as observed in Kurlat and Veldkamp (2015).[2]

Based on the above analysis, the degree of RWC affects welfare through the terms of Ψ_i and $\mathrm{Var}(\theta-p)$. Since we have shown above that the return volatility decreases with small γ, the welfare also decreases with small γ. However, when $\gamma>1$, the return volatility increases with γ, so that the monotonicity of the product $\Psi_i\,\mathrm{Var}(\theta-p)$ over large γ may be ambiguous and depends on model parameters. In fact, Proposition 2 demonstrates that, in the case where $\frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda}>1$, there are at most three (two) possibilities for the monotonicity pattern of high-precision (low-precision) investors' welfare within the interval $\left(0, \frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda}\right)$.

4. Endogenous information

In this section, we investigate the case of endogenous information. The market offers two types of signals for investors to choose from. One type of signal possesses high precision, denoted as $\tau_H>0$, while the other type exhibits lower precision, denoted as τ_L, where $0<\tau_L<\tau_H$. Investors face a decision of whether to incur a cost $c_1>0$ to acquire a high-precision signal y_i before engaging in trading. Here, $y_i=\theta+\epsilon_i$, and the noise component ϵ_i follows a normal distribution with mean zero and variance $\mathrm{Var}(\epsilon_i)=1/\tau_H$. Alternatively, investors can choose to incur a cost $c_2>0$ to obtain a low-precision signal y_j, where $y_j=\theta+\epsilon_j$ and $\mathrm{Var}(\epsilon_j)=1/\tau_L$. It is assumed that $c_1>c_2$ and both costs are externally determined.

Let $\mathcal{U}_i$ represent the monotonic transformation of investor i's ex ante welfare, defined as:

$$\mathcal{U}_i = -\frac{1}{\rho}\log\left(\frac{\mathbb{E}\left[\exp\left(-\rho\left(x_i^*(\theta-p)-\gamma\overline{W}\right)\right)\right]}{\mathbb{E}\left[\exp\left(\rho\gamma\overline{W}\right)\right]}\right), \tag{5}$$

where $\overline{W}=\int_0^1 x_j^*(\theta-p)dj$. The equilibrium price p is determined by Eq. (1) using the coefficients from Eqs. (2), (3), and (4).

We now employ the definition of $\mathcal{U}_i$ to formally establish the concept of an *endogenous equilibrium*, as outlined in García and Strobl (2011). To define an endogenous equilibrium, it is crucial to introduce the notion of the endogenous fraction of high-precision investors. Once the endogenous fraction is determined, a rational expectations equilibrium can be obtained with this fraction using Proposition 1. An interior equilibrium at the information acquisition stage occurs when a fraction $\lambda\in(0,1)$ satisfies

$$\mathcal{V}_L(\lambda)-c_2=\mathcal{V}_H(\lambda)-c_1,$$

where $\mathcal{V}_H\equiv\mathcal{U}_i$ for any high-precision investor $i\in[0,\lambda]$ and $\mathcal{V}_L\equiv\mathcal{U}_j$ for any low-precision investor $j\in(\lambda,1]$. Non-interior (or corner) equilibria are defined in a natural manner: $\lambda=0$ represents a corner equilibrium if $\mathcal{V}_L(0)-c_2\geq\mathcal{V}_H(0)-c_1$, and $\lambda=1$ represents a corner equilibrium if $\mathcal{V}_L(1)-c_2\leq\mathcal{V}_H(1)-c_1$.

Next, we proceed with the computation of the endogenous equilibrium. From equation (S25) in the Online Appendix, we have:

$$\mathbb{E}\left[\exp\left(-\rho\left(x_i^*(\theta-p)-\gamma\overline{W}\right)\right)\right]=\left(\frac{\Psi_i}{\Sigma_i}\,\mathrm{Var}(\theta-p)\right)^{-\frac{1}{2}},$$

where $\Psi_i=1-2\rho\gamma\beta_\lambda\Sigma_i$, $\Sigma_i^{-1}=\tau_\theta+\tau_H+\beta_\lambda^2\tau_z$ for $i\in[0,\lambda]$ and $\Sigma_i^{-1}=\tau_\theta+\tau_L+\beta_\lambda^2\tau_z$ for $i\in(\lambda,1]$.

The certainty equivalent of wealth for high-precision investors, as defined in Eq. (5), is given by:

$$\mathcal{V}_I(\lambda)=\frac{1}{2\rho}\log\left(\frac{\Psi_i}{\Sigma_i}\right)+M=\frac{1}{2\rho}\log\left(\tau_\theta+\tau_H+\beta_\lambda^2\tau_z-2\rho\gamma\beta_\lambda\right)+M,$$

where

$$M=\frac{1}{2\rho}\log(\mathrm{Var}(\theta-p))+\frac{1}{\rho}\log\left(\mathbb{E}\left[\exp\left(\rho\gamma\overline{W}\right)\right]\right).$$

Similarly, for low-precision investors, we have:

$$\mathcal{V}_U(\lambda)=\frac{1}{2\rho}\log\left(\frac{\Psi_i}{\Sigma_i}\right)+M=\frac{1}{2\rho}\log\left(\tau_\theta+\tau_L+\beta_\lambda^2\tau_z-2\rho\gamma\beta_\lambda\right)+M.$$

The endogenous fraction of high-precision investors, denoted by λ, is determined by solving the equation:

$$\frac{1}{2\rho}\log\left(\tau_\theta+\tau_H+\beta_\lambda^2\tau_z-2\rho\gamma\beta_\lambda\right)-\frac{1}{2\rho}\log\left(\tau_\theta+\tau_L+\beta_\lambda^2\tau_z-2\rho\gamma\beta_\lambda\right)=c_1-c_2,$$

which can be rewritten as:

$$\tau_\theta+\tau_H+\beta_\lambda^2\tau_z-2\rho\gamma\beta_\lambda=e^{2\rho(c_1-c_2)}\left(\tau_\theta+\tau_L+\beta_\lambda^2\tau_z-2\rho\gamma\beta_\lambda\right), \tag{6}$$

$$\Leftrightarrow \tau_z\beta_\lambda^2-2\rho\gamma\beta_\lambda-\widehat{C}=0, \tag{7}$$

where

$$\widehat{C}:=\frac{\tau_\theta+\tau_H-e^{2\rho(c_1-c_2)}(\tau_\theta+\tau_L)}{e^{2\rho(c_1-c_2)}-1}.$$

Furthermore, we can express λ in terms of β_λ using the relations $\beta_\lambda=\frac{\lambda\tau_H+(1-\lambda)\tau_L}{\rho}$ and $\tau_H>\tau_L$. Thus, we have $\lambda=\frac{\rho\beta_\lambda-\tau_L}{\tau_H-\tau_L}$.

The quadratic Eq. (7) has real roots if and only if $\rho^2\gamma^2+\widehat{C}\tau_z\geq 0$. If this inequality holds, then there exist two real solutions for β_λ given by:

$$\frac{\rho\gamma+\sqrt{\rho^2\gamma^2+\widehat{C}\tau_z}}{\tau_z}=:\beta_\lambda^*,\quad \frac{\rho\gamma-\sqrt{\rho^2\gamma^2+\widehat{C}\tau_z}}{\tau_z}=:\beta_\lambda^{**}.$$

We say an endogenous equilibrium is *stable* if small deviations in the equilibrium outcome will not change investors' trading strategy. The following result shows that β_λ^* can exclusively lead to a stable interior equilibrium.

Proposition 3. *Suppose information is endogenous, $\rho^2\gamma^2+\widehat{C}\tau_z>0$ and $\beta_\lambda^*\in\left(\frac{\tau_L}{\rho},\frac{\tau_H}{\rho}\right)$. Then the fraction of high-precision investors, given by*

$$\lambda=\frac{\rho\beta_\lambda^*-\tau_L}{\tau_H-\tau_L}=\frac{\rho\left(\frac{\rho\gamma+\sqrt{\rho^2\gamma^2+\widehat{C}\tau_z}}{\tau_z}\right)-\tau_L}{\tau_H-\tau_L},$$

represents the unique stable interior equilibrium.

Proposition 3 reveals that as the degree of RWC increases, the fraction of high-precision investors strictly rises. This finding is intuitive, as it suggests that when investors attach greater importance to others' wealth, more individuals within the investor population are motivated to seek high-precision signals. This enables them to stay competitive and strategically position themselves advantageously within the market.

[1] Some simple calculations show that $\frac{\partial^2(1-b)}{\partial\gamma^2}=\frac{2\tau_\theta\rho^2\beta_\lambda^2}{(\tau_\theta+\beta_\lambda\rho(1-\gamma)+\beta_\lambda^2\tau_z)^3}>0$ and $\frac{\partial^2 s}{\partial\gamma^2}=-\frac{2\tau_\theta\rho^2\beta_\lambda}{(\tau_\theta+\beta_\lambda\rho(1-\gamma)+\beta_\lambda^2\tau_z)^3}<0$.

[2] This effect is referred to as the "*return effect*" by He et al. (2021).

Proposition 4. *Suppose information is endogenous and the conditions in Proposition 3 hold. Then, the average wealth gap between L and H and each investor's welfare are both strictly decreasing in the coefficient of RWC γ.*

Recall that the average wealth gap is proportional to the return volatility $\mathrm{Var}(\theta - p)$. From Eqs. (7), (3) and (4), we have

$$1 - b = \frac{\tau_\theta}{\tau_\theta + \rho\beta_\lambda(1+\gamma) + \hat{C}} \quad \text{and} \quad s = \frac{\rho(1-\gamma) + \beta_\lambda \tau_z}{\tau_\theta + \rho\beta_\lambda(1+\gamma) + \hat{C}}.$$

The analysis reveals that $(1 - b)$ is strictly decreasing in γ, in contrast to its increasing trend with γ under exogenous information. Furthermore, the variable s may either strictly increase or strictly decrease in γ, depending on the model parameters (we omit the details here). However, regardless of whether γ increases from zero or one, the decrease in $(1 - b)$ outweighs the increase in s (if it increases), resulting in a strictly decreasing return volatility with respect to γ. Intuitively, when the degree of RWC rises, more information is produced and incorporated into asset prices, bringing the equilibrium price closer to the fundamental value.

Furthermore, the welfare of investor i is given by

$$-\left(\frac{\Psi_i}{\Sigma_i}\mathrm{Var}(\theta - p)\right)^{-\frac{1}{2}},$$

where $\Sigma_i = \mathrm{Var}[\theta|y_i, p] = 1/(\tau_\theta + \tau_i + \beta_\lambda^2\tau_z)$ and $\Psi_i = 1 - 2\rho\gamma\beta_\lambda\Sigma_i$. In contrast to the exogenous information case, β_λ increases with γ under endogenous information. As a result, the conditional variance Σ_i decreases with γ, leading to improved price informativeness measured by Σ_i^{-1}. Intuitively, the market price becomes more precise in predicting the fundamental value, resulting in reduced uncertainty about final payoffs. However, similar to the exogenous information case, the welfare loss $2\rho\gamma\beta_\lambda\Sigma_i$ due to RWC increases with γ, causing Ψ_i to decrease in γ. These two effects perfectly offset each other, resulting in a constant ratio Ψ_i/Σ_i. Consequently, the welfare of investors exhibits the same monotonicity as the average wealth gap.

Propositions 2 and 4 show that for the three scenarios: (i) endogenous information, (ii) exogenous information with informational inefficiency (i.e., $\frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda} \leq 1$), and (iii) exogenous information with informational efficiency (i.e., $\frac{\tau_\theta+\tau_L+\beta_\lambda^2\tau_z}{2\rho\beta_\lambda} > 1$) but limited γ values within the interval $[0, 1]$, it is impossible to simultaneously reduce the wealth gap and improve investor welfare by adjusting the degree of RWC in these scenarios. When certain coefficients of social comparison lead to a low wealth gap, they also result in low welfare. Therefore, from the perspective of social planners and policymakers, achieving an optimal balance between the wealth gap and welfare would require appropriate adjustments in the extent of social comparison within the population and improved transparency regarding wealth distribution within society.

5. Concluding remarks

Wealth gaps and investor welfare are vital indicators for measuring economic inequality and allocative efficiency, respectively. Achieving a low wealth gap and high welfare is a desirable goal in economies. In this paper, we employ a rational expectations equilibrium model to investigate the impact of RWC on the wealth gap and welfare.

Our analysis demonstrates that the effects of RWC on the wealth gap and welfare depend on whether information production is considered. When information is endogenous, a higher degree of RWC consistently leads to a decrease in both the wealth gap and investor welfare. However, in the case of exogenous information, the relationship between the degree of RWC and investor welfare can exhibit different patterns. It can either be monotonically decreasing or initially decreasing and then increasing, contingent upon the specific model parameters.

Our findings emphasize that in certain scenarios, it is impossible for a social planner to simultaneously reduce the wealth gap and increase investor welfare by simply manipulating the degree of RWC. This highlights the complexity of achieving desired outcomes and underscores the need for a comprehensive approach that goes beyond adjusting RWC alone.

Data availability

No data was used for the research described in the article

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2023.111376.

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