Following the actions of others: The simple average of strategies in a rational expectations economy

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Abstract

We study a rational expectations equilibrium economy where agents can learn from the actions of others by adopting the simple average of ex ante optimal strategies of their social network. When information is exogenous, large social networks benefit all agents if and only if agents are relatively homogeneous in terms of information precision. In contrast, a setting where both information acquisition and network formation are endogenous leads to small social networks of just two or three agents in equilibrium. However, each agent would benefit if larger networks were imposed on the entire economy by a central agent.

Keywords: Simple Average of Strategies, Rational Expectations Equilibrium, Welfare, Information Acquisition, Social Networks

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1 Introduction

Price theory prescribes that prices are determined in equilibrium of supply and demand. In classical models of financial markets, supply (noise trade) is exogenous and it is thus demand that determines prices. Demand is influenced by information about fundamentals, and how costly information is acquired and reflected in asset prices has been an important topic in economics going back to Grossman and Stiglitz (1980) and Hellwig (1980). A more recent literature integrates findings on information propagation through social networks into models with information acquisition and asset pricing, see Colla and Antonio (2010), Ozsoylev and Walden (2011), Han and Yang (2013), or Walden (2019). These studies are based on the assumption that agents voluntarily share information with other members of their social network.

In this paper, we study an alternative, simpler protocol of social learning which can intrinsically improve the allocative efficiency. Agents only observe the ex ante optimal strategies of other members in their social network and follow a simple heuristic to incorporate information contained in these strategies: They replicate the average of the ex ante optimal strategies of all other agents in their social network. Instead of learning about the signals received by others, agents in our model directly follow the actions of others. In many situations, agents might not be willing to disclose their own information, for example for privacy concerns. Actions are also typically more salient than signals, and learning from them is thus plausible from a positive perspective. Our aim is to introduce the idea of the *simple average of ex ante optimal strategies* (SAEAOS) into models of rational expectations economies, and to analyze its implications on the welfare of agents, as well as on information acquisition and other market quality measures.

In the literature on information aggregation, the heuristic of taking the simple average is widely regarded as an efficient approach to reduce noise in individuals' decisions and forecasts, and to form *wisdom of crowds* (DeMarzo et al. 2003; Golub and Jackson 2010; Jadbabie et al. 2012; Kahneman et al. 2021; Surowiecki 2004). For example, Kahneman et al. (2021, p. 261) state that "*The easiest way to aggregate several forecasts is to average them. Averaging is mathematically guaranteed to reduce noise: specifically, it divides it by the square root of the number of judgments averaged.*" As a reasonable first approach, we herein extrapolate this heuristic for an agent that only observes the ex ante actions of others, and aggregates the contained information by following their average. While averaging is natural and, as we will show, potentially improves the efficiency of equilibrium allocations, we do not claim that it is optimal. Building a theory of optimally aggregating the information contained in the actions of others goes beyond the scope of this paper.

Our model is based on the classical noisy rational expectations equilibrium economy of Hellwig (1980) populated by a large number of agents with constant absolute risk aversion. The market consists of a risk-free asset in perfect elastic supply and a risky asset in random, normal supply. The fundamentals of the risky asset and signals received by the agents are also normally distributed. We then extend this basic model by dividing the large economy into a large number of small, disjoint social groups. The key feature of our model is that, in each group, agents can form networks that jointly decide on a protocol of adopting the SAEAOS.

We start with the case where information is exogenous and characterize under what conditions agents are better off when adopting the SAEAOS of a given network. We find that the decision to join a network and adopt the SAEAOS thereof only depends on the structure of exogenous signal precisions and not on other model parameters. When agents' information precision is relatively homogenous, then larger social networks adopting the SAEAOS lead to better welfare for all agents involved. In contrast, when there is large variety in information precision across agents, those agents with large precision are not interested in joining networks with smaller average precision.

We then consider the case of endogenous information acquisition, first with exogenously imposed network structure and then in a setting where both information acquisition and network formation are endogenous. When the network structure is imposed exogenously and social groups are homogenous in terms of risk aversion, then endogenous information acquisition leads to homogenous information precision across each social network.

One of our main findings is that settings where agents can decide both on information acquisition and network formation lead to small social networks of just two or three agents. On the one hand, agents benefit from averaging their ex ante strategy with that of one or two other agents due to a reduction in noise, and small networks are thus superior to solitary action. On the other hand, and perhaps more surprisingly, larger network are not stable because each individual network has an incentive to disintegrate into smaller ones. Large networks generally lead to a reduction in information acquisition as agents attempt to free-ride on the information contained in the ex ante strategies of others and their own signal translates to a smaller weight in the SAEAOS that is eventually implemented. In such a situation, members of a large network are incentivized to disintegrate into smaller networks where they would increase information acquisition.

While large social networks are not stable when network formation is endogenous as just discussed, we find that everyone would benefit if the entire economy could agree to adopting the SAEAOS across all social groups. That is, if a social planner or central authority could impose a network structure ex ante, then the resulting expected welfare of each agent would be larger than in the setting where networks form endogenously. Furthermore, agents would benefit more from larger networks of adopting the SAEAOS. As a policy implication, our results indicate that, in some settings, economies can benefit from *greater transparency about actions*.

The underlying reason for this result is that a commitment to adopting the SAEAOS in all social groups of the economy leads to a reduction in information acquisition. Lower information acquisition corresponds to asset prices that are riskier. But lower information acquisition also increases the expected return of the risky asset, a consequence coined *return effect* in He et al. (2021). Furthermore, lower information acquisition also directly reduces the costs spent on acquiring information. Together, these effects outweigh the loss in welfare due to riskier assets.

In closing, we investigate how imposing the SAEAOS on the economy affects important market quality measures. Conforming with the intuition provided above, we find that information acquisition is indeed reduced. We further find that imposing the SAEAOS on all social groups in the economy: reduces market efficiency (the degree with which market prices reflect information on fundamentals); reduces market liquidity (i.e., increases the sensitivity of prices to variation in supply); reduces trading volume; and increases return volatility. These results conform with the findings of Han and Yang (2013), who study an economy where informed agents voluntarily share a noisy version of their private signal with other members of their social network.

The remainder of this paper is organized as follows. We review the related literature in Section 2. In Section 3, we introduce the model for the economy and the concept of adopting the SAEAOS within a social network. Our main results are in Sections 4 and 5, which treat the cases of exogenous and endogenous information acquisition respectively. We conclude the paper in Section 6. All proofs are delegated to the Appendix.

2 Related Literature

Our paper is related to the following three lines of research. First, our study directly contributes to the vast literature on the welfare analysis of rational expectation economies and Bayesian market games with incomplete information. Vives (1988) considers large Cournot markets where firms receive noisy private signals about random demands. He finds that incomplete information generally leads to a reduction in welfare and that the loss in welfare is increasing in the cost of information acquisition when information is endogenous. Morris and Shin (2002), Angeletos and Pavan (2004) and Angeletos and Pavan (2007) investigate quadratic Bayesian games with strategic complementarity or strategic substitutability where agents can receive noisy private information as well as exogenously given public information. They find that whether higher precision of public information improves welfare is ambiguous in general and specific to the strategic and external effects defined in the utility and social welfare functions. Different from the above literature in which public information is exogenously given, in market games studied in Vives (1997), Burguet and Vives (2000), Amador and Weill (2012), Vives (2017), Bayona (2018) and Heumann (2021) the public information is the price which is endogenously determined.[1] These works typically analyze whether an equilibrium allocation is socially optimal from the viewpoint of a social planner. Another important question of research is commonly investigated is how equilibrium welfare can be further improved by taking decentralized strategies that are required to be measurable with respect to their own private information (as well as the public information, if any). When there is a decentralized welfare benchmark, the social planner is only able to control how an agent's actions depend on her own information, but cannot make an agent's actions depend on other agents' private signals. In this line of research, private information cannot be transferred from one agent to another.

[1] Burguet and Vives (2000) and Amador and Weill (2012) respectively consider a dynamic model with private information and public information which is a noisy statistic of agents' past actions (playing a similar role as prices). They find that improving the precision of public information allows agents to reduce acquisition on their private signals and then potentially reduces social welfare.

Due to the externality of both learning and information, the socially-optimal solution generally deviates from market equilibrium solutions. Different from these studies, we herein consider an environment of social networks where agents have the option to follow the SAEAOS. This leads to unmeasurable strategies with respect to the private information of a single agent since they indirectly depend on the information obtained by other agents.

Second, our paper contributes to the recent strand of theoretical (Colla and Antonio 2010; Han and Yang 2013; Lou and Yang 2022; Manela 2014; Ozsoylev and Walden 2011; Walden 2019)[2] and experimental research (Halim et al. 2019) on the implications of information sharing on market outcomes.[3] Ozsoylev and Walden (2011) analyze how the network connectedness of a large economy influences price volatility, trading volume, welfare, and other measures of interest. They find that the ex ante certainty equivalent of agents is either globally decreasing, or initially increasing and eventually decreasing in network connectedness. Manela (2014) analyzes the effect of the speed of information diffusion on the welfare of agents and shows that the value of information is hump-shaped in the diffusion speed. Walden (2019) considers a dynamic model for a rational expectations economy with decentralized information diffusion through a general network. He shows that more central agents make higher profits, and, consistent with the findings in Colla and Antonio (2010) and Ozsoylev and Walden (2011), that agents that are close to each other have more positively correlated trades. While these papers assume that the information is exogenously given, Han and Yang (2013) and Halim et al. (2019) investigate the effect of social communication on market outcomes when information acquisition

[2]The another relevant line is information percolation in populations and markets where agents meet each other over time and exchange information to each other, and the information gathered is further shared at subsequent pairwise meetings (Duffie et al. 2010, 2009; Duffie and Manso 2007). Duffie et al. (2009) find that improving public signals for agents reduces their endogenous efforts in searching for other agents from whom they can share information, and then maybe reduce social welfare. While in most of the above literature, it is assumed that agents have no incentive to not share information with their neighbors since agents know that they and their neighbors have no price impact. Differently Indjejikian et al. (2014) and Goldstein et al. (2021) consider strategic settings where some agents have endogenous incentive to voluntarily leak their information to other agents to increase their welfare by impacting prices.

[3]There also has been some empirical work on the effects of social communication on trading behavior of investors, see for instance, Hong et al. (2004), Hong et al. (2005), Heimer (2016), Pool et al. (2015), Ozsoylev et al. (2014), etc.

is endogenous. Han and Yang (2013) show that social communication reduces the endogenous fraction of informed agents and thereby harms market efficiency, reduces trading volume, and improves welfare. Halim et al. (2019) show that social communication provides an incentive for agents to free ride on other agents' information and consequently reduces the overall information in the market. In contrast to these studies, agents in our model cannot directly observe their neighbors' signals. Instead, they can observe the actions of other agents in their network and adopt the SAEAOS if this is to their benefit. The welfare implications of this mechanism are quite different from that in Ozsoylev and Walden (2011) and Lou and Yang (2022). While the learning mechanism of direct information sharing in Ozsoylev and Walden (2011) and Lou and Yang (2022) cannot always lead to a Pareto improvement[4] depending on model parameters, our results show that for a relatively homogenous distribution of risk-aversion coefficients and signal precisions, the SAEAOS can always improve all agents' ex-ante welfare for both cases of exogenous information and endogenous information (with a mild condition).

Third, our work is also related to the literature on non-Bayesian learning over networks, for example, DeMarzo et al. (2003), Golub and Jackson (2010), Jadbabie et al. (2012) and Molavi et al. (2018). In this literature, the mechanism of averaging often plays a powerful role in reducing noises in agents' decisions (Kahneman et al. (2021)). Borrowing the averaging approach proposed by Degroot (1974), DeMarzo et al. (2003) study a learning model where agents receive independent noisy signals about the true value of nature. They communicate with their neighbors in a social network, and update beliefs by repeatedly taking the weighted average of the neighbors' opinions. They show that the believes in the network converge to a consensus belief, and that this consensus belief is correct if the network structure is balanced. Following DeMarzo et al. (2003), Golub and Jackson (2010) further show that agents' beliefs can be asymptotically accurate as the network becomes large even if it may not be optimal in finite societies. Different from these studies, we herein do not consider the accuracy of agents' opinions on the unknown state of nature, but instead consider an exchange economy and focus on the analysis of the effects of following others through their actions by adopting the SAEAOS.

[4]It is the case even if the coefficients of risk aversion and signal precision are the same across all agents in the economy.

3 The Economy

Our model builds on the finite-agent noisy rational expectations equilibrium economy of Hellwig (1980). There are n agents who are trading a risk-free and a risky asset. The risky asset has fundamental value $\theta \sim N(0, 1/\tau_\theta)$, $\tau_\theta > 0$. Let $W_{i,0}$ denote the initial wealth of agent i and x_i the holdings of the risky asset of agent i, $i = 1, \ldots, n$. The wealth of agent i at the end of the investment period is then given by $W_i(x_i) = w_{i,0} + x_i(\theta - p)$, where p is the price of the risky asset. This price is publicly observable by all agents. The preferences of the agents in our model are represented by CARA utility functions with risk aversion coefficient ρ_i, i.e., the utility agent i derives from wealth w is given by $U_i(w) = -\exp(-\rho_i w)$. We can therefore assume without further loss of generality that $w_{i,0} = 0$ for all $i = 1, \ldots, n$. The (stochastic) terminal utility agent i obtains from adopting strategy x_i is given by

$$U_i\left(W_i(x_i)\right) = -\exp\left(-\rho_i x_i(\theta - p)\right).$$

Each agent i observes a private signal $y_i = \theta + \epsilon_i$ about the fundamental θ, where the noise $\epsilon_i \sim N(0, 1/\tau_i)$, $\tau_i > 0$ denoting the precision of the information of agent i. The strategy of agent i is allowed to depend on both the private signal and the public price of the risky asset, i.e., $x_i = x_i(y_i, p)$. Noises in signals are assumed to be independent across agents. We will subsequently consider economies with *exogenous information* in Section 4, where τ_i is exogenously given for each agent i, and economies with *endogenous information* in Section 5, where the agent faces a cost $c(\tau)$ for obtaining a signal with precision τ and optimally chooses a precision that balances informativeness of the signal and resulting cost. To prevent prices from fully revealing, there is per-capita random supply u of noise traders in the market. The random per-capita supply satisfies $u \sim N(0, 1/\tau_u)$, $\tau_u > 0$, and is independent of other random variables θ and ϵ_i, $i = 1, ..., n$.[5]

Agents in our model are price takers. To justify this assumption, we adopt the island-

[5] A criticism from taking the approach of noise trading to prevent prices from being fully revealing is that one cannot immediately examine the welfare analysis for all agents. We still adopt such an approach for the sake of simplicity. Our main results on welfare implications do not depend on the amount of noise trading and can be consequently extended to an otherwise identical economy with random endowment (Diamond and Verrecchia (1981)), at least when the random endowment is small. Although the method of random endowment can also prevent prices from being fully revealing and at the same time, allows ones to examine the welfare analysis for

connection network setting as in Jackson (2008) or Han and Yang (2013) leading to a large economy.[6] We assume that the large economy consists of $w \in \mathbb{N}$ unconnected social groups denoted as $\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_w$. Each social groups is formed by n agents. Accordingly, the total number of agents in the large economy is given by wn. The following assumption imposes a degree of homogeneity across agents belonging to a given social group.

Assumption 1. *Agents within the same group have identical coefficients of risk aversion. When information is exogenous, we further assume that there is a finite number of profiles of coefficients of risk aversion and signal precisions in the economy, i.e.,* $(\rho_i, \tau_i) \in \{(\rho_1^\diamond, \tau_1^\diamond), ..., (\rho_m^\diamond, \tau_m^\diamond)\}$ *for all* i*, where* $m \in \mathbb{N}$ *and* $\{(\rho_1^\diamond, \tau_1^\diamond), ..., (\rho_m^\diamond, \tau_m^\diamond)\} \in \mathbb{R}_{>0}^{2m}$ *are given. Moreover, as* $w \to \infty$*, the fraction of agents with* $(\rho_k^\diamond, \tau_k^\diamond)$ *in all* w *groups converges to some* λ_k*, where* $0 < \lambda_k < 1$ *and* $\sum_{k=1}^m \lambda_k = 1$*. When information is endogenous, we only assume that there is a finite number of coefficients of risk aversion in the economy, i.e.,* $\rho_i \in \{\rho_1^\diamond, ..., \rho_m^\diamond\}$ *for all* i*, where* $m \in \mathbb{N}$ *and* $\{\rho_1^\diamond, ..., \rho_m^\diamond\} \in \mathbb{R}_{>0}^m$ *are given. Moreover, as* $w \to \infty$*, the fraction of agents with* $\rho_k^\diamond$ *in all* w *groups converges to some* λ_k*, where* $0 < \lambda_k < 1$ *and* $\sum_{k=1}^m \lambda_k = 1$*.*

Note that homogeneity of social groups solely refers to risk-aversion, and that differences in signal precision are possible within our model. Explicitly requiring that the number of profiles of risk aversion and signal precisions remains finite is only necessary when information acquisition is exogenous. In the case of endogenous information acquisition, it is sufficient to assume homogeneity of risk aversion across groups and a finite number of coefficients of risk aversion across the economy. We will later observe that this leads to social groups that are heterogenous in terms of both risk aversion and information precision when information is endogenous, cf. Section 5.

The key feature of our model is that agents can adopt the simple average of ex ante optimal strategies (SAEAOS) of their social network. Formally, for a group $\mathcal{S}_g$ and a subset $\mathcal{N} \subseteq \mathcal{S}_g$,

all agents, the analysis is more complicated (since there is an additional dimension of random endowment in agents' information sets and beliefs on the asset payoff).

[6]The replica network approach used in Walden (2019) for constructing large economies shares a similar spirit with the island-connection network approach.

we denote the SAEAOS of agents in the network $\mathcal{N}$ as

$$x^*_{\mathcal{N}} = \frac{1}{|\mathcal{N}|}\sum_{i\in\mathcal{N}} x^*_{i,g},$$

where $x^*_{i,g}$ denotes the optimal strategy for agent i in group g. Agents in a given group $\mathcal{S}_g$ can come together and decide to form a network $\mathcal{N} \subseteq \mathcal{S}_g$ whereafter each agent in $\mathcal{N}$ adopts $x^*_{\mathcal{N}}$, the SAEAOS of agents in $\mathcal{N}$. When $|\mathcal{N}| = 1$, each agent implements his or her own original strategy, while $|\mathcal{N}| = n$ means that all agents in the group join together and adopt the SAEAOS of the entire social group $\mathcal{S}_g$.[7] Throughout this paper, we will use the term *(social) group* to refer to the set of agents $\mathcal{S}_g$ that jointly form the large economy in the island-connection setting and to *(social) network* to refer to subsets $\mathcal{N} \subseteq \mathcal{S}_g$ of social groups in which agents adopt the SAEAOS.

Colla and Antonio (2010), Ozsoylev and Walden (2011), Han and Yang (2013) and Walden (2019) assume that signals are shared among agents over a social network. The learning mechanism through social interactions in our model and this literature is quite different: It is the action, not the signal, that is shared between agents, and agents learn from the actions of others by adopting the simple average of their strategies.[8] This is a case of bounded rationality:[9] Agents do not attempt to infer the signals of other agents in their network received from communicated ex ante strategies and they learn from others by taking the simple average of all strategies in the network instead of discriminating and weighting other agents ex ante strategies based on their attributes. In many situations, it is indeed difficult to infer the signals of others from their actions if one does not have precise knowledge about others' signal precision and risk aversion.

Furthermore, there are two complementary forces that can favor both the sharing of actions instead of signals and learning from actions instead of signals. First, agents often have concerns for privacy about information and a desire to protect the source of information. They thus

[7] In Appendix A, we will introduce a privacy preserving algorithm to implement the SAEOS where all agents can obtain the simple average of their demand strategies, while no agent knows the strategies of other agents.

[8] While the new learning mechanism differs from the direct information sharing in the literature, it generates different welfare implications as shown in the next two sections.

[9] A similar assumption of bounded rationality where agents are not able to extract information from prices was for example made in Mondria et al. (2022).

attempt to hide private signals in many scenarios. In contrast, it is typically difficult to hide actions as these are often public by nature. The second, and arguably more important, force is behavioral. Actions are more salient than signals, and sharing actions intuitively more natural. Consider for example a group of friends discussing their recent investment decisions. It seems more common that a person directly states that he/she recently purchased Stock A instead of telling about a signal the person received about the future price of Stock A that has a certain precision.

4 Exogenous Information

In this section, we assume that the signal precision of agents are exogenously given. We first introduce the notion of an equilibrium and review existing results on optimal strategies and equilibrium prices when each agent acts individually.

Definition 1. *An equilibrium with exogenous information is a tuple* $\left((x^*_{i,g})_{i=1,\dots,n;g=1,\dots,\infty}, p\right)$ *such that*

(i) for each $i = 1, \dots, n$ *and* $g = 1, \dots, \infty$*,* $x^*_{i,g}$ *maximizes conditional expected utility for agent* i *in group* g*, i.e.,*

$$x^*_{i,g}(y_{i,g}, p) \in \arg\max_x \mathbb{E}[U_{i,g}\left(W_{i,g}(x)\right)|y_{i,g}, p].$$

(ii) the market clears, i.e.,

$$\lim_{w\to\infty} \frac{1}{w}\sum_{g=1}^{w}\left(\frac{1}{n}\sum_{i=1}^{n} x^*_{i,g}(y_{i,g}, p)\right) = u.$$

Following the majority of the literature, we herein focus on *linear* equilibria, i.e., equilibria where strategies are linear functions of the signal and price and prices are linear in the signals and per-capital supply.[10] To simplify notation, we will drop the subscript g in the coefficient of risk aversion $\rho_{i,g}$, signal precision $\tau_{i,g}$, and other variables whenever we do not specify which group agent i belongs to or the meaning is clear from the context. Following the analysis in

[10] In this paper, we assume that all random variables have mean zero for notational convenience. Hence, there is no intercept in price function p.

Hellwig (1980), Ozsoylev and Walden (2011), and Han and Yang (2013), we see that, as w increases to infinity, the sequence of equilibrium prices of finite-agent economies converges in probability to

$$p = \frac{1}{\Delta + \frac{\tau_\theta}{\Delta\tau_u + \rho}}(\Delta\theta - u), \tag{1}$$

where

$$\rho = \left(\sum_{k=1}^{m} \frac{\lambda_k}{\rho_k^\diamond}\right)^{-1} \tag{2}$$

and

$$\Delta = \sum_{k=1}^{m} \lambda_k \frac{\tau_k^\diamond}{\rho_k^\diamond} \tag{3}$$

are the average risk aversion and the risk adjusted average signal precision in the economy. Interestingly, agents' private signals enter prices in terms of averaged signal precision. All other things being equal, the larger the risk adjusted average signal precision, the greater is the weight of the fundamental in determining prices. We further find that the agent i's equilibrium strategy in the limit of a large economy equals to

$$x_i^*(y_i, p) = \frac{\mathbb{E}[\theta|y_i, p] - p}{\rho_i \operatorname{Var}[\theta|y_i, p]} = \rho_i^{-1}\tau_i y_i - \left(\rho_i^{-1}\tau_i + \frac{\tau_\theta}{\Delta\tau_u + \rho}\right)p. \tag{4}$$

The first equality is the standard mean-variance portfolio strategy in the CARA-normality setting (see, e.g., Equations (6) and (11) in Grossman (1976)), and the second one follows from (1) and the projection theorem for normal random variables.

Additional computations yield the ex-ante welfare, i.e., the ex ante expected utility, of agent i at his or her equilibrium strategy x_i^*:[11]

$$\mathbb{E}[U_i(W_i(x_i^*))] = \mathbb{E}[-\exp\left(-\rho_i x_i^*(y_i, p)(\theta - p)\right)] = -\left(\operatorname{Var}(\theta - p)(\tau_\theta + \Delta^2\tau_u + \tau_i)\right)^{-\frac{1}{2}}. \tag{5}$$

We next present an important result on the ex ante expected utility that can be achieved when agents follow the SAEAOS. Note that, when agents within a subgroup adopt this protocol and information acquisition is exogenous, the market-clearing condition remains valid and

[11] See also the proof of Lemma 2 in Rahi and Zigrand (2018).

equilibrium prices are thus not affected.[12] Therefore, the expected utility under a regime where agents adopt the SAEAOS and the original equilibrium strategies are computed based on the same equilibrium price p given in (1).

PROPOSITION 1. *The (ex-ante) expected utility of the simple average $x^*_{\mathcal{N}}$ for agent $i \in \mathcal{N} \subset \mathcal{S}_g$ is given by* [13]

$$\mathbb{E}[U_i(W_i(x^*_{\mathcal{N}}))] = -\left(\mathrm{Var}(\theta - p)(\tau_\theta + \Delta^2\tau_u + \tau_{\mathcal{N}})\right)^{-\frac{1}{2}}, \tag{6}$$

where

$$\tau_{\mathcal{N}} = \left(2 - \frac{1}{|\mathcal{N}|}\right)\frac{\sum_{j\in\mathcal{N}}\tau_j}{|\mathcal{N}|}.$$

Furthermore, $\mathbb{E}[U_i(x^*_{\mathcal{N}})]$ is strictly increasing in $|\mathcal{N}|$ when agents in group $\mathcal{S}_g$ have the same signal precision.[14]

Comparing (6) in Proposition 1 and (5), we see that adopting the SAEAOS $x^*_{\mathcal{N}}$ in a subgroup $\mathcal{N} \subset \mathcal{S}_g$ dominates individual strategies $\{x^*_{i,g}, i \in \mathcal{N} \subset \mathcal{S}_g\}$ if and only if

$$\tau_{\mathcal{N}} > \tau_i,\ \ i \in \mathcal{N} \subset \mathcal{S}_g.$$

It is interesting to note that this condition only depends on the structure of signal precisions, but not on other model parameters.[15] We can interpret $\tau_{\mathcal{N}}$ as the threshold precision condition

[12] While the group size $|\mathcal{N}|$ has no effect on the equilibrium price in our economy, it has a large impact on the equilibrium price and consequently on the equilibrium statistics (for instance, price volatility $\sqrt{\mathrm{Var}(p)}$, market efficiency $1/\mathrm{Var}[\theta|p]$, return volatility $\sqrt{\mathrm{Var}(\theta - p)}$, etc.) in the economies with direct information sharing (Han and Yang 2013; Ozsoylev and Walden 2011).

[13] When agents in some group $\mathcal{S}_g$ have different risk aversion coefficients and signal precisions, the expected utility of the simple average $x^*_{\mathcal{N}}$ for agent $i \in \mathcal{N} \subset \mathcal{S}_g$ is given by the one in Proposition (1) by replacing $\tau_{\mathcal{N}}$ with

$$\tau^i_{\mathcal{N}} = \frac{2}{|\mathcal{N}|}\sum_{j\in\mathcal{N}}\frac{\rho_i}{\rho_j}\tau_j - \frac{1}{|\mathcal{N}|^2}\sum_{j\in\mathcal{N}}\frac{\rho_i^2}{\rho_j^2}\tau_j - \left(1 - \frac{1}{|\mathcal{N}|}\sum_{j\in\mathcal{N}}\frac{\rho_i}{\rho_j}\right)^2\frac{\rho^2}{\tau_u + \frac{(\Delta\tau_u+\rho)^2}{\tau_\theta}}.$$

[14] Proposition 1 also holds for other concave utility functions provided that the equilibrium exists, but with a potentially different threshold precision condition.

[15] As a byproduct, this property tells us that the welfare improvement result in Proposition 1 also holds for an otherwise identical economy with small random endowment (Diamond and Verrecchia (1981)), please also see Footnote 5 for more discussions.

for agent i to be indifferent between his or her original strategy $x^*_{i,g}$ and the simple average $x^*_{\mathcal{N}}$. Agent i has an incentive to follow the strategies of others when his or her own signal precision is below the threshold, and does not otherwise. Clearly, this threshold condition is satisfied when all agents have the same signal precision, and more generally when differences between agents' precision is small. Intuitively, when there are small differences between agents' precision, the simple average can efficiently reduce the noise and uncertainty for all agents making everyone better off. In contrast, if a given agent's precision is considerably larger than the average precision of a network, this agent does not have an incentive to join the network.

More specifically, for any given agent $i \in \mathcal{N}$, being part and adopting the SAEAOS of the subgroup $\mathcal{N}$ is worthwhile if and only if

$$\tau_i < \frac{2|\mathcal{N}| - 1}{|\mathcal{N}| - 1}\bar{\tau}_{\mathcal{N}\backslash\{i\}},$$

where $\bar{\tau}_{\mathcal{N}\backslash\{i\}} = \frac{1}{|\mathcal{N}|-1}\sum_{j\in\mathcal{N}\backslash\{i\}} \tau_j$ is the average precision that the subgroup would have without agent i. Note that $\frac{2|\mathcal{N}|-1}{|\mathcal{N}|-1}$ is decreasing in $|\mathcal{N}|$, takes the value 3 when $|\mathcal{N}| = 2$, and converges to 2 when $|\mathcal{N}|$ goes to infinity. Therefore, for any two agents it is worthwhile to form a network unless the signal precision of one agent is more than three times larger than that of the other. For a given agent to join and adopt the SAEAOS of an existing large network, his or her signal precision must not be larger than twice the average precision of that group.

The second part of Proposition 1 states that, when agents in a group are homogenous in terms of signal precision, then welfare is increasing in the size of social networks. In particular, when information is exogenous and both risk-aversion and information precision homogenous across social groups, everyone were to benefit when the entire social group would form a single network adopting the SAEAOS. It is worth to note that this welfare improvement result in our setting of adopting the SAEAOS does not depend on other model parameters (a relatively homogenous distribution of risk-aversion coefficients and signal precisions suffices) and is quite different from the welfare implication results in the setting of direct information sharing. For example, Proposition 11 (c) in Ozsoylev and Walden (2011) shows that the monotonicity of agents' welfare over network connectedness (which plays a similar role as the group size in the paper) highly depends on the model parameters, and generally is initially increasing and eventually decreasing even if the coefficients of risk-aversion and signal precision are the same across all agents.

5 Endogenous Information

In this section, we consider the case where signal precisions are determined endogenously. Agents face a cost $c(\tau)$ for acquiring information with precision τ and aim to optimally balance informativeness of the signal and resulting cost. We will assume that the cost function $c : [0, \infty) \to [0, \infty)$ is strictly convex, strictly increasing, continuously differentiable, and satisfies the conditions $\lim_{\tau\to 0} c'(\tau) = 0$ and $\lim_{\tau\to\infty} c'(\tau) = \infty$. Our goal is to explore whether agents will adopt the SAEAOS when they jointly decide on information acquisition and on whether to follow the actions of others.

Recall that from Assumption 1 agents of a given group are homogenous in terms of risk aversion and that there is a finite number of coefficients of risk aversion in the economy, i.e., $\rho_i \in \{\rho_1^\diamond, ..., \rho_m^\diamond\}$ for all i, for given $m \in \mathbb{N}$ and $\{\rho_1^\diamond, ..., \rho_m^\diamond\} \in \mathbb{R}^m_{>0}$. For each group $\mathcal{S}_g$ with coefficient of risk aversion $\rho_k^\diamond$ for some $k \in \{1, \dots, m\}$ we consider possible partitions of the group in networks of size r_k, where $r_k \in \mathbb{N}$ is a divisor of the group size n.

We study endogenization of both information and formation of networks that adopt the SAEAOS in two steps: First, we consider the case where only information acquisition is endogenous and the network structure is exogenously given. This is the mirror situation of the analysis in Section 4, where information acquisition was exogenous and we studied under the formation of networks. In the second step, we study the case where both information acquisition and formation of networks is endogenous.

Let $div(n) \subset \mathbb{N}$ denote the set of all divisors of n. An exogenously imposed network structure can be described by a vector $\mathbf{r} = (r_1, \dots, r_m) \in div(n)^m$ stipulating that groups with risk aversion coefficient $\rho_k^\diamond$ fragment into n/r_k networks containing r_k agents each.[16] For a given exogenous network structure $\mathbf{r} = (r_1, \dots, r_m) \in div(n)^m$, we denote by $\mathcal{N}(i, g)$ the network of

[16]We herein do not allow social groups to fragment into networks of different sizes. For example, a social group of size 6 can fragment into six networks consisting of a single agent, three networks of size 2, two networks of size 3, or a single network containing all agents of the group. But we do not allow the group to fragment into, say, two networks of size 4 and 2. This assumption is primarily made to simplify notation. But it also avoids conceptual definitions when introducing the notion of an equilibrium: What if, for example, in a network consisting of three agents, any two of them would be better off when excluding the third, but the agent that would eventually be excluded would prefer to remain the network consisting of all three agents.

agent $i \in \{1, \dots, n\}$ in group g.

Definition 2. *An equilibrium with endogenous information but exogenously imposed network structure* $\mathbf{r} \in div(n)^m$ *is a tuple* $\left(\left(\tau^*_{i,g}\right)_{i=1,\dots,n; g=1,\dots,\infty}, p \right)$ *such that*

(i) for each $i = 1, \dots, n$ *and* $g = 1, \dots, \infty$, $\tau^*_{i,g}$ *is the optimal precision given the precision of other agents and resulting optimal ex ante strategies, i.e.,*

$$\tau^*_{i,g} \in \arg\max_{\tau_{i,g}>0} \mathbb{E}\left[U_i\left(W_{i,g}(x^*_{\mathcal{N}(i,g)}(\tau_{i,g})) - c(\tau_{i,g}) \right) \right],$$

where

$$x^*_{\mathcal{N}(i,g)}(\tau_{i,g}) = \frac{1}{|\mathcal{N}(i,g)|} \left(\sum_{j \in \mathcal{N}(i,g) \setminus \{i\}} x^*_{j,g}(\tau^*_{j,g}) + x^*_{i,g}(\tau_{i,g}) \right),$$

is the SAEAOS,

$$x^*_{t,g}(\tau) = \frac{\mathbb{E}[\theta | y_{t,g}(\tau), p] - p}{\rho^\diamond_k \operatorname{Var}[\theta | y_{t,g}(\tau), p]}$$

is the optimal strategy of agent (t, g) *with signal* $y_{t,g}(\tau)$, $\rho^\diamond_k$ *is the risk aversion coefficient of agents in group* $\mathcal{S}_g$, *and* p *is the endogenous equilibrium price defined by (1) when* Δ *is replaced with* $\sum_{k=1}^m \lambda_k \frac{\tau^*_{i,k}}{\rho^\diamond_k}$, *here* $\tau^*_{i,k}$ *is the optimal signal precision of agents in groups with risk aversion coefficient* $\rho^\diamond_k$. *The notation* $y_{t,g}(\tau)$ *highlights that the precision of signal* $y_{t,g}$ *is* τ.

(ii) the market clears, i.e.,

$$\lim_{w \to \infty} \frac{1}{w} \sum_{g=1}^{w} \left(\frac{1}{n} \sum_{i=1}^{n} x^*_{i,g}(y_{i,g}, p) \right) = u.$$

Condition (i) of Definition 2 states that each agent's precision is optimal given the precision of other agents. An individual agent correctly anticipates that his/her chosen precision only affects his/her own ex ante strategy, but not the ex ante strategies of others. When networks are large, varying information precision therefore only has a small effect on the eventually implemented strategy, which is the SAEAOS of all agents in the network. Condition (ii) of Definition 2 is the usual market clearing conditions stating that, in equilibrium, supply must equal demand. Note that this condition is again not affected by adopting the SAEAOS.

The following proposition shows that an equilibrium with endogenous information but exogenous network structure exists, is unique, and leads to social groups that are homogenous not only in terms of risk aversion but also information precision.

PROPOSITION **2.** *For any exogenously given* $\mathbf{r} \in div(n)^m$*, there exists an equilibrium with endogenous information but exogenously imposed network structure* $\mathbf{r}$*,* $\left(\left(\tau^*_{i,g}\right)_{i=1,...,n;g=1,...,\infty}, p\right)$*. Moreover, the equilibrium is unique and satisfies* $\tau^*_{i,g} = \tau^*_{j,g} =: \tau^*_k(\mathbf{r})$ *for any* i, j *when the coefficient of risk aversion in group* $\mathcal{S}_g$ *is* $\rho^\diamond_k$*.*

As an immediate corollary to Proposition 2, we obtain that the assumption of a finite number of profiles of coefficients of risk aversion and signal precisions throughout the economy we imposed in Assumption 1 when information is exogenous automatically holds also for the endogenous case as long as the number of coefficients of risk aversion is finite.

The following proposition characterizes the unique equilibrium with endogenous information but exogenously imposed network structure. We also show the resulting welfare of each agent.

PROPOSITION **3.** *Let* $\mathbf{r} \in div(n)^m$*. The unique equilibrium with endogenous information but exogenously imposed network structure* $\mathbf{r}$ *is characterized by* $(\tau^*_k(\mathbf{r}))_{k=1,...,m}$ *which are jointly determined by the system of equations:*

$$2\rho^\diamond_k c'(\tau^*_k(\mathbf{r}))\left(\frac{r_k}{2-\frac{1}{r_k}}(\tau_\theta + \Delta^2_{\mathbf{r}}\tau_u) + r_k\tau^*_k(\mathbf{r})\right) = 1, \qquad k = 1, ..., m, \tag{7}$$

where $\Delta_{\mathbf{r}} = \sum_{k=1}^m \lambda_k \tau^*_k(\mathbf{r})/\rho^\diamond_k$ *and* $p_{\mathbf{r}} = \frac{1}{\Delta_{\mathbf{r}} + \frac{\tau_\theta}{\Delta_{\mathbf{r}}\tau_u + \rho}}(\Delta_{\mathbf{r}}\theta - u)$*. Moreover, the welfare (taking into account the cost of information acquisition) obtained in a group with risk aversion* $\rho^\diamond_k$ *and being split into networks composed of* r_k *agents is given by*

$$V_k(\mathbf{r}) = -\left(\exp\left(-2\rho^\diamond_k c(\tau^*_k(\mathbf{r}))\right)\mathrm{Var}(\theta - p_{\mathbf{r}})(\tau_\theta + \Delta^2_{\mathbf{r}}\tau_u + (2 - 1/r_k)\tau^*_k(\mathbf{r}))\right)^{-\frac{1}{2}}.$$

Next, we introduce the following notion of a local equilibrium. These will play a role in the definition of a fully endogenous equilibrium later on.

Definition 3. *Let* $g \in \mathbb{N}$ *and* $\mathcal{N} \subset \mathcal{S}_g$ *be a network. A local equilibrium of* $\mathcal{N}$ *given a network and signal structure* $\left(\mathbf{r}, (\tau_k(\mathbf{r}))_{k=1,...,m}\right)$ *and price* p *is a family* $(\hat{\tau}_{i,g})_{i\in\mathcal{N}}$ *such that for any* $i \in \mathcal{N}$*,*

$\hat{\tau}_{i,g}$ is the optimal precision given the precision of other agents and resulting optimal ex ante strategies, i.e.,

$$\hat{\tau}_{i,g} \in \arg\max_{\tau_{i,g}>0} \mathbb{E}\left[U_i\left(W_{i,g}(x^*_{\mathcal{N}}(\tau_{i,g})) - c(\tau_{i,g})\right)\right],$$

where

$$x^*_{\mathcal{N}}(\tau_{i,g}) = \frac{1}{|\mathcal{N}|}\left(\sum_{j\in\mathcal{N}\setminus\{i\}} x^*_{j,g}(\hat{\tau}_{j,g}) + x^*_{i,g}(\tau_{i,g})\right)$$

is the SAEAOS with

$$x^*_{t,g}(\hat{\tau}) = \frac{\mathbb{E}[\theta|y_{t,g}(\hat{\tau}),p] - p}{\rho^\diamond_k \operatorname{Var}[\theta|y_{t,g}(\hat{\tau}),p]}$$

being the optimal strategy of agent t,g with signal $y_{t,g}(\tau)$ and $\rho^\diamond_k$ the risk aversion coefficient of agents in group $\mathcal{S}_g$.

A slight modification of Proposition 2 shows that local equilibria exist, are unique, and lead to homogeneous information precision across a network.

PROPOSITION 4. *Let $\left(\mathbf{r}, (\tau_k(\mathbf{r}))_{k=1,\ldots,m}\right)$ be a given network and signal structure, $p\in\mathbb{R}$, $g\in\mathbb{N}$, and $\mathcal{N}\subset\mathcal{S}_g$. Then there exists a local equilibrium of $\mathcal{N}$ given the network and signal structure $\left(\mathbf{r}, (\tau_k(\mathbf{r}))_{k=1,\ldots,m}\right)$ and price p, say $(\hat{\tau}_{j,g})_{j\in\mathcal{N}}$. Moreover, this local equilibrium is unique and satisfies $\hat{\tau}_{i,g} = \hat{\tau}_{j,g}$ for any $i,j\in\mathcal{N}$.*

We will denote the unique and uniform local equilibrium of a network $\mathcal{N}\subset\mathcal{S}_g$ for a given network and signal structure $\left(\mathbf{r}, (\tau_k(\mathbf{r}))_{k=1,\ldots,m}\right)$ and price p by $\hat{\tau}_{\mathcal{N}}$. A fully endogenous equilibrium, where both information acquisition and network formation are determined endogenously, is defined as follows.

Definition 4. *A fully endogenous equilibrium is a tuple $\left(\mathbf{r}, (\tau^*_k(\mathbf{r}))_{k=1,\ldots,m}, p\right)$ such that*

(i) setting $\tau^*_{i,g} = \tau^*_k(\mathbf{r})$ for any i,g, where k is such that the coefficient of risk aversion coefficient in $\mathcal{S}_g$ is $\rho^\diamond_k$, $\left(\left(\tau^*_{i,g}\right)_{i=1,\ldots,n;g=1,\ldots,\infty}, p\right)$ is an equilibrium with endogenous information but exogenous network structure $\mathbf{r}$, and

(ii) for any group $\mathcal{S}_g$, any possible network $\mathcal{N} \subset \mathcal{S}_g$ and agent $i \in \mathcal{N}$,

$$\mathbb{E}\left[U_{i,g}\left(W_{i,g}\left(x^*_{\mathcal{N}}(\hat{\tau}_{\mathcal{N}})\right) - c\left(\hat{\tau}_{\mathcal{N}}\right)\right)\right] \leq V_k(\mathbf{r}),$$

where $\hat{\tau}_{\mathcal{N}}$ is the local equilibrium of $\mathcal{N}$ given $\left(\mathbf{r}, (\tau^*_k(\mathbf{r}))_{k=1,\dots,m}, p\right)$, and $x^*_{\mathcal{N}}(\hat{\tau}_{\mathcal{N}}) = \sum_{i\in\mathcal{N}} x^*_{i,g}(\hat{\tau}_{\mathcal{N}})/|\mathcal{N}|$.

Definition 4 makes two requirements on fully endogenous equilibria. First, given the network structure, each agent correctly anticipates the adaptation of the SAEAOS in his or her network and optimally acquires information given this anticipation. Optimal information acquisition will translate to corresponding demands and determine prices. Second, there is no potential network that has an incentive to deviate from the network structure.

The following proposition identifies the network size of fully endogenous equilibria under the assumption of a quadratic cost function. Together with Proposition 3, this will give a fully characterization of equilibria with endogenous information acquisition and formation of social networks. Let $div(n) = \{d_1, \dots, d_{|div(n)|}\}$ denote the set of all divisors of n as before with $d_1 = 1$, $d_i < d_{i+1}$, and $d_{|div(n)|} = n$.

Proposition 5. *Suppose the cost function of information acquisition is of the form $c(\tau) = \alpha\tau^2$, where $\alpha > 0$.*[17] *Then there exists a fully endogenous equilibrium with network structure $\mathbf{r} = (r_1, \dots, r_m)$ given according to the following case distinction:*
(i) if $d_2 \in \{2, 3\}$, i.e., group sizes are even or odd but divisible by three, then $r_k = d_2$ for $k = 1, \dots, m$;
(ii) if $d_2 \notin \{2, 3\}$, i.e., group sizes are neither divisible by two nor three, then $r_k \in \{d_1, d_2\}$ and $r_k = r_j$ for $k, j = 1, \dots, m$.

Interestingly, the optimal network size emerging in a fully endogenous equilibrium is uniform across social groups, $r_k = r_\ell$ for all $k, \ell = 1, \dots, m$. In particular, network size in a fully endogenous equilibrium does not depend on the risk-aversion prevalent in a given social group. To abbreviate notation, we will denote the optimal information acquisition of an agent with

[17] The assumption of a quadratic cost function is common in the literature, see for example, Gao and Liang (2013), He et al. (2021) and Goldstein and Yang (2017). In our case, when the cost function takes a more general form of $c(\tau) = \alpha\tau^\ell$, $\ell \geq 2$, we can similarly show that $r_k \leq (\ell + 3)/2$, i.e., both endogenous information acquisition and network formation will lead to small social networks with size not greater than $(\ell + 3)/2$.

risk aversion $\rho_k^\diamond$ in an economy with uniform network size d_i by $\tau_k^*(d_i)$, i.e., $\tau_k^*(d_i) = \tau_k^*(\mathbf{r})$ with $\mathbf{r} = (d_i, \ldots, d_i)$ for $i = 1, \ldots, |div(n)|$ and $k = 1, \ldots, m$. We further denote the risk adjusted average signal precision in such an economy by $\Delta_{d_i} = \sum_{k=1}^m \lambda_k \tau_k^*(d_i)/\rho_k^\diamond$ and denote the welfare $V_k((d_i, \ldots, d_i))$ by $V_k(d_i)$.

Our analysis shows that both cases can occur in case (ii) of Proposition 5 when the group size is neither even nor divisible by three. When the factor α in the cost function $c(\tau) = \alpha\tau^2$ is either small or large, we can show that solitary action is optimal, i.e., $r_k = d_1 = 1$, if and only if $d_2 \notin \{2, 3, 5\}$. Otherwise, the smallest possible network is optimal, i.e., $r_k = d_2$, $k = 1, \ldots, m$. For example, if the group size is $n = 5 \times 7 = 35$, then the network size in a fully endogenous equilibrium is $r_k = d_2 = 5$. If the group size is $n = 11 \times 13 = 143$, then the network size in a fully endogenous equilibrium is $r_k = d_1 = 1$.[18]

Proposition 5 shows that fully endogenous equilibria typically lead to small social networks. Forming a network with one or two other agents is superior to solitary action. Perhaps more surprisingly, larger networks are not stable as agents have an incentive to disintegrate into smaller ones. The intuition behind this is that agents generally reduce information acquisition when joining larger networks as, one the one hand, they can free-ride on the information captured in the ex ante strategies of other agents in the network, and, on the other hand, their own ex ante optimal strategy receives a relatively small weight in the SAEAOS of a large network. Although reducing information acquisition leads to a reduction in the cost required to acquire information, when an increasing number of agents simultaneously attempts to free-ride on other agents' information, the benefit from cost saving is off-set by the loss in information available across the network. Anticipating this, a large network thus has an incentive to disintegrate into smaller networks where agents would then increase information acquisition.

To summarize, large social networks are not stable when networks are formed endogenously because each individual network has an incentive to disintegrate. We will next show that, when the *entire economy* simultaneously could agree to adopting the SAEAOS within each social group, then all agents in all networks were to benefit. Furthermore, information acquisition in

[18]If one were to allow social groups to fragment into networks of unequal as discussed in Footnote 16, then networks would be of the size 2 with one agent remaining solitary if the group size is odd.

such an economy would be reduced.

Proposition **6.** *(i) Imposing the SAEAOS for larger social networks across the entire economy reduces information acquisition, i.e.,* $\Delta_{d_{i+1}} < \Delta_{d_i}$ *for* $i = 1, \ldots, |div(n)| - 1$*;*

(ii) Suppose all agents in the economy have the same coefficient of risk aversion ρ *and that* $(\tau^*(n)/\rho)^2 \tau_u \geq \tau_\theta$*.*[19] *Then imposing the SAEAOS for larger social networks across the entire economy increases welfare, i.e.,* $V(d_i) < V(d_{i+1})$ *for* $i = 1, \ldots, |div(n)| - 1$*.*

Part (i) in Proposition 6 shows that adopting the SAEAOS will reduce agents' incentive to acquire information. This results from two separate mechanisms: Agents are tempted to free ride on the strategies of other agents in their group and their own information translates to a smaller effect in the SAEAOS than it would in their own, individual strategy. The first mechanism conforms with the finding that information sharing crowds out information production (Halim et al. 2019; Han and Yang 2013) while the second is new to the best of our knowledge. Part (ii) in Proposition 6 shows that all agents where to benefit when a protocol of adopting the SAEAOS for larger social networks across the entire economy were imposed. However, as noted above, each individual network would have an incentive to deviate from this protocol and instead form smaller social networks. Therefore, the economy where all agents adopt the SAEAOS is not stable.

The intuition behind this surprising result is as follows. In an economy where larger networks commit to the protocol of adopting the simple average agents reduce their information acquisition. On the one hand, high level of information in the economy implies that prices are precise in predicting the fundamental, and uncertainty about the final payoff is thus low. This results in risk averse agents facing smaller trading risks and thus experiencing higher expected utilities. But on the other hand, reduction in risk will also lead to lower expected returns (Kurlat and Veldkamp (2015)), called *return effect*, a term coined by He et al. (2021). In sum-

[19]When all traders in the large economy are homogeneous in their risk aversion, we omit the subscript k from all the notations. Note that here we can show by contradiction from (7) that $\tau^*(n) \to 0$ as $\tau_u \to \infty$ and as a consequence, from (7) again we have $(\tau^*(n)/\rho)^2 \tau_u \to \infty$ as $\tau_u \to \infty$. Similar to the case of exogenous information, this result tells us that the welfare improvement result in Part (ii) of Proposition 6 for the case of endogenous information also holds for an otherwise identical economy with small random endowment (Diamond and Verrecchia (1981)), please also see Footnote 5 for more discussions.

mary, the risky asset in an economy with high information acquisition contains less risk, but also offers a lower expected return. But the second effect dominates the first one, and hence all agents would benefit when larger networks commit to the protocol of adopting the simple average. When agents reduce information acquisition when adopting the SAEAOS, they thus directly benefit due to a reduction in costs and indirectly due to a more favorable risk-return profile of the risky asset. It is worth noting that Proposition 3 in Lou and Yang (2022) provides a precise albeit complicated necessary and sufficient condition on the monotonicity of agents' welfare over network connectedness in the setting of information sharing. In contrast to Lou and Yang (2022), Part (ii) in Proposition 6 provides a condition ensuring welfare improvement that is easily verified.

As discussed, large social networks have an incentive to disintegrate when social networks are formed endogenously. Large social networks adopting the SAEAOS would thus have to be enforced by a social planner or central authority. In the reminder of this section, we investigate the impact of imposing the SAEAOS for larger networks across the entire economy on important market quality measures: *Market efficiency* is measured by $1/\operatorname{Var}[\theta|p]$ (Han and Yang 2013; Ozsoylev and Walden 2011) and refers to the degree with which market prices reflect information on fundamentals. *Market liquidity* is measured by $\frac{1}{\partial p/\partial u}$. High market liquidity implies that a shock in supply or noise trading is absorbed without moving the price much (Han and Yang (2013)). *Average trading volume* is measured by $\lim_{w\to\infty}\frac{1}{nw}\sum_{g=1}^{w}\sum_{i=1}^{n}\mathbb{E}|x_{i,g}^{*}|$, and *return volatility* by $\sqrt{\operatorname{Var}(\theta-p)}$. We have the following result.

PROPOSITION **7.** *Imposing the protocol of adopting the SAEAOS for larger networks within each social group on the entire economy will*

(i) reduce market efficiency;

(ii) increase return volatility;

(iii) reduce market liquidity if $\Delta_n > (\sqrt{\tau_\theta\tau_u}-\rho)/\tau_u$*, and increase market liquidity if* $\Delta_1 < (\sqrt{\tau_\theta\tau_u}-\rho)/\tau_u$*;*

(iv) reduce average trading volume when all agents in the economy have the same risk aversion coefficient.

We next compare these results with the findings of Han and Yang (2013), who study a rational expectations equilibrium model where agents can share their signals with others in an exogenous social network. Han and Yang (2013) show that, when the information structure is endogenous, increasing network connectedness incentivizes agents to reduce information acquisition, harms market efficiency, reduces liquidity if the market is sufficiently informationally efficient, decreases trading volume and increases agents' welfare. The results in Propositions 6 and 7 are consistent with the findings in Han and Yang (2013). However, they are generated under a different mechanism of social interaction. In Han and Yang (2013), social communication refers to the case where informed agents voluntarily share a noisy version of their private signals to all other agents within the same network, and network connectedness is defined as the size of these networks. In our model, social interaction results in agents following the actions of others by adopting the SAEAOS. That is, it is the action, not signal, that is shared between agents in our model. Furthermore, while the results in Han and Yang (2013) and our paper for the case of endogenous information are consistent, they differ for the case of exogenous information. For example, Han and Yang (2013) numerically show that in the exogenous case increasing network connectedness will reduce agents' welfare. We make the opposite observation in our model as adopting the SAEAOS leads to improvements in welfare and larger improvements when more agents adopt SAEAOS when information is exogenous (Proposition 1). In this case, adopting the SAEAOS keeps overall information and equilibrium prices unchanged, and benefit agents by reducing the noise present in individuals' demands.

6 Conclusions

We contribute to the emerging literature studying the implications of social communication and learning for market outcomes in a rational expectations equilibrium economy. Different from the popular approach of direct information sharing where agents may be worse off, we herein consider the case where agents only observe the actions of the members of their social network and learn from these through the simple, heuristic protocol of adopting the SAEAOS of their social network.

Whether adopting the SAEAOS in a large social network benefits all members thereof crucially depends on the homogeneity of information precision across the network when information acquisition is exogenously fixed. If information precision is relatively homogenous across a social network, then all agents benefit from adopting the SAEAOS. In contrast, if there is considerable heterogeneity between agents' information precision, then those agents with higher precision were better off when forming a network excluding those with low information precision or even just following their own ex ante optimal strategies and ignoring the actions of others.

We find that social networks are always homogenous in terms of information precision when information acquisition is endogenous under the assumption that social groups are homogenous in terms of risk-aversion. More surprisingly, a setting where both information acquisition and network formation is endogenous leads to small social networks in equilibrium. Both solitary action and large social networks are not stable as agents have an incentive to form, respectively disintegrate into, small social networks of two or three agents. Despite this, each agent would benefit if larger networks were imposed on the entire economy by a central agent or social planner. Our results indicate that the proposed protocol of SAEAOS can intrinsically improve agents' welfare. Imposing large networks on the networks would reduce information acquisition, market efficiency, market liquidity, and trading volume, while increasing return volatility. These affects on market quality measures coincide with the findings of Han and Yang (2013), where agents share signals instead of actions.

There are several interesting directions for future research. In the model of this paper, we assume that agents are homogenous in terms of risk-aversion within each social group. It would be more realistic if we were to allow for some heterogeneity in risk-aversion within social groups. Many of our results depend on this assumption of homogeneous risk aversion, and an interesting open question is thus to study whether qualitatively similar results can be obtained when relaxing this assumption.

Agents in our model learn from the actions of others in their social networks through adaptation of the SAEAOS. While this is an intuitive, heuristic approach, it would be interesting to explore other forms of learning from the actions of others. One could for example discriminate between the actions of leaders and followers in social networks.

In our model, strategies of agents correspond to the amount invested in a single risky asset. It would be interesting to consider an economy containing multiple risky assets and allow for a more general interpretation of strategies as investment portfolios. When information acquisition is costly, a setup with multiple risky assets typically leads to under-diversification in the optimal strategy of a single agent (van Nieuwerburgh and Veldkamp, 2010). Adopting the SAEAOS could thus lead to additional benefits in terms of diversification, and it would be interesting to explore the implications of this protocol on information acquisition, asset prices, and welfare.

A recent literature studies investment in social networks where agents' preferences depend on the outcomes of others (Genicot and Ray, 2017; Lou et al., 2021). In the model of this paper, agents learn from members of their social networks by adapting the SAEAOS, but are otherwise not influenced by the strategies or outcomes of others. It would be interesting to study a model that combines the two features of social interaction: On the one hand, agents can learn from the strategies of other members in their network; on the other hand, their preferences are interested by the outcomes of others.

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Appendix A: The Implementation of SAEAOS

In this appendix, we provide a privacy preserving algorithm for agents $\mathcal{N} = \{1, 2, ..., n\}$ to compute their SAEAOS.[20] Suppose that agents' linear strategies are given by $x_i^*(y_i, p) = a_i y_i -$

[20] The idea of the designed algorithm has been used in the well-known algorithm on *calculating the average salary without disclosing the individual salary*, for example, see http://findnerd.com/list/view/How-to-know-the-average-without-disclosing-the-salaries/15268/ or https://www.geeksforgeeks.org/puzzle-26-know-average-salary-without-disclosing-individual-salaries/ This method shares the same idea with secure computations of Yao (1982).

$b_i p$, where a_i, b_i are constants, $i = 1, ..., n$. Then the SAEAOS is accordingly given by

$$\frac{1}{n}\sum_{i=1}^{n} x_i^*(y_i, p) = \frac{1}{n}\sum_{i=1}^{n} a_i y_i - \frac{1}{n}\sum_{i=1}^{n} b_i p,$$

which also specifies a demand for every price p.

Algorithm 1 Privacy Preserving Simple Averaging Algorithm

Initialization: the intercepts $(a_1 y_1, ..., a_n y_n)$, and the coefficients $(b_1, ..., b_n)$ of the linear simply averaged strategy $\sum_i x_i^*(y_i, \cdot)/n$, $n \geq 3$.

Algorithm:

Step 1: For $i = 1, ..., n$, agent i first adds a random noise δ_i to $a_i y_i$ and further adds the noisy sum $a_i y_i + \delta_i$ into the sum told by her predecessor. Agent i then tells the sum to agent $i + 1$ (here $n + 1 \pmod{n}$=1); finally agent 1 receives a noisy sum:

$$\sum_{i=1}^{n}(a_i y_i + \delta_i);$$

Step 2: For $i = 1, ..., n - 1$, agent i first subtracts his own random noise δ_i from the sum told by her predecessor agent, then tells the sum to agent $i + 1$ (here $n + 1 \pmod{n}$=1); agent n subtracts his own random noise δ_n from the sum told by agent $n - 1$ and gets an exact sum:

$$\sum_{i=1}^{n} a_i y_i.$$

Finally, agent n tells the simple average $(\sum_{i=1}^{n} a_i y_i)/n$ to all the other agents in the group.

Step 3: Repeat the above two steps for $(b_1, ..., b_n)$, agents get the simple average $\sum_{i=1}^{n} b_i/n$.

Output: the simply averaged strategy

$$\frac{1}{n}\sum_{i=1}^{n} x_i^*(y_i, \cdot)$$

of the original demand strategies $(x_1^*(y_1, \cdot), ..., x_n^*(y_n, \cdot))$

By construction, the *Privacy Preserving Simple Averaging Algorithm* 1 outputs the simple average of agents' strategies, while no agent knows the equilibrium strategies of other agents. This protects the private information of each individual agent in the network.

Note that in Algorithm 1, all agents truthfully report their intercept and linearity coefficient disturbed by a random number which will be subtracted later so that the actual ones computed in the final sum are exact to her successor agent. That is, agents in the algorithm are not strategic. We next consider a strategic setting where agents tell a noisy version of their truthful intercepts and linearity coefficients when computing the simple average in Algorithm 1. Instead of truthful subtractions in *Step 2* of Algorithm 1, we now suppose that agents are allowed to be strategic in the sense that they do not subtract the added noise in *Step 1*, resulting in the simple average being a noisy version of the true one. We use $\tilde{x}_i^*$ to denote the strategy contributed by agent i to the final simple average (i.e., the simple average is $\sum_{i=1}^n \tilde{x}_i^*/n$), where $\tilde{x}_i^* = x_i^*$ if agent i is telling the truth, and $x_i^* + \delta_i$ if not (we still use the notation δ_i to denote the noise for simplicity; δ_i denotes either the random noise used when computing the simple average of intercepts or the random noise used when computing the simple average of linearity coefficients). Note that when agent i is not telling the truth, although all the other agents do not know that the strategy x_i^* appearing in the final simple average is biased while agent i clearly knows this and then has an incentive to subtract the noise added by herself when computing the simple average. Since agents within the same group have the same risk aversion coefficient (Assumption 1), we ignore the subscript and write U_i as U for simplicity.

Proposition **8.** *The followings hold:*

(i) For any i, *and any given* $\tilde{x}_j^*, j \neq i$, *it holds that*

$$\mathbb{E}\Big[U\Big(W_j\Big(\Big(\sum_{j\neq i} \tilde{x}_j^* + x_i^* + \delta_i\Big)/n\Big)\Big)\Big] < \mathbb{E}\Big[U\Big(W_j\Big(\Big(\sum_{j\neq i} \tilde{x}_j^* + x_i^*\Big)/n\Big)\Big)\Big];$$

(ii) For any i, r, *and any given* $\tilde{x}_j^*, j \in \mathcal{N} \setminus \{i, r\}$, *it holds that*

$$\mathbb{E}\Big[U\Big(W_i\Big(\Big(\sum_{j\in\mathcal{N}\setminus\{i,r\}} \tilde{x}_j^* + x_r^* + \delta_r + x_i^*\Big)/n\Big)\Big)\Big] < \mathbb{E}\Big[U\Big(W_i\Big(\Big(\sum_{j\in\mathcal{N}\setminus\{i,r\}} \tilde{x}_j^* + x_r^* + x_i^*\Big)/n\Big)\Big)\Big];$$

(iii) $\mathbb{E}[U(W((\sum_{i=1}^n \tilde{x}_i^*)/n))] \leq \mathbb{E}[U(W((\sum_{i=1}^n x_i^*)/n))]$, *where the inequality is strict if* $\tilde{x}_i^* \neq x_i^*$ *for some* i.

Part (i) of Proposition 8 shows that for any given strategies of the other agents, compared with the benchmark setting where each agent reports the truth, the non-truthful telling of one agent will always make all the other agents' welfare worse off. At the same time, the welfare of the agent not reporting truthfully cannot improve (note that the welfare of agent i is still given by $\mathbb{E}[U(W_i((\sum_{j\neq i}\tilde{x}_j^* + x_i^*)/n))]$ even if she lies because agent i will subtract the noise added by herself when computing the simple average). Part (ii) reveals that when one agent lies, say agent i, and the other agent, say agent r, knows that agent i lies, then agent r can punish agent i and make him worse off. Indeed, note again that when agent i is lying, her welfare is given by $\mathbb{E}\big[U\big(W_i\big(\big(\sum_{j\in\mathcal{N}\setminus\{i,r\}}\tilde{x}_j^* + x_r^* + \delta_r + x_i^*\big)/n\big)\big)\big]$ if agent r also lies, and by $\mathbb{E}\big[U\big(W_i\big(\big(\sum_{j\in\mathcal{N}\setminus\{i,r\}}\tilde{x}_j^* + x_r^* + x_i^*\big)/n\big)\big)\big]$ if not because agent i will subtract the noise added by herself when computing the simple average no matter whether agent r lies. Part (iii) tells us a fact that all agents clearly know that when there are two agents who lie, all agents will be worse off compared with the benchmark where all agents are telling the truth. Hence, the three parts of Proposition 8 together indicate that it is incentive compatible in the sense that all agents are willing to tell the truth and no incentive to lie when computing the simple average of demand strategies.

Appendix B: Proofs of All Propositions

The following lemma is used to compute the expected utility of a quadratic function (see the result on page 382 in Vives (2008) or Lemma A.1 in the Appendix in Marín and Rahi (1999)).

LEMMA 1. *Suppose that z is an n-dimensional normal random vector with mean 0 and positive definite variance-covariance matrix Σ, B is a symmetric $n\times n$ matrix, b is a n-dimensional vector, and c is a constant. If the matrix $(\Sigma^{-1} - 2B)$ is positive definite, then*

$$\mathbb{E}[\exp(z'Bz + b'z + c)] = (\det(I_n - 2\Sigma B))^{-\frac{1}{2}} \exp\left(\frac{1}{2}b'(I_n - 2\Sigma B)^{-1}\Sigma b + c\right),$$

where I_n denotes the identity matrix in $\mathbb{R}^n$ and $\det(\cdot)$ is the determinant operator.

Proof of Proposition 1

Let ρ denote the common risk aversion coefficient of agents in $\mathcal{S}_g$ and consider agent $i \in \mathcal{N}$. The SAEAOS of all agents' equilibrium strategies can be expressed as

$$x^*_{\mathcal{N}} = \frac{1}{|\mathcal{N}|}\sum_{j\in\mathcal{N}} x^*_{j,g} = \rho^{-1}\left(\bar{\tau}_{\mathcal{N}}\theta + \xi_{\mathcal{N}} - \left(\bar{\tau}_{\mathcal{N}} + \frac{\tau_\theta}{1+\rho^{-1}\Delta\tau_u}\right)p\right),$$

where $\bar{\tau}_{\mathcal{N}} = \sum_{j\in\mathcal{N}} \tau_j/|\mathcal{N}|$, $\xi_{\mathcal{N}} = \sum_{j\in\mathcal{N}} \tau_j\epsilon_j/|\mathcal{N}|$.

We intend to apply Lemma 1 for $z = (\theta - p, \xi_{\mathcal{N}}, p)'$. The variance-covariance matrix Σ and B are given by

$$\Sigma = \begin{pmatrix} \phi & 0 & \varsigma \\ 0 & \mathrm{Var}(\xi_{\mathcal{N}}) & 0 \\ \varsigma & 0 & \frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2} \end{pmatrix}, \quad B = \begin{pmatrix} \bar{\tau}_{\mathcal{N}} & \frac{1}{2} & -\frac{\rho\beta}{2} \\ \frac{1}{2} & 0 & 0 \\ -\frac{\rho\beta}{2} & 0 & 0 \end{pmatrix},$$

where

$$\beta = \frac{\tau_\theta}{\Delta\tau_u + \rho}, \quad \phi = \frac{\beta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2}, \quad \varsigma = \frac{\Delta\beta/\tau_\theta - 1/\tau_u}{(\Delta+\beta)^2}.$$

In order to apply Lemma 1, we need that $\Sigma^{-1} + 2B$ is positive definite. We first show the following claim. Suppose $z'Bz = \hat{z}'\hat{B}\hat{z}$, for some symmetric matrix $\hat{B}$, where z, $\hat{z}$ are two normal random vectors. Let Γ be invertible such that $\hat{z} = \Gamma z$ holds and let Σ and $\hat{\Sigma}$ denote the respective positive definite variance-covariance matrices of z and $\hat{z}$, respectively. Clearly, we have $\hat{\Sigma} = \Gamma\Sigma\Gamma'$. We claim that $\Sigma^{-1} + 2B$ is positive definite if and only if $\hat{\Sigma}^{-1} + 2\hat{B}$ is positive definite. First, from $\hat{z}'\hat{B}\hat{z} = z'\Gamma'\hat{B}\Gamma z = z'Bz$, we have $B = \Gamma'\hat{B}\Gamma$. Then it follows that $\Sigma^{-1} + 2B = \Gamma'\hat{\Sigma}^{-1}\Gamma + 2\Gamma'\hat{B}\Gamma = \Gamma'(\hat{\Sigma}^{-1} + 2\hat{B})\Gamma$, which implies the claim.

Observe that we can alternatively write $\rho x^*_{\mathcal{S}_g}(\theta - p)$ as $\hat{z}'\hat{B}\hat{z}$ for some normal random vector $\hat{z}$ and symmetric matrix $\hat{B}$. In fact, from the expressions $p = (\Delta\theta - u)/(\Delta+\beta)$ (see Equation (1)) and $\theta - p = (\beta\theta + u)/(\Delta + \beta)$, we have

$$\begin{aligned}
\rho x^*_{\mathcal{S}_g}(\theta - p) &= \Big(\bar{\tau}_{\mathcal{N}}(\theta - p) + \xi_{\mathcal{N}} - \rho\beta p\Big)(\theta - p) \\
&= \frac{1}{\Delta+\beta}\Big(\bar{\tau}_{\mathcal{N}}\frac{\beta\theta+u}{\Delta+\beta} + \xi_{\mathcal{N}} - \rho\beta\frac{\Delta\theta - u}{\Delta+\beta}\Big)(\beta\theta+u) \\
&= \frac{1}{\Delta+\beta}\Big(\frac{(\bar{\tau}_{\mathcal{N}} - \rho\Delta)\beta}{\Delta+\beta}\theta + \frac{\bar{\tau}_{\mathcal{N}} + \rho\beta}{\Delta+\beta}u + \xi_{\mathcal{N}}\Big)(\beta\theta+u),
\end{aligned}$$

which can be written as $\hat{z}'\hat{B}\hat{z}$ with $\hat{z} = (\theta, u, \xi_{\mathcal{N}})$ and

$$\hat{B} = \frac{1}{\Delta+\beta}\begin{pmatrix} \frac{(\bar{\tau}_{\mathcal{N}}-\rho\Delta)\beta^2}{\Delta+\beta} & \frac{(\bar{\tau}_{\mathcal{N}}+\rho(\beta-\Delta)/2)\beta}{\Delta+\beta} & \frac{\beta}{2} \\ \frac{(\bar{\tau}_{\mathcal{N}}+\rho(\beta-\Delta)/2)\beta}{\Delta+\beta} & \frac{\bar{\tau}_{\mathcal{N}}+\rho\beta}{\Delta+\beta} & \frac{1}{2} \\ \frac{\beta}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$

Let $\hat{\Sigma}$ denote the variance-covariance matrix of random vector $\hat{z}$. Some simple calculations give

$$\hat{\Sigma}^{-1} + 2\hat{B} = \begin{pmatrix} \tau_\theta + \frac{2(\bar{\tau}_{\mathcal{N}}-\rho\Delta)\beta^2}{(\Delta+\beta)^2} & \frac{(2\bar{\tau}_{\mathcal{N}}+\rho(\beta-\Delta))\beta}{(\Delta+\beta)^2} & \frac{\beta}{\Delta+\beta} \\ \frac{(2\bar{\tau}_{\mathcal{N}}+\rho(\beta-\Delta))\beta}{(\Delta+\beta)^2} & \tau_u + \frac{2(\bar{\tau}_{\mathcal{N}}+\rho\beta)}{(\Delta+\beta)^2} & \frac{1}{\Delta+\beta} \\ \frac{\beta}{\Delta+\beta} & \frac{1}{\Delta+\beta} & \frac{1}{\bar{\tau}_{\mathcal{N}}} \end{pmatrix}.$$

By some simple but tedious derivations, we can show that $\hat{\Sigma}^{-1} + 2\hat{B}$ is positive definite. We omit the details here.

From the expressions $p = (\Delta\theta - u)/(\Delta+\beta)$ and $\theta - p = (\beta\theta + u)/(\Delta+\beta)$ again, we see that

$$(\theta - p, \xi_{\mathcal{N}}, p)' = \begin{pmatrix} \frac{\beta}{\Delta+\beta} & \frac{1}{\Delta+\beta} & 0 \\ 0 & 0 & 1 \\ \frac{\Delta}{\Delta+\beta} & -\frac{1}{\Delta+\beta} & 0 \end{pmatrix} (\theta, u, \xi_{\mathcal{N}})'$$

is an invertible transformation, by the above claim, we know that matrix $\Sigma^{-1} + 2B$ is positive definite.

Using Lemma 1 with $z = (\theta - p, \xi_{\mathcal{N}}, p)'$, the matrices Σ, B, and setting $b = 0$ and $c = 0$, we obtain

$$\mathbb{E}\left[-\exp(-\rho x^*_{\mathcal{N}}(\theta - p))\right] = -(\det(I_3 + 2\Sigma B))^{-\frac{1}{2}} = -(\det(I_3 + 2B\Sigma))^{-\frac{1}{2}},$$

where

$$I_3 + 2B\Sigma = \begin{pmatrix} 1+2\omega & \mathrm{Var}(\xi_{\mathcal{N}}) & 2\gamma \\ \phi & 1 & \varsigma \\ -\rho\beta\phi & 0 & 1-\rho\beta\varsigma \end{pmatrix}$$

with

$$\omega = \bar{\tau}_{\mathcal{N}}\phi - \frac{\rho\beta\varsigma}{2}, \quad \gamma = \bar{\tau}_{\mathcal{N}}\varsigma - \frac{\rho\beta}{2}\frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2}.$$

Expanding the determinant $\det(I_3 + 2B\Sigma)$ along the first row yields

$$\begin{aligned}\det(I_3 + 2B\Sigma) &= (1 + 2\bar{\tau}_{\mathcal{N}}\phi - \rho\beta\varsigma)(1 - \rho\beta\varsigma) - \mathrm{Var}(\xi_{\mathcal{N}})\phi + 2\gamma\rho\beta\phi \\ &= (1-\rho\beta\varsigma)^2 - (\rho\beta)^2\phi\frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2} + 2\bar{\tau}_{\mathcal{N}}\phi(1-\rho\beta\varsigma) - \mathrm{Var}(\xi_{\mathcal{N}})\phi + 2\bar{\tau}_{\mathcal{N}}\varsigma\rho\beta\phi \\ &= (1-\rho\beta\varsigma)^2 - (\rho\beta)^2\phi\frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2} + (2\bar{\tau}_{\mathcal{N}} - \mathrm{Var}(\xi_{\mathcal{N}}))\phi.\end{aligned}$$

Similarly, we can show that the expected utility at $x^*_{i,g}$ is given by

$$\mathbb{E}\big[-\exp\left(-\rho x^*_{i,g}(\theta - p)\right)\big] = (1-\rho\beta\varsigma)^2 - (\rho\beta)^2\phi\frac{\Delta^2/\tau_\theta + 1/\tau_u}{(\Delta+\beta)^2} + (2\tau_i - \mathrm{Var}(\tau_i\epsilon_i))\phi.$$

Therefore, each agent's welfare by adopting the simple average will be the same as that by taking the original equilibrium strategy if $2\tau_i - \mathrm{Var}(\tau_i\epsilon_i) = 2\bar{\tau}_{\mathcal{N}} - \mathrm{Var}(\xi_{\mathcal{N}})$, or equivalently, $\tau_i = 2\sum_{j\in\mathcal{N}}\tau_j/|\mathcal{N}| - \sum_{j\in\mathcal{N}}\tau_j/|\mathcal{N}|^2$. The first conclusion then follows from the alternative expression (5) of the expected utility at $x^*_{i,g}$.

When agents in group $\mathcal{S}_g$ have the same signal precision τ, $\tau_{\mathcal{N}} = (2 - 1/|\mathcal{N}|)\tau$ is strictly increasing in $|\mathcal{N}|$, and hence $\mathbb{E}[U_i(x^*_{\mathcal{N}})]$ is strictly increasing in $|\mathcal{N}|$. □

Proof of Proposition 2

It follows from (6) that

$$\begin{aligned}&\mathbb{E}\left[U_i\left(W_{i,g}(x^*_{\mathcal{N}(i,g)}(\tau_{i,g})) - c(\tau_{i,g})\right)\right] \\ &= -\exp\left(\rho^\diamond_k c(\tau_{i,g})\right)\left(\mathrm{Var}(\theta - p_{\mathbf{r}})\left(\tau_\theta + \Delta^2_{\mathbf{r}}\tau_u + \left(2 - \frac{1}{r_k}\right)\frac{\sum_{j\in\mathcal{N}(i,g)\setminus\{i\}}\tau^*_{j,g} + \tau_{i,g}}{r_k}\right)\right)^{-\frac{1}{2}} \\ &= -\left(\mathrm{Var}(\theta - p_{\mathbf{r}})\exp\left(-2\rho^\diamond_k c(\tau_{i,g})\right)\left(\tau_\theta + \Delta^2_{\mathbf{r}}\tau_u + \left(2 - \frac{1}{r_k}\right)\frac{\sum_{j\in\mathcal{N}(i,g)\setminus\{i\}}\tau^*_{j,g} + \tau_{i,g}}{r_k}\right)\right)^{-\frac{1}{2}}.\end{aligned} \tag{8}$$

By taking derivative with respect to $\tau^*_{i,g}$ for both sides of (8), we see that $\tau^*_{i,g}$ is determined by

$$c'(\tau^*_{i,g}) = \frac{\frac{2}{r_k} - \frac{1}{r_k^2}}{2\rho^\diamond_k\left(\tau_\theta + \Delta^2_{\mathbf{r}}\tau_u + (2 - 1/r_k)\frac{\sum_{j\in\mathcal{N}(i,g)\setminus\{i\}}\tau^*_{j,g} + \tau^*_{i,g}}{r_k}\right)}, \tag{9}$$

from which we conclude that $\tau_{i,g}^* = \tau_{j,g}^*$, denoted as $\tau_k^*(\mathbf{r})$, for any $i, j \in \mathcal{S}_g$. It then follows from (9) that $\{\tau_k^*(\mathbf{r})\}_{k=1}^m$ satisfies

$$c'(\tau_k^*(\mathbf{r})) = \frac{\frac{2}{r_k} - \frac{1}{r_k^2}}{2\rho_k^\diamond\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + (2 - 1/r_k)\tau_k^*(\mathbf{r})\right)} = \frac{1}{2\rho_k^\diamond\left(\frac{r_k}{2-\frac{1}{r_k}}(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u) + r_k\tau_k^*(\mathbf{r})\right)}, \qquad k = 1, ..., m. \tag{10}$$

We next show existence of an equilibrium by showing that the system of equations (10) has a solution. Recall that for each $k \in \{1, ..., m\}$, λ_k denotes the non-negative fraction of groups with risk aversion coefficient $\rho_k^\diamond$ in the limit economy. Observing (10), we define the mapping $\boldsymbol{f} = (f_1, f_2, ..., f_m)$, $f_k : (0, \infty)^m \to (0, \infty)$ as follows:

$$f_k(\tau) = (c')^{-1}\left(\frac{1}{2\rho_k^\diamond\left(\frac{r_k}{2-\frac{1}{r_k}}(\tau_\theta + (\Delta(\boldsymbol{\tau}))^2\tau_u) + r_k\tau_k\right)}\right),$$

where $\boldsymbol{\tau} = (\tau_1, ..., \tau_m)$, $\Delta(\boldsymbol{\tau}) = \sum_{k=1}^m \lambda_k\tau_k/\rho_k^\diamond$. Let $d_{max} = 1/\left(2\tau_\theta \min_{1\le k\le m} \frac{\rho_k^\diamond r_k}{2-\frac{1}{r_k}}\right)$ and

$$d_{min} = \frac{1}{2}\left(\max_{1\le k\le m}\left(\rho_k^\diamond\frac{r_k}{2-\frac{1}{r_k}}\right)\left(\tau_\theta + \left((c')^{-1}(d_{max})\sum_{k=1}^m \frac{\lambda_k}{\rho_k^\diamond}\right)^2\tau_u\right) + \max_{1\le k\le m}(\rho_k^\diamond r_k)(c')^{-1}(d_{max})\right)^{-1}.$$

We then can see that

$$(c')^{-1}(d_{min}) \le f_k(\tau) \le (c')^{-1}(d_{max})$$

for any $\tau \in (0, \infty)^m$ with $|\tau_k| \le (c')^{-1}(d_{max})$, $k = 1, ..., m$. Hence, the mapping $\boldsymbol{f}$ maps the convex, compact set $[(c')^{-1}(d_{min}), (c')^{-1}(d_{max})]^m$ into itself. Note that the two numbers of $(c')^{-1}(d_{min})$ and $(c')^{-1}(d_{max})$ are well-defined due to the conditions $\lim_{\tau\to 0} c'(\tau) = 0$ and $\lim_{\tau\to\infty} c'(\tau) = \infty$. In addition, the mapping $\boldsymbol{f}$ is also continuous over $(0, \infty)^m$. Hence applying Brouwer's Fixed Point Theorem to the mapping $\boldsymbol{f}(\cdot)$ will lead to a fixed point, or the solution to (10).

Finally, we show uniqueness. Suppose that both $\{\tau_k^*(\mathbf{r})\}_{k=1}^m$ and $\{\hat{\tau}_k^*(\mathbf{r})\}_{k=1}^m$ are solutions to (10). Then we first claim that $\hat{\Delta}_{\mathbf{r}} := \sum_{k=1}^m \lambda_k\hat{\tau}_k^*(\mathbf{r})/\rho_k^\diamond = \Delta_{\mathbf{r}}$. Otherwise, if $\hat{\Delta}_{\mathbf{r}} > \Delta_{\mathbf{r}}$, then $\hat{\tau}_k^*(\mathbf{r}) < \tau_k^*(\mathbf{r})$ for every $k = 1, ..., m$ from (10), and consequently, $\hat{\Delta}_{\mathbf{r}} < \Delta_{\mathbf{r}}$, a contradiction. A similar contradiction also arises if $\hat{\Delta}_{\mathbf{r}} < \Delta_{\mathbf{r}}$. Hence the claim $\hat{\Delta}_{\mathbf{r}} = \Delta_{\mathbf{r}}$ follows and then $\{\tau_k^*(\mathbf{r})\}_{k=1}^m$ is uniquely determined by (10). The proof is completed. □

Proof of Proposition 3

Follows directly from (8) and (10). □

Proof of Proposition 4

Follows directly from the arguments in the proof of Proposition 2. □

Proof of Proposition 5

Consider any possible network $\mathcal{N} \subset \mathcal{S}_g$ with risk aversion coefficient $\rho_k^{\diamond}$, and denote $K_s = \mathbb{E}\left[U_{i,g}\left(W_{i,g}\left(x^*_{\mathcal{N}}(\tau^*_{\mathcal{N}})\right) - c\left(\tau^*_{\mathcal{N}}\right)\right)\right]$, where $s = |\mathcal{N}|$. Due to the assumption of a quadratic cost function, we have similar to (8) and (9) that

$$K_s = -\left(\mathrm{Var}(\theta - p_{\mathbf{r}}) \exp\left(-2\rho_k^{\diamond} c(\tau^*_{\mathcal{N}}(s))\right)\left(\tau_\theta + \Delta_{\mathbf{r}}^2 \tau_u + \left(2 - \frac{1}{s}\right)\tau^*_{\mathcal{N}}(s)\right)\right)^{-\frac{1}{2}},$$

where $\tau^*_{\mathcal{N}}(s)$ satisfies

$$4\alpha\rho_k^{\diamond}\tau^*_{\mathcal{N}}(s)\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2 - \frac{1}{s}\right)\tau^*_{\mathcal{N}}(s)\right) = \frac{2}{s} - \frac{1}{s^2}. \tag{11}$$

To show the conclusion, it suffices to show that $K_{d_j} > K_{d_{j+1}}$ for $j = 2, \ldots, |div(n)| - 1$, that $\bar{K}_2 > \bar{K}_3 > \bar{K}_1$, and that the order of the K_j's does not depend on $\rho_k^{\diamond}$. For the first statement, by letting s be a fictitious, continuous variable taking values in $[1, n]$, it suffices to show that K_s, or equivalently,

$$\bar{K}_s = \exp\left(-2\rho_k^{\diamond} c(\tau^*_{\mathcal{N}}(s))\right)\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2 - \frac{1}{s}\right)\tau^*_{\mathcal{N}}(s)\right)$$

is strictly decreasing in $s \in [3, n]$.

We first show that $\bar{K}_s$ is strictly decreasing in $s \in [3, n]$. Taking derivative with respect to s on both sides of (11), we have

$$\begin{aligned} &4\alpha\rho_k^{\diamond}\left(\frac{\partial\tau^*_{\mathcal{N}}(s)}{\partial s}\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2 - \frac{1}{s}\right)\tau^*_{\mathcal{N}}(s)\right) + \tau^*_{\mathcal{N}}(s)\left(\frac{\tau^*_{\mathcal{N}}(s)}{s^2} + \left(2 - \frac{1}{s}\right)\frac{\partial\tau^*_{\mathcal{N}}(s)}{\partial s}\right)\right) \\ &= -\frac{2}{s^2}\left(1 - \frac{1}{s}\right). \end{aligned}$$

Consequently,

$$\frac{\partial \tau_{\mathcal{N}}^*(s)}{\partial s} = \frac{\frac{-\frac{2}{s^2}\left(1-\frac{1}{s}\right)}{4\alpha\rho_k^\diamond} - \frac{(\tau_{\mathcal{N}}^*(s))^2}{s^2}}{\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + 2\left(2-\frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)}. \tag{12}$$

Moreover, we have

$$\begin{aligned}
\frac{\partial \bar{K}_s}{\partial s} &\propto -4\alpha\rho_k^\diamond \tau_{\mathcal{N}}^*(s)\frac{\partial \tau_{\mathcal{N}}^*(s)}{\partial s}\left(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + \left(2-\frac{1}{s}\right)\tau_{\mathcal{N}}^*(s)\right) + \frac{\tau_{\mathcal{N}}^*(s)}{s^2} + \left(2-\frac{1}{s}\right)\frac{\partial \tau_{\mathcal{N}}^*(s)}{\partial s} \\
&= \frac{\tau_{\mathcal{N}}^*(s)}{s^2} + \left(2-\frac{1}{s}\right)\left(1-\frac{1}{s}\right)\frac{\partial \tau_{\mathcal{N}}^*(s)}{\partial s} \\
&= \frac{\tau_{\mathcal{N}}^*(s)}{s^2} + \left(2-\frac{1}{s}\right)\left(1-\frac{1}{s}\right)\frac{-\frac{2}{s^2}(1-\frac{1}{s}) - 4\alpha\rho_k^\diamond\frac{(\tau_{\mathcal{N}}^*(s))^2}{s^2}}{4\alpha\rho_k^\diamond(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + 2(2-1/s)\tau_{\mathcal{N}}^*(s))} \\
&\propto \frac{\frac{1}{s^2}(\frac{2}{s}-\frac{1}{s^2})}{\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + (2-1/s)\tau_{\mathcal{N}}^*(s)} + \left(2-\frac{1}{s}\right)\left(1-\frac{1}{s}\right)\frac{-\frac{2}{s^2}(1-\frac{1}{s}) - 4\alpha\rho_k^\diamond\frac{(\tau_{\mathcal{N}}^*(s))^2}{s^2}}{\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + 2(2-1/s)\tau_{\mathcal{N}}^*(s)} \\
&< \frac{\frac{1}{s^2}(\frac{2}{s}-\frac{1}{s^2})}{\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + (2-1/s)\tau_{\mathcal{N}}^*(s)} - \frac{(2-\frac{1}{s})(1-\frac{1}{s})\frac{2}{s^2}(1-\frac{1}{s})}{\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + 2(2-1/s)\tau_{\mathcal{N}}^*(s)} \\
&\propto (\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + 2(2-1/s)\tau_{\mathcal{N}}^*(s))/s - 2(1-1/s)^2(\tau_\theta + \Delta_{\mathbf{r}}^2\tau_u + (2-1/s)\tau_{\mathcal{N}}^*(s)),
\end{aligned} \tag{13}$$

where the first equality and the second $\propto$ follow from (11), and the second equality from (12). We see that the term in (13) is negative if $(1-1/s)^2 > 1/s$, which is true when $s \geq 3$. Hence $\bar{K}_s$ is strictly decreasing in s for $s \geq 3$.

We next show that $\bar{K}_2 > \bar{K}_3 > \bar{K}_1$. Let $\hat{K}_s = \log(\bar{K}_s)$, $s = 1, 2, 3$. We have

$$\hat{K}_s = \log\left(\bar{\tau}_\theta + \bar{\tau}_{\mathcal{N}}^*(s)\right) - 2\rho_k^\diamond\alpha(\tau_{\mathcal{N}}^*(s))^2,$$

where $\bar{\tau}_\theta = \tau_\theta + \Delta_{\mathbf{r}}^2\tau_u$, $\bar{\tau}_{\mathcal{N}}^*(s) = (2-1/s)\tau_{\mathcal{N}}^*(s)$. From (11), we have

$$4\rho_k^\diamond\alpha\bar{\tau}_{\mathcal{N}}^*(s)(\bar{\tau}_\theta + \bar{\tau}_{\mathcal{N}}^*(s)) = \frac{1}{s}\left(2-\frac{1}{s}\right)^2,$$

so that

$$\bar{\tau}_{\mathcal{N}}^*(s) = \frac{1}{2}\left(-\bar{\tau}_\theta + \sqrt{(\bar{\tau}_\theta)^2 + \frac{\frac{1}{s}(2-\frac{1}{s})^2}{\rho_k^\diamond\alpha}}\right).$$

Hence,

$$\begin{aligned}
\hat{K}_s &= \log\left(\bar{\tau}_\theta + \bar{\tau}_{\mathcal{N}}^*(s)\right) - 2\rho_k^\diamond\alpha(\tau_{\mathcal{N}}^*(s))^2 \\
&= \log\left(\bar{\tau}_\theta + \bar{\tau}_{\mathcal{N}}^*(s)\right) - \frac{2\rho_k^\diamond\alpha}{(2-1/s)^2}(\bar{\tau}_{\mathcal{N}}^*(s))^2
\end{aligned}$$

$$
\begin{aligned}
&= \log\left(\frac{\bar{\tau}_\theta + \sqrt{(\bar{\tau}_\theta)^2 + \frac{\frac{1}{s}\left(2-\frac{1}{s}\right)^2}{\rho_k^\diamond\alpha}}}{2}\right) - \frac{2\rho_k^\diamond\alpha}{(2-1/s)^2}\frac{2(\bar{\tau}_\theta)^2 + \frac{\frac{1}{s}\left(2-\frac{1}{s}\right)^2}{\rho_k^\diamond\alpha} - 2\bar{\tau}_\theta\sqrt{(\bar{\tau}_\theta)^2 + \frac{\frac{1}{s}\left(2-\frac{1}{s}\right)^2}{\rho_k^\diamond\alpha}}}{4} \\
&= \log\left(1 + \sqrt{1 + \frac{\frac{1}{s}\left(2-\frac{1}{s}\right)^2}{\rho_k^\diamond\alpha(\bar{\tau}_\theta)^2}}\right) - \log\left(2/\bar{\tau}_\theta\right) \\
&\quad - \frac{1}{2(2-1/s)^2}\left(2\rho_k^\diamond\alpha(\bar{\tau}_\theta)^2 + \frac{1}{s}\left(2-\frac{1}{s}\right)^2 - 2\sqrt{(\rho_k^\diamond\alpha(\bar{\tau}_\theta)^2)^2 + \frac{1}{s}\left(2-\frac{1}{s}\right)^2\rho_k^\diamond\alpha(\bar{\tau}_\theta)^2}\right) \\
&=: h(s,a) - \log(2/\bar{\tau}_\theta),
\end{aligned}
$$

where

$$
h(s,a) = \log\left(1 + \sqrt{1 + \frac{1}{s}\frac{(2-\frac{1}{s})^2}{a}}\right) - \frac{2 + \frac{1}{s}\frac{(2-\frac{1}{s})^2}{a} - 2\sqrt{1 + \frac{1}{s}\frac{(2-\frac{1}{s})^2}{a}}}{2\frac{(2-1/s)^2}{a}}
$$

with $a = \rho_k^\diamond\alpha(\bar{\tau}_\theta)^2$. As a result,

$$
h(2,a) - h(3,a) = \log\left(\frac{1 + \sqrt{1 + \frac{9}{8a}}}{1 + \sqrt{1 + \frac{25}{27a}}}\right) - \left(\frac{1}{12} + \left(\frac{4}{9} - \frac{9}{25}\right)a - \frac{4a}{9}\sqrt{1 + \frac{9}{8a}} + \frac{9a}{25}\sqrt{1 + \frac{25}{27a}}\right),
$$

and

$$
h(3,a) - h(1,a) = \log\left(\frac{1 + \sqrt{1 + \frac{25}{27a}}}{1 + \sqrt{1 + \frac{1}{a}}}\right) - \left(-\frac{1}{3} + \left(\frac{9}{25} - 1\right)a - \frac{9a}{25}\sqrt{1 + \frac{25}{27a}} + a\sqrt{1 + \frac{1}{a}}\right).
$$

We first show that $h(3,a) > h(1,a)$ for any $a > 0$ and thus $\bar{K}_3 > \bar{K}_1$. We have

$$
\begin{aligned}
&\frac{d(h(3,a) - h(1,a))}{da} \\
&= -\frac{\frac{1}{2}\frac{\frac{25}{27a^2}}{\sqrt{1+\frac{25}{27a}}}}{1 + \sqrt{1 + \frac{25}{27a}}} + \frac{\frac{1}{2}\frac{\frac{1}{a^2}}{\sqrt{1+\frac{1}{a}}}}{1 + \sqrt{1 + \frac{1}{a}}} + \frac{16}{25} + \frac{9}{25}\frac{1}{2}\frac{2a + \frac{25}{27}}{\sqrt{a^2 + \frac{25}{27}a}} - \frac{1}{2}\frac{2a+1}{\sqrt{a^2+a}} \\
&\propto \frac{9}{25}\frac{2a + \frac{25}{27}}{\sqrt{a^2 + \frac{25}{27}a}} - \frac{25}{27}\frac{1}{a^2 + \frac{25}{27}a + a\sqrt{a^2 + \frac{25}{27}a}} + \frac{1}{a^2 + a + a\sqrt{a^2+a}} - \frac{2a+1}{\sqrt{a^2+a}} + \frac{32}{25} \\
&= \frac{\frac{9}{25}(2a + \frac{25}{27})\left(a + \sqrt{a^2 + \frac{25}{27}a}\right) - \frac{25}{27}}{a^2 + \frac{25}{27}a + a\sqrt{a^2 + \frac{25}{27}a}} + \frac{1 - (2a+1)(a + \sqrt{a^2+a})}{a^2 + a + a\sqrt{a^2+a}} + \frac{32}{25}
\end{aligned}
$$

$$
\begin{aligned}
&\propto \frac{9}{25}\left(2a+\frac{25}{27}\right)\left(a+\sqrt{a^2+\frac{25}{27}a}\right)(\sqrt{a^2+a}+a+1) \\
&\quad -(2a+1)\left(a+\sqrt{a^2+a}\right)\left(\sqrt{a^2+\frac{25}{27}a}+a+\frac{25}{27}\right)-\frac{25}{27}\left(\sqrt{a^2+a}+a\right) \\
&\qquad +\left(\sqrt{a^2+\frac{25}{27}a}+a\right)+\frac{32}{25}\sqrt{a^2+a}\left(a+\sqrt{a^2+a}\right)\left(\sqrt{a^2+\frac{25}{27}a}+a+\frac{25}{27}\right).
\end{aligned}
$$

Then

$$
\begin{aligned}
&\frac{d(h(3,a)-h(1,a))}{da} \\
&\propto \frac{9}{25}\left(2a+\frac{25}{27}\right)\left(a+\sqrt{a^2+\frac{25}{27}a}\right)(\sqrt{a^2+a}+a+1) \\
&\quad -(2a+1)\left(a+\sqrt{a^2+a}\right)\left(\sqrt{a^2+\frac{25}{27}a}+a\right)-\frac{50}{27}(a+1)\left(\sqrt{a^2+a}+a\right) \\
&\qquad +\sqrt{a^2+\frac{25}{27}a}+a+\frac{32}{25}\left(a\sqrt{a^2+\frac{25}{27}a}+a\left(a+\frac{25}{27}\right)\right)\left(\sqrt{a^2+a}+a+1\right) \\
&=(\sqrt{a^2+a}+a)\left(-\frac{2}{3}\sqrt{a^2+\frac{25}{27}a}-\frac{4}{3}a-\frac{50}{27}\right)+\left(2a+\frac{4}{3}\right)\sqrt{a^2+\frac{25}{27}a}+2a^2+\frac{68}{27}a \\
&=\frac{2}{3}\left(2(a+1)\sqrt{a^2+\frac{25}{27}a}+a^2+a-\sqrt{a^2+a}\left(2a+\frac{25}{9}+\sqrt{a^2+\frac{25}{27}a}\right)\right)
\end{aligned}
$$

and therefore

$$
\begin{aligned}
\frac{d(h(3,a)-h(1,a))}{da} &\propto 2\sqrt{a+1}\sqrt{a+\frac{25}{27}}+\sqrt{a^2+a}-\left(2a+\frac{25}{9}+\sqrt{a^2+\frac{25}{27}a}\right) \\
&\propto 4(a+1)\left(a+\frac{25}{27}\right)+a^2+a+4(a+1)\sqrt{a^2+\frac{25}{27}a} \\
&\qquad -\left(4a^2+\frac{625}{81}+\frac{100}{9}a+a^2+\frac{25}{27}a+\left(4a+\frac{50}{9}\right)\sqrt{a^2+\frac{25}{27}a}\right) \\
&< 0.
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
\lim_{a\to\infty}(h(3,a)-h(1,a)) &= \frac{1}{3}+\lim_{a\to\infty}\left(a\left(1-\sqrt{1+\frac{1}{a}}\right)-\frac{9a}{25}\left(1-\sqrt{1+\frac{25}{27a}}\right)\right) \\
&= \frac{1}{3}+\lim_{z\to 0}\left(\frac{1-\sqrt{1+z}}{z}-\frac{9}{25}\frac{1-\sqrt{1+\frac{25z}{27}}}{z}\right)
\end{aligned}
$$

$$= \frac{1}{3} - \frac{1}{2} + \frac{1}{6} = 0,$$

and

$$\lim_{a \to 0} (h(3,a) - h(1,a)) = \sqrt{\frac{25}{27}} + \frac{1}{3} > 0.$$

Hence $h(3,a) > h(1,a)$ for any $a > 0$. Consequently, $\bar{K}_3 > \bar{K}_1$.

Finally, we show that $h(2,a) > h(3,a)$ for any $a > 0$ and thus $\bar{K}_2 > \bar{K}_3$. We have

$$\frac{d(h(2,a) - h(3,a))}{da}$$
$$= -\frac{\frac{1}{2}\frac{\frac{9}{8a^2}}{\sqrt{1+\frac{9}{8a}}}}{1 + \sqrt{1 + \frac{9}{8a}}} + \frac{\frac{1}{2}\frac{\frac{25}{27a^2}}{\sqrt{1+\frac{25}{27a}}}}{1 + \sqrt{1 + \frac{25}{27a}}} - \frac{19}{225} + \frac{4}{9}\frac{1}{2}\frac{2a + \frac{9}{8}}{\sqrt{a^2 + \frac{9}{8}a}} - \frac{9}{25}\frac{1}{2}\frac{2a + \frac{25}{27}}{\sqrt{a^2 + \frac{25}{27}a}}$$
$$\propto \frac{4}{9}\frac{2a + \frac{9}{8}}{\sqrt{a^2 + \frac{9}{8}a}} - \frac{9}{8}\frac{1}{a^2 + \frac{9}{8}a + a\sqrt{a^2 + \frac{9}{8}a}} + \frac{25}{27}\frac{1}{a^2 + \frac{25}{27}a + a\sqrt{a^2 + \frac{25}{27}a}} - \frac{9}{25}\frac{2a + \frac{25}{27}}{\sqrt{a^2 + \frac{25}{27}a}} - \frac{38}{225}.$$

Therefore,

$$\frac{d(h(2,a) - h(3,a))}{da}$$
$$\propto \frac{\frac{4}{9}(2a + \frac{9}{8})\left(a + \sqrt{a^2 + \frac{9}{8}a}\right) - \frac{9}{8}}{a^2 + \frac{9}{8}a + a\sqrt{a^2 + \frac{9}{8}a}} + \frac{\frac{25}{27} - \frac{9}{25}(2a + \frac{25}{27})\left(a + \sqrt{a^2 + \frac{25}{27}a}\right)}{a^2 + \frac{25}{27}a + a\sqrt{a^2 + \frac{25}{27}a}} - \frac{38}{225}$$
$$\propto \frac{4}{9}\left(2a + \frac{9}{8}\right)\left(a + \sqrt{a^2 + \frac{9}{8}a}\right)\left(\sqrt{a^2 + \frac{25}{27}a} + a + \frac{25}{27}\right)$$
$$- \frac{9}{25}\left(2a + \frac{25}{27}\right)\left(a + \sqrt{a^2 + \frac{25}{27}a}\right)\left(\sqrt{a^2 + \frac{9}{8}a} + a + \frac{9}{8}\right)$$
$$- \frac{9}{8}\left(\sqrt{a^2 + \frac{25}{27}a} + a + \frac{25}{27}\right) + \frac{25}{27}\left(\sqrt{a^2 + \frac{9}{8}a} + a + \frac{9}{8}\right)$$
$$- \frac{38}{225}\sqrt{a^2 + \frac{25}{27}a}\left(a + \sqrt{a^2 + \frac{25}{27}a}\right)\left(\sqrt{a^2 + \frac{9}{8}a} + a + \frac{9}{8}\right)$$
$$\propto \frac{4}{9}\left(2a + \frac{9}{8}\right)\left(a + \sqrt{a^2 + \frac{9}{8}a}\right)\left(\sqrt{a^2 + \frac{25}{27}a} + a + \frac{25}{27}\right)$$
$$- \frac{9}{25}\left(2a + \frac{25}{27}\right)\left(a + \sqrt{a^2 + \frac{25}{27}a}\right)\left(\sqrt{a^2 + \frac{9}{8}a} + a + \frac{9}{8}\right)$$
$$- \frac{9}{8}\left(\sqrt{a^2 + \frac{25}{27}a} + a\right) + \frac{25}{27}\left(\sqrt{a^2 + \frac{9}{8}a} + a\right)$$

$$
\begin{aligned}
&- \frac{38}{225}\sqrt{a^2+\frac{25}{27}a}\left(a+\sqrt{a^2+\frac{25}{27}a}\right)\left(\sqrt{a^2+\frac{9}{8}a}+a+\frac{9}{8}\right) \\
\propto\ & \frac{4}{9}\left(2a+\frac{9}{8}\right)\left(a+\sqrt{a^2+\frac{9}{8}a}\right)\left(\sqrt{a^2+\frac{25}{27}a}+a+\frac{25}{27}\right) \\
&- \frac{9}{25}\left(2a+\frac{25}{27}\right)\left(a+\sqrt{a^2+\frac{25}{27}a}\right)\left(\sqrt{a^2+\frac{9}{8}a}+a\right) \\
&- \frac{9}{8}\left(\frac{18}{25}a+\frac{4}{3}\right)\left(\sqrt{a^2+\frac{25}{27}a}+a\right)+\frac{25}{27}\left(\sqrt{a^2+\frac{9}{8}a}+a\right) \\
&- \frac{38}{225}\left(\sqrt{a^2+\frac{25}{27}a}+a+\frac{25}{27}\right)\left(a\sqrt{a^2+\frac{9}{8}a}+a^2+\frac{9}{8}a\right).
\end{aligned}
$$

Then

$$
\begin{aligned}
&\frac{d(h(2,a)-h(3,a))}{da} \\
&\propto \left(\sqrt{a^2+\frac{25}{27}a}+a\right)\left(\frac{1}{6}\sqrt{a^2+\frac{9}{8}a}-\frac{5}{6}a-\frac{3}{2}\right)+\frac{25}{27}\left(\frac{162}{225}a+\frac{3}{2}\right)\sqrt{a^2+\frac{9}{8}a}+\frac{25}{27}\left(\frac{162}{225}a^2+\frac{131}{100}a\right) \\
&= \left(\sqrt{a^2+\frac{25}{27}a}+a\right)\left(\frac{1}{6}\sqrt{a^2+\frac{9}{8}a}-\frac{5}{6}a-\frac{3}{2}\right)+\frac{25}{27}\left(\frac{18}{25}a+\frac{3}{2}\right)\sqrt{a^2+\frac{9}{8}a}+\frac{25}{27}\left(\frac{18}{25}a^2+\frac{131}{100}a\right) \\
&= \sqrt{a^2+\frac{25}{27}a}\left(\frac{1}{6}\sqrt{a^2+\frac{9}{8}a}-\frac{5}{6}a-\frac{3}{2}\right)+\left(\frac{5}{6}a+\frac{25}{18}\right)\sqrt{a^2+\frac{9}{8}a}-\frac{1}{6}a^2-\frac{31}{108}a \\
&= \left(\frac{1}{6}\sqrt{a^2+\frac{25}{27}a}+\frac{5}{6}a+\frac{25}{18}\right)\sqrt{a^2+\frac{9}{8}a}-\left(\left(\frac{5}{6}a+\frac{3}{2}\right)\sqrt{a^2+\frac{25}{27}a}+\frac{1}{6}a^2+\frac{31}{108}a\right) \\
&\propto \left(\sqrt{a^2+\frac{25}{27}a}+5a+\frac{25}{3}\right)\sqrt{a^2+\frac{9}{8}a}-\left((5a+9)\sqrt{a^2+\frac{25}{27}a}+a^2+\frac{31}{18}a\right) \\
&\propto \left(\sqrt{a^2+\frac{25}{27}a}+5a+\frac{25}{3}\right)^2\left(a^2+\frac{9}{8}a\right)-\left((5a+9)\sqrt{a^2+\frac{25}{27}a}+a^2+\frac{31}{18}a\right)^2 \\
&= \left(a^2+\frac{9}{8}a\right)\left(a^2+\frac{25}{27}a+25\left(a^2+\frac{25}{9}+\frac{10}{3}a\right)+10\left(a+\frac{5}{3}\right)\sqrt{a^2+\frac{25}{27}a}\right) \\
&\quad - (25a^2+81+90a)\left(a^2+\frac{25}{27}a\right)-a^2\left(a^2+\frac{31^2}{18^2}+\frac{31}{9}a\right) \\
&\quad - (10a+18)\left(a^2+\frac{31}{18}a\right)\sqrt{a^2+\frac{25}{27}a}.
\end{aligned}
$$

Hence,

$$
\frac{d(h(2,a)-h(3,a))}{da}
$$

$$
\begin{aligned}
&\propto \left(\left(\frac{335}{12} - \frac{317}{9} \right) a^2 + \left(\frac{225}{12} - 31 \right) a \right) \sqrt{a^2 + \frac{25}{27}a} + \mu_3 a^3 + \mu_2 a^2 + \mu_1 a \\
&\propto \left(\left(\frac{335}{12} - \frac{317}{9} \right) a + \frac{225}{12} - 31 \right) \sqrt{a^2 + \frac{25}{27}a} + \mu_3 a^2 + \mu_2 a + \mu_1 \\
&= -(7.3056a + 12.25)\sqrt{a^2 + \frac{25}{27}a} + \mu_3 a^2 + \mu_2 a + \mu_1 \\
&=: w(a),
\end{aligned}
$$

where

$$
\begin{aligned}
\mu_3 &= \frac{91 \times 25}{27} + \frac{9 \times 13}{4} - 90 - \frac{625}{27} - \frac{31}{9} \approx -3.0833 < 0, \\
\mu_2 &= \frac{625}{9} + \frac{91 \times 25}{24} - 81 - \frac{25 \times 90}{27} - \frac{31^2}{18^2} \approx -3.0633 < 0, \\
\mu_1 &= \frac{625}{8} - \frac{25 \times 81}{27} \approx 3.1250 > 0.
\end{aligned}
$$

It is clear that $\lim_{a\to 0} w(a) = \mu_1 > 0$, $\lim_{a\to\infty} w(a) = -\infty$, and $w(\cdot)$ is strictly decreasing in a. Hence we can conclude that $h(2, a) - h(3, a)$ first increases and eventually decreases in a. Since we also have that $\lim_{a\to 0} h(2, a) - h(3, a) = \log\left(\frac{9}{5}\sqrt{\frac{3}{8}}\right) - \frac{1}{12} > 0$, and $\lim_{a\to\infty} h(2, a) - h(3, a) = 0$, we have that $h(2, a) - h(3, a) > 0$ for any $a > 0$. This implies that $\bar{K}_2 > \bar{K}_3$.

Finally, it is clear from the above arguments that the order of the K_j's does not depend on $\rho_k^\diamond$. □

Proof of Proposition 6

We first show part (i). Assume by contradiction that $\Delta_{d_{i+1}} \geq \Delta_{d_i}$. From (7) we see that $\tau_k^*(d_{i+1}) < \tau_k^*(d_i)$ for every $k = 1, ..., m$, and consequently $\Delta_{d_{i+1}} < \Delta_{d_i}$, a contradiction. Hence, $\Delta_{d_{i+1}} < \Delta_{d_i}$.

We now show part (ii). Since all agents in the economy have the same risk aversion coefficient ρ, we see that $\tau = \rho\Delta$. From Proposition 3,

$$
V(d_i) = -\left(\exp\{-2\rho c(\rho\Delta_{d_i})\}\, \mathrm{Var}(\theta - p_{d_i})(\tau_\theta + \Delta_{d_i}^2 \tau_u + (2 - 1/d_i)\rho\Delta_{d_i})\right)^{-\frac{1}{2}},
$$

where $\Delta_{d_i} = \sum_{k=1}^m \lambda_k \tau_k^*(d_i)/\rho_k^\diamond$, p_{d_i} is given by (1) with the replacement of Δ with Δ_{d_i}. Note that from (7) we have $\Delta_{d_{i+1}} < \Delta_{d_i}$, so in order to show $V(d_i) < V(d_{i+1})$, it suffices to show that for any given $1 \le s \le n$, $\mathrm{Var}(\theta - p)(\tau_\theta + \Delta^2\tau_u + (2 - 1/s)\rho\Delta)$ is strictly decreasing in Δ since if it is, then

$$\begin{aligned}
V(d_{i+1}) &= -\Big(\exp\{-2\rho c(\rho\Delta_{d_{i+1}})\} \,\mathrm{Var}(\theta - p_{d_{i+1}})\left(\tau_\theta + \Delta_{d_{i+1}}^2\tau_u + (2 - 1/d_{i+1})\rho\Delta_{d_{i+1}}\right)\Big)^{-\frac{1}{2}} \\
&> -\left(\exp\{-2\rho c(\rho\Delta_{d_i})\} \,\mathrm{Var}(\theta - p_{d_i})\left(\tau_\theta + \Delta_{d_i}^2\tau_u + (2 - 1/d_{i+1})\rho\Delta_{d_i}\right)\right)^{-\frac{1}{2}} \\
&> -\left(\exp\{-2\rho c(\rho\Delta_{d_i})\} \,\mathrm{Var}(\theta - p_{d_i})\left(\tau_\theta + \Delta_{d_i}^2\tau_u + (2 - 1/d_i)\rho\Delta_{d_i}\right)\right)^{-\frac{1}{2}} \\
&= V(d_i).
\end{aligned}$$

We next show the strict monotonicity. Recall $\beta = \frac{\tau_\theta}{\Delta\tau_u + \rho}$, so that $\frac{\partial\beta}{\partial\Delta} = -\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2}$ and $\mathrm{Var}(\theta - p) = \frac{\frac{\beta^2}{\tau_\theta} + \frac{1}{\tau_u}}{(\Delta+\beta)^2}$. Then we have

$$\begin{aligned}
&\frac{\partial}{\partial\Delta}\left(\mathrm{Var}(\theta - p)(\tau_\theta + \Delta^2\tau_u + (2 - 1/s)\rho\Delta\right) \\
&= \frac{\partial\,\mathrm{Var}(\theta - p)}{\partial\Delta}\left(\tau_\theta + \Delta^2\tau_u + (2 - 1/s)\rho\Delta\right) + \mathrm{Var}(\theta - p)\,(2\Delta\tau_u + (2 - 1/s)\rho) \\
&= -\frac{2}{(\Delta+\beta)^3}\left(\frac{\Delta}{\Delta\tau_u + \rho}\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2} + \frac{1}{\tau_u}\right)\left(\tau_\theta + \Delta^2\tau_u + (2 - 1/s)\rho\Delta\right) \\
&\qquad + \frac{1}{(\Delta+\beta)^2}\left(\frac{\tau_\theta}{(\Delta\tau_u+\rho)^2} + \frac{1}{\tau_u}\right)(2\Delta\tau_u + (2 - 1/s)\rho) \\
&\propto \frac{\tau_\theta}{(\Delta\tau_u+\rho)^2}\left(2\Delta\tau_u + (2 - 1/s)\rho - \frac{2\Delta\tau_u\,(\tau_\theta + \Delta^2\tau_u + (2 - 1/s)\rho\Delta)}{(\Delta\tau_u + \rho)(\Delta+\beta)}\right) \\
&\quad + \frac{1}{\tau_u}\left(2\Delta\tau_u + (2 - 1/s)\rho - \frac{2(\tau_\theta + \Delta^2\tau_u + (2 - 1/s)\rho\Delta)}{\Delta+\beta}\right) \\
&= \frac{\tau_\theta}{(\Delta\tau_u+\rho)^2}\frac{(2\Delta\tau_u + (2 - 1/s)\rho)(\Delta^2\tau_u + \Delta\rho + \tau_\theta) - 2\Delta\tau_u\,(\tau_\theta + \Delta^2\tau_u + (2 - 1/s)\rho\Delta)}{\Delta^2\tau_u + \Delta\rho + \tau_\theta} \\
&\quad + \frac{1}{\tau_u}\frac{(2\Delta\tau_u + (2 - 1/s)\rho)(\Delta^2\tau_u + \Delta\rho + \tau_\theta) - 2(\tau_\theta + \Delta^2\tau_u + (2 - 1/s)\rho\Delta)(\Delta\tau_u + \rho)}{\Delta^2\tau_u + \Delta\rho + \tau_\theta} \\
&= \frac{\tau_\theta}{(\Delta\tau_u+\rho)^2}\frac{2\Delta^2\tau_u\rho + (2 - 1/s)(\Delta\rho^2 + \tau_\theta\rho - \Delta^2\rho\tau_u)}{\Delta^2\tau_u + \Delta\rho + \tau_\theta} \\
&\quad + \frac{1}{\tau_u}\frac{-2\tau_\theta\rho + (2 - 1/s)(\rho\tau_\theta - \Delta\rho^2 - \Delta^2\tau_u\rho)}{\Delta^2\tau_u + \Delta\rho + \tau_\theta} \\
&\propto \tau_u\tau_\theta[2\Delta^2\tau_u\rho + (2 - 1/s)(\Delta\rho^2 + \tau_\theta\rho - \Delta^2\rho\tau_u)] \\
&\quad + (\Delta^2\tau_u^2 + \rho^2 + 2\Delta\tau_u\rho)[-2\tau_\theta\rho + (2 - 1/s)(\rho\tau_\theta - \Delta\rho^2 - \Delta^2\tau_u\rho)],
\end{aligned}$$

which is less than

$$(\Delta^2\tau_u^2 + \rho^2 + 2\Delta\tau_u\rho)(2 - 1/s)(-\Delta\rho^2 - \Delta^2\tau_u\rho) + \tau_u\tau_\theta(2 - 1/s)(\Delta\rho^2 + \tau_\theta\rho),$$

which is negative under the condition $\Delta^2\tau_u \geq \tau_\theta$. The conclusion follows from the fact that $\Delta_{d_i}^2\tau_u \geq \tau_\theta$ holds for any i since we assume that $\Delta_n^2\tau_u \geq \tau_\theta$ in this proposition and have shown that $\Delta_{d_{i+1}} < \Delta_{d_i}$ for any $i = 1, \ldots, |div(n)| - 1$. $\square$

Proof of Proposition 7

Market Efficiency. From (1) we have $1/\operatorname{Var}[\theta|p] = \tau_\theta + \Delta^2\tau_u$, so that imposing the protocol of adopting the SAEAOS for larger networks within each social group on the entire economy reduces market efficiency.

Return volatility. Recall that $\operatorname{Var}(\theta - p) = \left(\frac{\beta^2}{\tau_\theta} + \frac{1}{\tau_u}\right) / (\Delta + \beta)^2$, where $\beta = \frac{\tau_\theta}{\Delta\tau_u + \rho}$. Direct computations lead to

$$
\begin{aligned}
\frac{\partial \operatorname{Var}(\theta - p)}{\partial \Delta} &= \frac{\frac{2\beta}{\tau_\theta}\frac{\partial\beta}{\partial\Delta}(\Delta+\beta)^2 - 2(\frac{\beta^2}{\tau_\theta} + \frac{1}{\tau_u})(\Delta+\beta)(1 + \frac{\partial\beta}{\partial\Delta})}{(\Delta+\beta)^4} \\
&= \frac{2}{(\Delta+\beta)^3}\left(\frac{\beta}{\tau_\theta}\frac{\partial\beta}{\partial\Delta}(\Delta+\beta) - \left(\frac{\beta^2}{\tau_\theta} + \frac{1}{\tau_u}\right)\left(1 + \frac{\partial\beta}{\partial\Delta}\right)\right) \\
&= \frac{2}{(\Delta+\beta)^3}\left(\frac{\beta}{\tau_\theta}\frac{\partial\beta}{\partial\Delta}\Delta - \frac{\beta^2}{\tau_\theta} - \frac{1}{\tau_u} - \frac{1}{\tau_u}\frac{\partial\beta}{\partial\Delta}\right) \\
&= \frac{2}{(\Delta+\beta)^3}\left(\left(\frac{\Delta}{\Delta\tau_u + \rho} - \frac{1}{\tau_u}\right)\frac{\partial\beta}{\partial\Delta} - \frac{1}{\tau_u} - \frac{\tau_\theta}{(\Delta\tau_u + \rho)^2}\right) \\
&= -\frac{2}{(\Delta+\beta)^3}\left(\frac{\Delta}{\Delta\tau_u + \rho}\frac{\tau_\theta\tau_u}{(\Delta\tau_u + \rho)^2} + \frac{1}{\tau_u}\right) \\
&< 0,
\end{aligned}
$$

where we use the relation $\frac{\partial\beta}{\partial\Delta} = -\frac{\tau_\theta\tau_u}{(\Delta\tau_u+\rho)^2}$. Hence the conclusion (ii) follows.

Market Liquidity. We have $\frac{1}{\partial p/\partial u} = \Delta + \frac{\tau_\theta}{\Delta\tau_u+\rho}$ and

$$
\frac{\partial\left(\Delta + \frac{\tau_\theta}{\Delta\tau_u+\rho}\right)}{\partial\Delta} = 1 - \frac{\tau_\theta\tau_u}{(\Delta\tau_u + \rho)^2},
$$

which is positive if and only if $\Delta > (\sqrt{\tau_\theta\tau_u} - \rho)/\tau_u$. Note that we have shown that $\Delta_{d_{i+1}} < \Delta_{d_i}$ for $i = 1, \ldots, |div(n)| - 1$ in Proposition 6. Hence the conclusion (iii) follows.

Trading Volume. From (4) and the formula that $\mathbb{E}|z| = \sigma\sqrt{2/\pi}$ if $z \sim N(0, \sigma^2)$, for $i \in \mathcal{S}_g$ (with the parameter $(\rho_k^\diamond, \tau_k^\diamond)$) we have

$$
\mathbb{E}|x_{i,g}^*| = \sqrt{\frac{2}{\pi}}\sqrt{\frac{\tau_k^\diamond}{(\rho_k^\diamond)^2} + \frac{\frac{\tau_\theta}{(\Delta\tau_u+\rho)^2}(\tau_k^\diamond/\rho_k^\diamond - \Delta)^2 + (\tau_k^\diamond/\rho_k^\diamond + \frac{\tau_\theta}{\Delta\tau_u+\rho})^2/\tau_u}{\left(\Delta + \frac{\tau_\theta}{\Delta\tau_u+\rho}\right)^2}},
$$

so that the average trading volume is given by

$$\sqrt{\frac{2}{\pi}}\sum_{k=1}^{m}\lambda_k\sqrt{\frac{\tau_k^{\diamond}}{(\rho_k^{\diamond})^2}+\frac{\frac{\tau_\theta}{(\Delta\tau_u+\rho)^2}(\tau_k^{\diamond}/\rho_k^{\diamond}-\Delta)^2+(\tau_k^{\diamond}/\rho_k^{\diamond}+\frac{\tau_\theta}{\Delta\tau_u+\rho})^2/\tau_u}{\left(\Delta+\frac{\tau_\theta}{\Delta\tau_u+\rho}\right)^2}}.$$

When all agents in the economy have the same risk aversion coefficient ρ (and then the same endogenous signal precision), the average trading volume is simplified as

$$n\sqrt{2/\pi}\sqrt{\Delta/\rho+1/\tau_u}.$$

Moreover, simple average reduces trading volume. Consequently, imposing the protocol of adopting the SAEAOS for larger networks within each social group on the entire economy reduces trading volume. □

Proof of Proposition 8

Denote $\breve{x}_i^* = \sum_{j\neq i}\tilde{x}_j^* + x_i^*$. By the strict concavity of function $U(\cdot)$, we have

$$U(W_j((\breve{x}_i^*+\delta_i)/n)) \leq U(W_j(\breve{x}_i^*/n)) + \nabla U(W_j(\breve{x}_i^*/n))\delta_i/n,$$

and the inequality becomes strict if $\delta_i \neq 0$. Taking the expectation on both sides of the preceding inequality and noting the fact that δ_i is independent of the other random variables in the model, the part (i) follows. The proofs of parts (ii) and (iii) are similar to that of part (i). □

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