Can institutional investors always beat individual investors? *

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Abstract

This paper explores the performance dynamics of institutional and individual investors in an imperfectly competitive market. We find that institutional investors with information advantage consistently achieve higher expected trading profits than sophisticated individual investors who internalize their price impact. However, when the noise-trading volume and noise-to-signal ratio are sufficiently large, this institutional investor underperforms naive individual investors who act as price-takers. The aggressive trading behavior of naive investors, driven by their failure to internalize price impact, compels institutional investors to reduce their trading aggressiveness. Our findings reveal that the irrationality of naive traders can, under certain conditions, erode the advantages typically associated with information-driven trading strategies.

Keywords: Information advantage, price impact, institutional investor, trading profits

JEL Classification: D82; D85; G11; G14

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1 Introduction

The demand side of financial markets is typically divided between institutional investors, who are distinguished by their ability to produce information, and individual investors, who are generally less informed. In classical perfectly competitive markets (Grossman and Stiglitz 1980), investors with an information advantage (institutional investors) consistently achieve higher expected trading profits than their less-informed counterparts (individual investors). However, it is well-documented that large financial institutions possess significant market influence. At the same time, with algorithms becoming an essential feature of institutional order executions, the individual traders' order flow might even have a larger average trade size than other flows (Boehmer et al. 2021).[1] In light of these developments, this paper explores whether the institutional investor who has an information advantage can still outperform less-informed individual investors within an imperfectly competitive market.

We consider a financial market where a single risky asset is traded by a finite number of investors, who differ in both their information advantages and their levels of rationality, alongside noise traders. There are three types of investors. The first is an *institutional* investor, who possesses complete information about the market. The remaining investors have access to only one piece of asymmetric information but differ in how they use it. Among these, *sophisticated* individual investors behave strategically and internalize the impact of their demands on the asset price when making optimal demand schedules. In contrast, *naive* individual investors are unaware of their price impact and consider themselves price-takers, assuming that their trades do not influence market prices.[2] The interaction between the demand schedules of all investors, along with the presence of noise trading, determines the endogenous equilibrium price, which reflects the aggregation of all market information and noise.

We analyze the expected trading profits of different investors, focusing on the interplay be-

[1]Previous studies always treat individual investors as small competitive traders (Kacperczyk et al. 2023) and many researchers use trade size as a proxy for retail order flow (Campbell et al. 2009). However, with algorithms becoming popular in the early 2000s, institutional investors start to split their trades. Hence, a trade-size partition has become far less useful as a proxy for retail order flow. Furthermore, evidence suggests that retail investors tend to have a meaningful impact on the returns of stocks with small market capitalization (see Kumar and Lee 2006), and it is well known that the market for such stocks is imperfectly competitive.

[2]The institutional investor tends to be experienced and can calculate their price impact correctly. By contrast, individual investors may not realize that their demands will impact asset prices, or even if they do, however, they cannot calculate it correctly due to limitations, such as limited investment experiences and understanding of the market environment.

tween two key effects: the positive information effect, captured by information efficiency (Rahi and Zigrand 2018; Lou and Rahi 2023), and the negative risk effect, characterized by market-implied risk aversion—defined as the sum of price impact and conditional uncertainty about the payoff. The institutional investor, with superior access to information, naturally benefits from a stronger information effect. However, the magnitude of the risk effect varies depending on the rationality of individual investors and market conditions, particularly the volume of noise trading and the noise-to-signal ratio for information. Our analysis shows that while the institutional investor consistently outperforms sophisticated individual investors across all market conditions, there are scenarios—specifically when noise-trading volume and the noise-to-signal ratio are sufficiently large—where the institutional investor may underperform naive individual investors.

When all individual investors are sophisticated, our analytical and numerical analysis reveals that the institutional investor consistently outperforms them. In scenarios where the noise-trading volume is sufficiently large, excessive noise is incorporated into the price, making it less informative for predicting fundamentals. As a result, sophisticated individual investors tend to overlook the informational content of the price. The institutional investor has weaker risk effect. Otherwise, if individual investors exhibit a weaker negative risk effect, they are more inclined to provide liquidity, which leads to a lower price impact for the institutional investor. This contradicts to previous assumption because the institutional investor has lower conditional variance. As noise-trading volume gradually decreases, the institutional investor begins to experience a more substantial negative risk effect. Nevertheless, unlike in perfectly competitive markets, the presence of imperfect competition ensures that the institutional investor's information advantage persists. Consequently, the positive information effect consistently dominates the trade-off, allowing the institutional investor to consistently outperform sophisticated individual investors.

The results differ significantly when individual investors are naive and behave as price-takers. Our analysis demonstrates that the institutional investor cannot outperform naive individual investors when the noise-trading volume and the noise-to-signal ratio for information are sufficiently large. This outcome is illustrated in Table 1. Naive individual investors, perceiving themselves as price-takers, mistakenly believe that their trading does not influence

Table 1: **Intuition for Results on Expected Trading Profits.** We simplify the main model to a two-player game (the institutional investor (I) v.s. the individual investor) with two strategies (aggressive (A) or conservative (C) trading strategies). The individual investor can be either sophisticated (S) or naive (N). The payoff matrix on the left represents the real payoffs recognized by investor I and investor S, while the "fictional" payoff matrix on the right reflects the mistakes made by investor N in estimating payoffs resulting from failing to internalize his price impact. The payoff matrix demonstrates three characteristics in the main model. First, each investor's trade has a price impact. Strategy A influences the equilibrium price and imposes negative externalities on other investors, as $\pi^S(A,A) < \pi^S(C,A)$ and $\pi^S(A,C) < \pi^S(C,C)$ for investor S and $\pi^I(A,A) < \pi^I(A,C)$ and $\pi^I(C,A) < \pi^I(C,C)$ for investor I, because more trading drives the price up more. Second, the institutional investor has an information advantage over the individual investors. Investor I tends to trade more aggressively when investor S trades conservatively, i.e., $\pi^I(A,C) > \pi^I(C,C)$. Conversely, without an information advantage, investor S stays conservative even investor I trades conservatively, i.e., $\pi^S(C,A) < \pi^S(C,C)$. Combining previous two characteristics, when one of the investors has imposed an aggressive trading strategy already, following up with aggressive trading is not optimal even if the investor has an information advantage (that is, $\pi^I(A,A) < \pi^I(C,A)$ and $\pi^S(A,A) < \pi^S(A,C)$), which will impose too much impact on equilibrium price. Third, the naive individual investor is irrational. Investor N does not internalize the impact of his own trading on the market. This leads to a belief that aggressive trading always yields higher payoffs, regardless of the trading behavior of the other player (investor N believes $\pi^N(A,A) > \pi^N(A,C)$ and $\pi^N(C,A) > \pi^N(C,C)$).

I \ S	A	C
A	1 2	2 7
C	4 3	5 6

I \ N	A	C
A	'3' 2	2 7
C	'6' 3	5 6

the equilibrium price. As a result, they tend to trade more aggressively—buying heavily on positive signals and selling on negative ones. This behavior is depicted by the "fictional" payoff matrix in the right table, where the payoffs for naive investors ($\pi^N(A,A) > \pi^N(A,C)$ and $\pi^N(C,A) > \pi^N(C,C)$) reflect their beliefs in the non-impact of their trades. Recognizing the irrationality of naive investors, the institutional investor anticipates their aggressive trading behavior and is consequently forced to reduce his own trading aggressiveness, despite having an informational advantage. The institutional investor's potential gain from additional information is outweighed by the decreased share of profits due to the heightened sensitivity of the price to these aggressive trades. In essence, the irrationality of naive individual investors acts as a commitment device, embedding their aggressive trading into the market dynamics.

The institutional investor's advantage lies in having more information, but this is counterbalanced by the disadvantage that arises when naive individual investors commit to aggressive trading. When the noise-to-signal ratio for information becomes sufficiently large, the institutional investor's negative risk effect intensifies, and the information advantage cannot compensate for the relatively conservative trading strategy that is forced upon them by the naive individual investors. Furthermore, as noise-trading volume increases, the equilibrium price variance also rises, diminishing the institutional investor's positive information effect. Under these conditions, naive individual investors outperform the institutional investor. However, despite their success, the aggressive trading driven by the naive investors' irrationality imposes negative externalities on all market participants, leading to lower expected trading profits across the board compared to scenarios where individual investors are sophisticated.

Related Literature. Our imperfectly competitive market equilibrium is based on the seminal framework of Kyle (1989) (see Zhou (2022), Kacperczyk et al. (2023), Glebkin et al. (2023) and Anthropelos and Robertson (2024) for recent extensions) and is most related to Nezafat and Schroder (2023) that theoretically establish the existence of zero-precision symmetric equilibrium in an imperfectly competitive market.[3] Two main differences exist between our study and Nezafat and Schroder (2023). First, while all traders in Nezafat and Schroder (2023) are rational, we introduce the irrationality for some individual investors; that is, naive individual investors are not aware of their price impact and consider themselves to be price-takers. Second, while the focus of Nezafat and Schroder (2023) is on the zero-precision equilibrium existence with cost-free signals, our focus is on whether unconsciousness may strictly dominate rationality and survive in the long run.[4] Since unconsciousness acting likes a commitment device in a

[3]The model in Nezafat and Schroder (2023) encompasses two stages: an information-acquisition stage and a trading stage. In the information-acquisition stage, each investor chooses a signal precision to maximize his expected utility at the trading stage while considering the price impact in the trading stage and the impact of his precision choice on the trading strategies of other investors in the market. The reduced payoff uncertainty resulting from a more precise private signal increases the price sensitivity (as well as the signal sensitivity) of the deviating investor's demand, and this increased price sensitivity reduces the price impact of the conforming investors. Moreover, the decline in the conforming investors' price impact increases their demand-function price sensitivities, further reducing price impact. Lower price impact (i.e., more liquid markets) generated by the deviating investor's improved signal causes all rational investors to trade more aggressively (i.e., increase the absolute size of their trades), reducing the stock's equilibrium absolute risk premium. A zero-precision equilibrium arises when the utility cost of the lower risk premium exceeds the utility benefit from more precise private information.

[4]Furthermore, Proposition 2 in Nezafat and Schroder (2023) shows that when noise-trading volume is suffi-

standard Cournot model, institutional investors underperform naive individual investors who consider themselves to be price-takers when noise-trading volume and noise-to-signal ratio for information are sufficiently large.

We further contribute to the emerging literature on behavioral rational expectations equilibrium.[5] Eyster et al. (2019) model a financial market where some traders of a risky asset do not fully appreciate what prices convey about others' private information. Malikov and Pasquariello (2022) define quantitative investing as myopic via its reliance on a backtested trading strategy; that is, quantitative investors are unaware that other investors are aware of their existence. We share the similar feature that some traders irrationally neglect rational elements in the financial market; however, the focuses differ. Regarding economic mechanism, we are closely related to Kyle and Wang (1997), who show that overconfidence may strictly dominate rationality and survive in the long run because overconfidence acts like a commitment device in a standard Cournot duopoly model. In our study, this commitment device comes from naive investors' unconsciousness about price impact.

2 The Model

Assets: We consider a Kyle (1989)-type economy with imperfect competition. The financial market consists of a risk-free asset, with a normalized price and payoff of 1, and a risky asset with price p and a random payoff $\theta \sim \mathcal{N}(0, 1/\tau_\theta)$, $\tau_\theta > 0$. To prevent the price from being fully revealing, there is also per-capita random demand by noise traders $u \sim \mathcal{N}(0, 1/\tau_u)$, $\tau_u = 1/\sigma_u^2$, where u is independent of other random variables.[6]

ciently large, the deviating trader will benefit from a positive-precision signal only when the absolute expected noise trading is small (i.e., the mean of the noise trading is small). This differs from our Proposition 3, which does not depend on the mean of noise trading (that is, our main results also hold for a large mean of noise trading).

[5]Banerjee et al. (2009) and Banerjee (2011) combine REE and disagreement frameworks to allow investors to underestimate the precision of other investors' private information. Basak and Buffa (2019) study the decision-making of a financial institution in the presence of novel implementation friction that gives rise to operational risk. A more sophisticated model generates a more informative signal about an investment opportunity by relying on the latest IT infrastructure and advanced data analytics. However, using these technologies makes it more prone to operational errors. Mondria et al. (2022) propose an optimal inattention–style variant of partial cursedness in which each trader observes the price but employs a noisy signal to infer the information that it contains and can pay a cost to reduce the noise. They endogenize traders' sophistication levels, showing that sophistication acquisition can exhibit complementarities.

[6]Here we assume zero mean for random variables θ and u for simplicity of exposition. In fact, the main

Preference: There are $n \geq 3$ investors,[7] which are divided into three groups as introduced later. The utility of investor i who buys $x_i \in \mathbb{R}$ units of the risky asset at price $p \in \mathbb{R}$ is given by

$$-\exp\{-\rho x_i(\theta - p)\},$$

where ρ is the CARA risk aversion parameter. Here without loss of generality we assume that all investors have zero initial wealth due to the CARA assumption abstract wealth effect.

Institutional investors: We assume that there is only one investor ($i = 1$) who possesses all the information in the economy.[8] Throughout this paper, we refer to the investor with an information advantage as the institutional investor. Specifically, each individual investor $i = 2, ..., n$ can observe a private signal $y_i = \theta + \epsilon_i$, $\epsilon_i \sim \mathcal{N}(0, 1/\tau_\epsilon)$, $\tau_\epsilon > 0$. There is also a public signal $y_1 = \theta + \epsilon_1$, $\epsilon_1 \sim \mathcal{N}(0, 1/\tau_\epsilon)$, in the economy. The idiosyncratic noise $\{\epsilon_1, ..., \epsilon_n\}$ are mutually independent and also independent of other random variables in the model. The institutional investor possesses all the information in the market, that is, the signal set of the institutional investor is given by $\{y_1, y_2, ..., y_n\}$. The institutional investor behaves strategically since he tends to be experienced in financial markets and can calculate and estimate his price impact correctly. Following the similar analysis in Kyle (1989), the optimal demand of the institutional investor is given by

$$x_1^* = \frac{\mathbb{E}[\theta|y_1, y_2, ..., y_n, p] - p}{\lambda_1 + \xi_1}, \quad \xi_1 = \rho \operatorname{Var}[\theta|y_1, y_2, ..., y_n, p], \tag{1}$$

where $\lambda_1 > 0$, which will be generated endogenously, denotes the price impact of institutional investor. The institutional investor realizes that his demand has an impact on the equilibrium price, and anticipates such an impact when making optimal demand schedules.

Sophisticated individual investors: The individual investors are assumed to be strategic or behave as price-takers. We refer to strategic individual investors as *sophisticated* individual investors, and price-taking individual investors as *naive* individual investors. The information

results continue to hold for more general nonzero-mean cases.

[7]When $n = 2$, the linear equilibrium does not exist, see Equation (39) in the appendix. This is in line with the Proposition 5.1 in Kyle (1989) which states that when there are no uninformed investors in the market, there exists a linear equilibrium only when the number of informed investors is greater than or equals three.

[8]Our results also hold for a more general setting where there are many investors who have all the information in the market.

set of the individual investor $i = 2, ..., n$ is $\{y_1, y_i\}$.[9] The optimal demand of the sophisticated individual investor $i = 2, ..., m+1$ is given by

$$x_i^* = \frac{\mathbb{E}[\theta|y_1, y_i, p] - p}{\lambda_s + \xi_s}, \quad \xi_s = \rho \operatorname{Var}[\theta|y_1, y_i, p], \tag{2}$$

where $\lambda_s > 0$, which will be generated endogenously, denotes the price impact of sophisticated individual investors $2, ..., m+1$, respectively. Previous studies always treat individual investors as small competitive traders (Kacperczyk et al. 2023) and many researchers use trade size as a proxy for retail order flow (Campbell et al. 2009). However, with algorithms becoming an important feature of institutional order executions in the early 2000s, a trade-size partition has become far less useful as a proxy for retail order flow. The retail order flow might even have a larger average trade size than other flow (Boehmer et al. 2021).

Naive individual investors: A key feature of the model is that some individual investors consider themselves price-takers. Naive individual investors do not realize that their demands will have an impact on asset prices, or even if they do, but are unable to calculate it correctly due to some limitations, for example limited investment experience, limited understanding on the market environment, etc. The optimal demand of the naive individual investor $j = m+2, ..., n$ is given by[10]

$$x_j^* = \frac{\mathbb{E}[\theta|y_1, y_j, p] - p}{\xi_n}, \quad \xi_n = \rho \operatorname{Var}[\theta|y_1, y_j, p]. \tag{3}$$

Even though naive investors consider themselves to be price-takers, their trading actually have price impact on the equilibrium price as indicated by the market-clearing condition (4). In this sense, we are related to the emerging literature on behavioral rational-expected equilibrium.

Equilibrium definition: As standard in the literature, in this paper we consider linear equilibria. That is, the price of the risky asset is a linear function of investors' signals and the noise demand. A *linear Bayesian Nash equilibrium* is defined as a linear price function p, together

[9]The model can also be alternatively nested into the framework of information sharing (Colla and Antonio 2010; Ozsoylev and Walden 2011; Han and Yang 2013; Lou and Yang 2023), where the information network is represented by a star, where investor 1 is the central node who initially has access to a private signal y_1, and the other investor i is the non-central node who initially possesses individual private signal y_i, $i = 2, ..., n$.

[10]Due to symmetry, ξ_s and ξ_n does not depend on the specific index i and j respectively.

with the above given optimal demands $x_i^*, i = 1, ..., n$, such that the market-clearing condition

$$x_1^* + \sum_{i=2}^{m+1} x_i^* + \sum_{j=m+2}^{n} x_j^* + nu = 0 \tag{4}$$

holds almost surely.[11] Equation (4) indicates that the naive individual investors also have price impact. In this paper, we are interested in the question of whether the institutional investor can beat the individual investors in the sense that the institutional investor has higher trading profits than individual investors.

3 Equilibrium Characterization

In this section, we establish the existence of linear Bayesian Nash equilibria by taking the *first conjecture-then-verification* method widely used in the literature. Suppose that the equilibrium price takes the linear form of [12]

$$p = \pi_1 y_1 + \pi_s \sum_{i=2}^{m+1} y_i + \pi_n \sum_{j=m+2}^{n} y_j + \gamma u. \tag{5}$$

Since the sophisticated and naive individual investors have different awareness about their price impact on equilibrium price, they have different demand sensitivities to their price information, resulting the different price coefficients π_s and π_n.

Based on the equilibrium price form (5), from the perspective of the sophisticated individual investor who knows $\{y_1, y_i, p\}$, the price information p is equivalent to

$$\begin{aligned} s_{p,i}^s &= \frac{p - \pi_1 y_1 - \pi_s y_i}{\pi_s(m-1) + \pi_n(n-m-1)} \\ &=: \frac{1}{\pi_s(m-1) + \pi_n(n-m-1)} \left[\pi_s \sum_{r=2, r\neq i}^{m+1} y_r + \pi_n \sum_{j=m+2}^{n} y_j + \gamma u \right] \end{aligned}$$

[11]Alternatively, we can interpret $-u$ as the per-capita random supply in the market.

[12]Here we postulate that the coefficients on the signals $y_2, ..., y_{m+1}$ $(y_{m+2}, ..., y_n)$ in the conjectured linear equilibrium price p are the same because sophisticated (naive) individual investors are symmetric in the model. Moreover, since we assume that all random variables have mean zero without loss of generality, there is no intercept in the conjectured price function p.

$$=: \theta + v_s \sum_{r=2, r\neq i}^{m+1} \epsilon_r + q_s \sum_{j=m+2}^{n} \epsilon_j + z_s u,$$

where

$$\begin{aligned} v_s &= \frac{\pi_s}{\pi_s(m-1) + \pi_n(n-m-1)}, \\ q_s &= \frac{\pi_n}{\pi_s(m-1) + \pi_n(n-m-1)}, \\ z_s &= \frac{\gamma}{\pi_s(m-1) + \pi_n(n-m-1)}. \end{aligned}$$

Form the projection theorem for normal random variables, we have

$$\mathbb{E}[\theta | y_1, y_i, p] = \frac{\tau_\epsilon (y_1 + y_i) + \Theta^s s^s_{p,i}}{\tau_\theta + 2\tau_\epsilon + \Theta^s} =: \alpha^s_1 y_1 + \alpha^s_o y_i + \beta^s p,$$

where

$$\Theta^s = \left[(v_s^2(m-1) + q_s^2(n-m-1))/\tau_\epsilon + z_s^2/\tau_u \right]^{-1}, \tag{6}$$

$$\alpha^s_1 = \frac{\tau_\epsilon - \Theta^s \frac{\pi_1}{\pi_s(m-1)+\pi_n(n-m-1)}}{\tau_\theta + 2\tau_\epsilon + \Theta^s}, \tag{7}$$

$$\alpha^s_o = \frac{\tau_\epsilon - \Theta^s v_s}{\tau_\theta + 2\tau_\epsilon + \Theta^s}, \tag{8}$$

$$\beta^s = \frac{\Theta^s}{\tau_\theta + 2\tau_\epsilon + \Theta^s} \frac{1}{\pi_s(m-1) + \pi_n(n-m-1)}. \tag{9}$$

The parameters α^s_1, α^s_o and β^s measure sophisticated individual investor's expectation sensitivity to public information, private information, and price.

Similarly, based on the equilibrium price form (5), from the perspective of the naive individual investor who knows $\{y_1, y_j, p\}$, the price information is equivalent to

$$\begin{aligned} s^n_{p,j} &= \frac{p - \pi_1 y_1 - \pi_n y_j}{\pi_s m + \pi_n(n-m-2)} \\ &=: \frac{1}{\pi_s m + \pi_n(n-m-2)} \left[\pi_s \sum_{i=2}^{m+1} y_i + \pi_n \sum_{r=m+2, r\neq j}^{n} y_r + \gamma u \right] \\ &=: \theta + v_n \sum_{i=2}^{m+1} \epsilon_i + q_n \sum_{r=m+2, r\neq j}^{n} \epsilon_r + z_n u, \end{aligned}$$

where

$$
\begin{aligned}
v_n &= \frac{\pi_s}{\pi_s m + \pi_n(n-m-2)}, \\
q_n &= \frac{\pi_n}{\pi_s m + \pi_n(n-m-2)}, \\
z_n &= \frac{\gamma}{\pi_s m + \pi_n(n-m-2)}.
\end{aligned}
$$

Form the projection theorem for normal random variables, we have

$$
\mathbb{E}[\theta|y_1, y_j, p] = \frac{\tau_\epsilon(y_1 + y_j) + \Theta^n s_{p,j}^n}{\tau_\theta + 2\tau_\epsilon + \Theta^n} =: \alpha_1^n y_1 + \alpha_o^n y_j + \beta^n p,
$$

where

$$
\begin{aligned}
\Theta^n &= \left[(v_n^2 m + q_n^2(n-m-2))/\tau_\epsilon + z_n^2/\tau_u \right]^{-1}, \\
\alpha_1^n &= \frac{\tau_\epsilon - \Theta^n \frac{\pi_1}{\pi_s m + \pi_n(n-m-2)}}{\tau_\theta + 2\tau_\epsilon + \Theta^n}, \\
\alpha_o^n &= \frac{\tau_\epsilon - \Theta^n q_n}{\tau_\theta + 2\tau_\epsilon + \Theta^n}, \\
\beta^n &= \frac{\Theta^n}{\tau_\theta + 2\tau_\epsilon + \Theta^n} \frac{1}{\pi_s m + \pi_n(n-m-2)}.
\end{aligned}
$$

The parameters α_1^n, α_o^n and β^n measure naive individual investor's expectation sensitivity to public information, private information, and price.

In terms of the institutional investor who observes $\{y_1, y_2, ..., y_n, p\}$, based on the equilibrium price form (5) and applying the projection theorem for normal random variables, we have

$$
\mathbb{E}[\theta|y_1, y_2, ...y_n, p] = \mathbb{E}[\theta|y_1, y_2, ...y_n] = \frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon} \sum_{i=1}^{n} y_i,
$$

where we use the fact that the price p is redundant for the institutional investor given the full information in the market.

Moreover, the conditional uncertainty about the fundamental of the institutional investor, strategic individual investors, naive individual investors are respectively given by

$$\mathrm{Var}[\theta|y_1,...,y_n,p]=\mathrm{Var}[\theta|y_1,...,y_n]=\frac{1}{\tau_\theta+n\tau_\epsilon}, \tag{10}$$

$$\mathrm{Var}[\theta|y_1,y_i,p]=\frac{1}{\tau_\theta+2\tau_\epsilon+\Theta^s}, \tag{11}$$

$$\mathrm{Var}[\theta|y_1,y_j,p]=\frac{1}{\tau_\theta+2\tau_\epsilon+\Theta^n}. \tag{12}$$

The market-clearing condition (4) indicates

$$\frac{\frac{\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}\sum_{r=1}^n y_r-p}{\lambda_1+\xi_1}+\sum_{i=2}^{m+1}\frac{\alpha_1^s y_1+\alpha_o^s y_i+\beta^s p-p}{\lambda_s+\xi_s}+\sum_{j=m+2}^{n}\frac{\alpha_1^n y_1+\alpha_o^n y_j+\beta^n p-p}{\xi_n}+nu=0,$$

which implies

$$p=\left[\frac{1}{\lambda_1+\xi_1}+\frac{m(1-\beta^s)}{\lambda_s+\xi_s}+\frac{(n-m-1)(1-\beta^n)}{\xi_n}\right]^{-1}$$
$$\times\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}\sum_{r=1}^n y_r+\sum_{i=2}^{m+1}\frac{\alpha_1^s y_1+\alpha_o^s y_i}{\lambda_s+\xi_s}+\sum_{j=m+2}^{n}\frac{\alpha_1^n y_1+\alpha_o^n y_j}{\xi_n}+nu\right].$$

Matching coefficients over both RHSs of the preceding price function and the conjectured price function (5) leads to

$$\gamma=n\left[\frac{1}{\lambda_1+\xi_1}+\frac{m(1-\beta^s)}{\lambda_s+\xi_s}+\frac{(n-m-1)(1-\beta^n)}{\xi_n}\right]^{-1}, \tag{13}$$

$$\pi_1=\frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{m\alpha_1^s}{\lambda_s+\xi_s}+\frac{(n-m-1)\alpha_1^n}{\xi_n}\right], \tag{14}$$

$$\pi_s=\frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{\alpha_o^s}{\lambda_s+\xi_s}\right], \tag{15}$$

$$\pi_n=\frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{\alpha_o^n}{\xi_n}\right]. \tag{16}$$

Moreover, the price impact parameters should satisfy

$$\lambda_1=\left[\frac{m(1-\beta^s)}{\lambda_s+\xi_s}+\frac{(n-m-1)(1-\beta^n)}{\xi_n}\right]^{-1}, \tag{17}$$

$$\lambda_s=\left[\frac{1}{\lambda_1+\xi_1}+\frac{(m-1)(1-\beta^s)}{\lambda_s+\xi_s}+\frac{(n-m-1)(1-\beta^n)}{\xi_n}\right]^{-1}. \tag{18}$$

Similar to Kyle (1989), Equations (17) and (18) show that each investor's price impact equals the reciprocal of the sum of the price sensitivities of all the other investors. Now, we get a system of equilibrium equations (13)-(18) with variables π_s, π_n, π_1, γ, λ_1 and λ_s. Furthermore, even though the naive individual investors treat themselves as price-takers, they indeed have price impact. We summarize all the variables we need to establish the equilibrium in Table 2:

Table 2: Notations

Symbol	**Definition**
Exogenous	
θ	Random payoff of the risky asset, $\theta \sim N(0, 1/\tau_\theta)$
u	Per-capita random demand of noise traders, $u \sim N(0, 1/\tau_u)$
ρ	Risk aversion parameter
n	Total number of investors
m	Number of sophisticated individual investors
$\epsilon_r, r = 1, ..., n$	Idiosyncratic noise of signal y_r
y_1	Public signal, $y_1 = \theta + \epsilon_1$
$y_r, r = 2, ..., n$	Private signal of investor r, $y_r = \theta + \epsilon_r$
x_1	Demand of institutional investor
$x_i, i = 2, ..., m+1$	Demand of sophisticated individual investor i
$x_j, j = m+2, ..., n$	Demand of naive individual investor j
ξ_1	Risk aversion adjusted conditional variance for institutional investor
ξ_s	Risk aversion adjusted conditional variance for sophisticated individual investors
ξ_n	Risk aversion adjusted conditional variance for naive individual investors
Endogenous	
p	Price of the risky asset
π_1	Price sensitivity to public information
π_s	Price sensitivity to private information of sophisticated individual investors
π_n	Price sensitivity to private information of naive individual investors
γ	Price sensitivity to demand of noise traders
λ_1	Price impact of the institutional investor
λ_s	Price impact of each sophisticated individual investor
α_1^s	Sophisticated individual investor's expectation sensitivity to public information
α_1^n	Naive individual investor's expectation sensitivity to public information
α_o^s	Sophisticated individual investor's expectation sensitivity to private information
α_o^n	Naive individual investor's expectation sensitivity to private information
β^s	Sophisticated individual investor's expectation sensitivity to price
β^n	Naive individual investor's expectation sensitivity to price

We will first discuss two special cases of all individual investors behaving strategically and behaving as price-takers in Section 4 and Subsection 5.1, respectively, and then consider the more general case of the coexistence of three types of investors in Subsection 5.3.

4 Sophisticated Individual Investors

Applying $m = n - 1$, this section investigates whether the institutional investor can beat sophisticated individual investors who behave strategically and take into account the impact of their demands on the asset price.

4.1 Equilibrium

We first establish a linear Bayesian Nash equilibrium and then define expected trading profits.

PROPOSITION **1.** *There exists a linear Bayesian Nash equilibrium determined by the following system of equations*

$$\gamma = n\left[\frac{1}{\lambda_1+\xi_1}+\frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}\right]^{-1}, \tag{19}$$

$$\pi_1 = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{(n-1)\alpha_1^s}{\lambda_s+\xi_s}\right], \tag{20}$$

$$\pi_s = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{\alpha_o^s}{\lambda_s+\xi_s}\right], \tag{21}$$

$$\lambda_1 = \left[\frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}\right]^{-1}, \tag{22}$$

$$\lambda_s = \left[\frac{(n-2)(1-\beta^s)}{\lambda_s+\xi_s}+\frac{1}{\lambda_1+\xi_1}\right]^{-1}, \tag{23}$$

where

$$\alpha_1^s = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}\frac{\pi_1}{\pi_s}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}},$$

$$\alpha_o^s = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}, \tag{24}$$

$$\beta^s = \frac{\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\frac{1}{(n-2)\pi_s}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}, \tag{25}$$

$$z_s = \frac{\gamma}{(n-2)\pi_s}.$$

The equilibrium system (19)-(23) can be directly obtained from the system (13)-(18) by setting $m = n-1$ and removing the equation (16) which corresponds to naive individual investors. The expected trading profits of the institutional investor, i.e., the investor 1, are given by

$$\Pi_1 := \mathbb{E}[(\theta - p)x_1^*] = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_2, ..., y_n, p]}{\lambda_1 + \xi_1}, \tag{26}$$

where we use the relation in (1) and the law of total variance. Similarly, from (2) the (expected) trading profits of sophisticated individual investor $i = 2, ..., n$ are given by

$$\Pi_s := \mathbb{E}[(\theta - p)x_i^*] = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_i, p]}{\lambda_s + \xi_s}, i = 2, ..., n. \tag{27}$$

Following the definition of informational efficiency in the literature (Rahi and Zigrand 2018; Lou and Rahi 2023), here we can analogously define a measure of informational efficiency for predicting the asset return $\theta - p$:

$$\Psi_1 := \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_2, ..., y_n, p]}{\mathrm{Var}(\theta - p)} = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta|y_1, y_2, ..., y_n]}{\mathrm{Var}(\theta - p)}. \tag{28}$$

Similarly, also define

$$\Psi_s := \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_i, p]}{\mathrm{Var}(\theta - p)} = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta|y_1, y_i, p]}{\mathrm{Var}(\theta - p)}, i = 2, ..., n. \tag{29}$$

Equations (26) and (27) indicate that investors' trading profits are determined by both the information effect, described by information efficiency, and risk effect, characterized by the market-implied risk aversion (i.e., the sum of price impact λ and the risk-aversion-adjusted conditional variance about the fundamental ξ). From (26)-(29), we can see that the institutional investor can beat sophisticated individual investors, i.e., $\Pi_1 > \Pi_s$, if and only if $\frac{\Psi_1}{\lambda_1+\xi_1} > \frac{\Psi_s}{\lambda_s+\xi_s}$, or equivalently, $\frac{\lambda_s+\xi_s}{\lambda_1+\xi_1} > \frac{\Psi_s}{\Psi_1}$.

Because the institutional investor obtains more information, the information efficiency is higher, that is $\Psi_1 > \Psi_s$. Hence, when we were considering a competitive setting, it is clear that the institutional investor can always beat individual investors.[13] The analysis becomes more complicated as illustrated below in an imperfectly competitive setting. Price impact kicks in and plays an important role in imperfectly competitive economies. Besides the information advantage, if additionally, the institutional investor also has lower market-implied risk aversion, then the institutional investor will have a higher trading profit. However, if the institutional investor has large enough market-implied risk aversion, whether he has a higher trading profit depends on which one of the two conflicting effects arising information advantage and illiquidity is the dominant one. Following, we focus on the risk effect through examining the market-implied risk aversion for both institutional and sophisticated individual investors and then discuss the expected trading profits.

4.2 Market-implied risk aversion: Risk effect

Price impact refers to the change in the asset price caused by investors' trading. A high price impact means that a demand shock will drive the price higher and then potentially reduce investors' trading profits. In the studied linearity framework, the price impact of one investor is determined by the reciprocal of the price sensitivity of the demand functions of his counterparties in the market (Kyle (1989)). When his counterparties demand less sensitive to the price, or in other words, have less elastic demand, any deviation by the investor from his equilibrium demand at any given price requires a greater price adjustment in order for the market to absorb it, and thus the investor has large price impact. Intuitively, less elastic demand of his counterparties implies that his counterparties are unwilling to provide liquidity to the investor, and make the investor facing an illiquid market.

Besides the price impact parameter, the risk-aversion-adjusted conditional variance also has an effect on investors' demands. A higher price impact and a larger risk-aversion-adjusted conditional variance tends to reduce investors' demands. Throughout this paper, we refer to the sum of the price impact and the risk-aversion-adjusted conditional variance about the payoff

[13] When all investors behave as price-takers, the trading profits of the institutional investor and individual investors are given by Equations (26) and (27) with the setting of $\lambda_1 = 0$ and $\lambda_s = 0$, respectively. It is easy to see that the institutional investor faces a lower conditional uncertainty than individual investors, i.e., $\xi_1 < \xi_s$. Moreover, because the institutional investor has an information advantage over individual investors, he has a higher trading profit than individual investors in a competitive setting.

as the *market-implied* risk aversion.

Proposition **2.** *Suppose that all individual investors are sophisticated, we have*

(i) $\lambda_1 > \lambda_s$.

(ii) $\lambda_1 + \xi_1 \geq \lambda_s + \xi_s$ *if* $(n-1)(1-\beta^s) \leq 1$. *Furthermore, if* $(n-1)(1-\beta^s) > 1$, *then* $\lambda_1 + \xi_1 < \lambda_s + \xi_s$ *if and only if*

$$\xi_1 < \left[1 - \frac{\beta^s}{(n-1)(n-2)(1-\beta^s)^2}\right]\xi_s, \tag{30}$$

and $\lambda_1 + \xi_1 > \lambda_s + \xi_s$ *otherwise.*

Proposition 2 (i) shows that the institutional investor always has a higher price impact. The intuition is clearest if we assume, for the sake of argument, that investors ignore the information in the price.[14] When $\xi_s = \xi_1$, $\lambda_s = \lambda_1$ due to the symmetry. Suppose the institutional investor has more precise information.[15] The institutional investor's demand is more sensitive to the price, which implies a lower price impact faced by individual investors. This further makes the individual investors' demands more price sensitive and decreases the price impact faced by the institutional investor. The reduced amount of the price impact faced by the institutional investor is less than that faced by individual investors, so that $\lambda_1 > \lambda_s$.[16] The above explanations also apply to the setting that the individual investors learn from the price, when the inference sensitivity β^s is small. However, when the inference sensitivity β^s is large,[17] the individual investors have very low willingness to provide liquidity and the institutional

[14]Specifically, let us first consider the benchmark that the sophisticated individual investors $2, ..., n$ ignore the information contained in the price, i.e., $\beta^s = 0$. In this case, the two price impact parameters λ_1, λ_s are coupled with each other and completely determined by investors' conditional uncertainty about the payoff, see Equations (22) and (23) (with the setting of $\beta^s = 0$).

[15]We start the benchmark with the equal informative signal, that is $\xi_s = \xi_1 = \rho\,\mathrm{Var}[\theta|y_1, y_j]$ (here we regard ξ_s and ξ_1 as exogenously given parameters), which implies that $\lambda_s = \lambda_1$ due to the symmetry. We now lower the value of ξ_1 (and fix the value of ξ_s) such that $\xi_1 < \xi_s$. It indeed holds that $\xi_1 < \xi_s$ in our model.

[16]Kyle (1989) shows that each trader's price impact equals the reciprocal of the sum of the price sensitivities of all the other investors. Because the institutional investor has a lower risk-aversion-adjusted conditional variance about the fundamental, the decrease in his price impact has a stronger influence on his price sensitivity, that is $\partial\left((\lambda_1 + \xi_1)^{-1}\right)/\partial\lambda_1 > \partial\left((\lambda_s + \xi_s)^{-1}\right)/\partial\lambda_s$, which leads to a relatively larger impact on other investors' price impact.

[17]Our analysis shows that β^s is small and close to zero as τ_u is sufficiently small, and β^s is large and close to its upper bound $(2n-4)/(2n-3)$ as τ_u is sufficiently large, see the proof of Proposition 3.

investor faces a more illiquid market than individual investors, resulting in a larger price impact for the institutional investor.

Proposition 2 (ii) indicates that only when sophisticated individual investors rely less on price to infer fundamentals and are less informative at the same time, the risk effect for the institutional investor is relatively weaker. Suppose individual investors neglect the information content conveyed by price. The price impacts are characterized as

$$\lambda_1 = \left(\frac{n-1}{\lambda_s + \xi_s}\right)^{-1}, \quad \lambda_s = \left(\frac{n-2}{\lambda_s + \xi_s} + \frac{1}{\lambda_1 + \xi_1}\right)^{-1}.$$

The institutional investor has a weaker risk effect when his information is more precise. Otherwise, if the individual investors have lower market-implied risk aversion $\lambda_s + \xi_s$, they are more willing to provide liquidity to the institutional investor. This results in a lower price impact λ_1 for the institutional investor, which contradicts to previous assumption because the institutional investor has more information and then a lower conditional variance. Therefore, the more informed institutional investor has a weaker negative risk effect when individual investors neglect the information content of the price. The above augments also apply to the setting that the individual investors learn from the price but the inference sensitivity β^s is small.

4.3 Expected trading profits: information v.s. risk effect

This subsection focuses on the impact of noise-trading volume. The parameter τ_u^{-1} represents the uncertainty of noise trading, and is used to measure the trading volume by noise traders (Kovalenkov and Vives 2014; Nezafat and Schroder 2023).[18] Proposition 3 discusses and Figure 1 numerically illustrates how the noise-trading volume impacts the relative profits of the institutional and sophisticated individual investors.

Proposition **3.** *Suppose that τ_u is either sufficiently small or sufficiently large. Then the institutional investor can beat the sophisticated individual investors.*

When noise-trading volume is sufficiently large (i.e., τ_u is sufficiently small), too much noise is incorporated into the price. In this case, the price is less informative to predict the

[18] The noise-trading volume is usually defined as the expected absolute value of the traded amount by noise traders. We can see that $\mathbb{E}|nu| = \sigma_u n\sqrt{2/\pi}$ and consequently, it is reasonable to interpret σ_u as noise-trading volume.

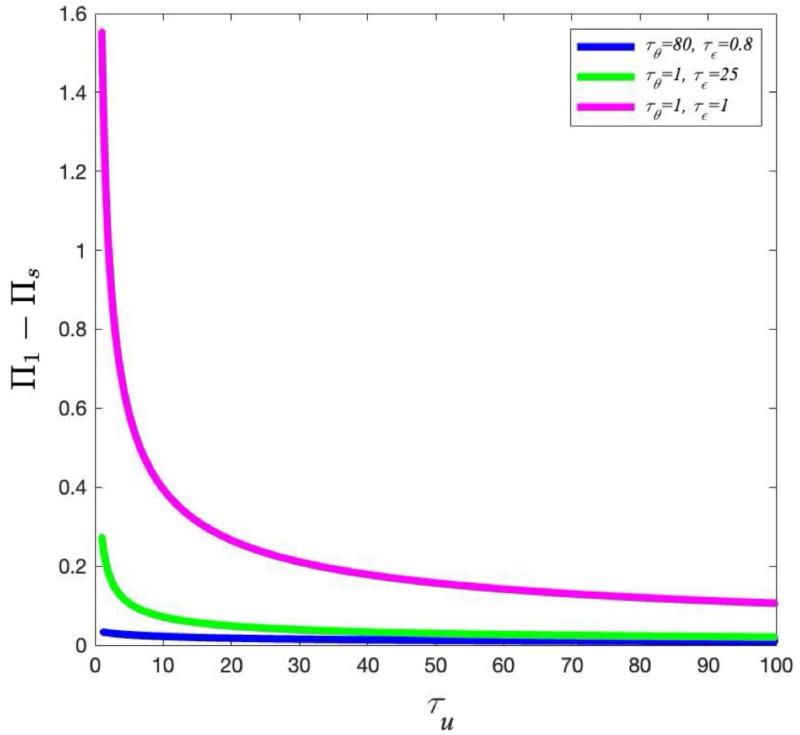


Figure 1: **The effect of noise-trading volume on difference between trading profits.** This figure plots how noise-trading volume affects the difference between trading profits of the institutional investor Π_1 and the sophisticated individual investors Π_s under three sets of parameters $\{\tau_\theta, \tau_\epsilon\}$. Here the risk aversion parameter is $\rho = 2$ and the total number of investors in the market is $n = 10$.

fundamental and then the individual investors will ignore the information contained in the price when making optimal demands, i.e., $\beta^s \to 0$ (Eyster et al. 2019). As explained earlier in (30) due to the lower conditional variance of the institutional investor, the market-implied risk aversion for the institutional investor, in this case, is smaller than that for the sophisticated individual investors, or in other words the institutional investor faces a more liquid market. Therefore, the institutional investor has a relatively weaker negative risk effect. Hence, the institutional investor, who also has a more significant information effect, faces a larger trading opportunity and has a higher trading profit than the sophisticated individual investors.

When noise-trading volume is sufficiently small (i.e., τ_u is sufficiently large), sophisticated individual investors heavily rely on price to infer fundamental information (the inference sensitivity β^s tends to its allowable upper bound) so that the institutional investor has a higher market-implied risk aversion and a relatively stronger risk effect. However, different from the result in perfectly competitive markets, the price will not reveal all the information in the market due to imperfect competition,[19] so that the information advantage of the institutional investor survives. Furthermore, more informed signal directly influences the information effect while it indirectly affects risk effect through the interaction between the institutional and individual investors. Hence, the information advantage effect of the institutional investor dominates the risk effect arising from the higher market-implied risk aversion, and the institutional investor thus has a higher trading profit than the sophisticated individual investors.

Observation **1.** *The institutional investor can always beat the sophisticated individual investors for intermediate values of* τ_u.

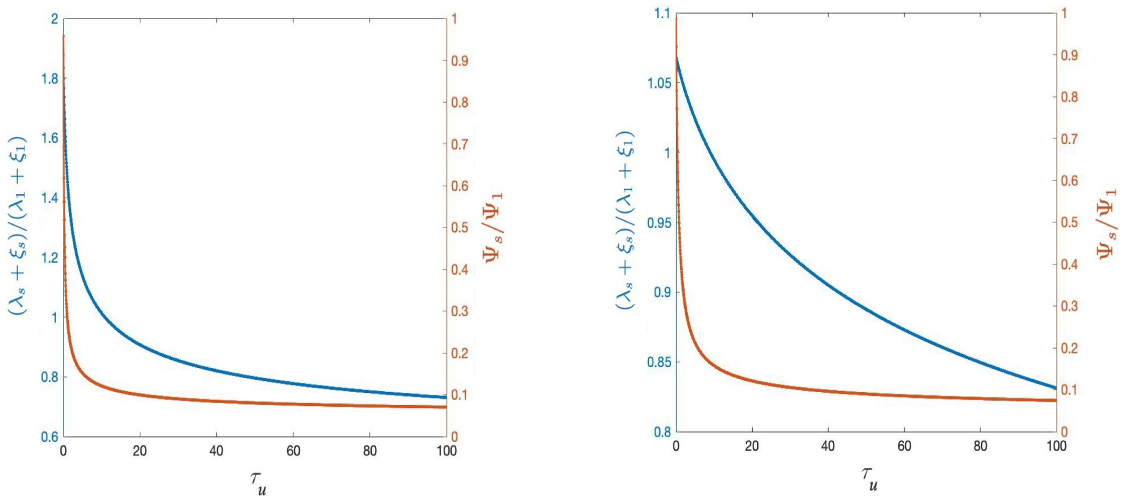


Figure 2: **The impact of noise-trading volume on ratio of market-implied risk aversion and ratio of informational efficiency.** This figure plots how noise-trading volume affects the ratio of market-implied risk aversion $(\lambda_s + \xi_s)/(\lambda_1 + \xi_1)$ and the ratio of informational efficiency Ψ_s/Ψ_1. Each panel contains two y-axes, where the left y-axis (shown in blue) represents the ratio of market-implied risk aversion and the right y-axis (shown in orange) represents the ratio of informational efficiency. The x-axis represents the value of τ_u from 0.01 to 100. In the left panel, $\tau_\theta = 25$ and $\tau_\epsilon = 5$, whereas, in the right panel, $\tau_\theta = 100$ and $\tau_\epsilon = 1$. The other parameters are $\rho = 2$ and $n = 10$.

While Proposition 3 shows that the institutional investor can beat sophisticated individual investors for two extreme settings of sufficiently large and small noise trading, Figure 1 shows that the results in Proposition 3 also hold for intermediate values of τ_u. That is, the information advantage of the institutional investor always dominates the risk effect so that the institutional investor can always beat sophisticated individual investors. We further decompose the two

[19]Proposition 7.2 in Kyle (1989) indicates that the price in his model never reveal more than one-half the private precision of informed speculators.

components, i.e., the information and risk effect in Figure 2. First, as shown by Figure 2, the relative risk effect of the institutional investor over sophisticated individual investors increases with the precision of noise trading τ_u. When noise-trading volume is large (small) (i.e., τ_u is small (large)), the institutional investor has a relatively weaker (stronger) negative risk effect than those sophisticated individual investors. Second, due to information advantage, the institutional investor always has higher information effect as indicated by the fact that the value of Ψ_s/Ψ_1 is always lower than 1. Third, as τ_u increases, on the one hand, sophisticated individual investors infer more information from price ($\mathrm{Var}[\theta|y_1, y_i, p]$ decreases); on the other hand, the variance of the asset return decreases as well ($\mathrm{Var}(\theta - p)$ decreases). Hence, the relative strength of institutional investor's information effect further amplifies, which renders institutional investor always beat sophisticated individual investors.

5 Naive individual investors

Applying $m = 0$, this section investigates whether the institutional investor can beat naive individual investors who are unaware of their price impact and consider themselves price-takers.

5.1 Equilibrium and expected trading profits

We first establish a linear Bayesian Nash equilibrium and then define expected trading profits.

PROPOSITION 4. *There exists a linear Bayesian Nash equilibrium determined by the following system of equations*

$$\gamma = n \left[\frac{1}{\lambda_1 + \xi_1} + \frac{(n-1)(1-\beta^n)}{\xi_n} \right]^{-1}, \tag{31}$$

$$\pi_1 = \frac{\gamma}{n} \left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1 + \xi_1)} + \frac{(n-1)\alpha_1^n}{\xi_n} \right], \tag{32}$$

$$\pi_n = \frac{\gamma}{n} \left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1 + \xi_1)} + \frac{\alpha_o^n}{\xi_n} \right], \tag{33}$$

$$\lambda_1 = \left[\frac{(n-1)(1-\beta^n)}{\xi_n} \right]^{-1}, \tag{34}$$

where

$$\alpha_1^n = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_n^2}{\tau_u}}\frac{\pi_1}{\pi_n}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}},$$

$$\alpha_o^n = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_n^2}{\tau_u}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}}, \tag{35}$$

$$\beta^n = \frac{\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}\frac{1}{(n-2)\pi_n}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}}, \tag{36}$$

$$z_n = \frac{\gamma}{(n-2)\pi_n}.$$

The equilibrium system (31)-(34) can be directly obtained from the system (13)-(18) by setting $m = 0$ and removing the equation (15) which corresponds to sophisticated individual investors. The (expected) trading profits of naive individual investors are given by

$$\Pi_n := \mathbb{E}[(\theta - p)x_j^*] = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_j, p]}{\xi_n}, j = 2, ..., n. \tag{37}$$

We also define informational efficiency for predicting the asset return $\theta - p$ for naive individual investors:

$$\Psi_n := \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_j, p]}{\mathrm{Var}(\theta - p)}, j = 2, ..., n. \tag{38}$$

Similar to Equations (26) and (27), naive individual investor's trading profits are determined by both the information effect, described by information efficiency, and risk effect, which is however characterized only by the risk-aversion-adjusted conditional variance about the fundamental. From (26), (28), (37) and (38), we can see that the institutional investor can beat naive individual investors, i.e., $\Pi_1 > \Pi_n$, if and only if $\frac{\Psi_1}{\lambda_1+\xi_1} > \frac{\Psi_n}{\xi_n}$, or equivalently, $\frac{\xi_n}{\lambda_1+\xi_1} > \frac{\Psi_n}{\Psi_1}$. Proposition 5 characterizes the condition under which the institutional investor underperforms the naive individual investors.

PROPOSITION **5.** *Suppose that τ_u is sufficiently small, the institutional investor underperforms the naive individual investors if $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$.*

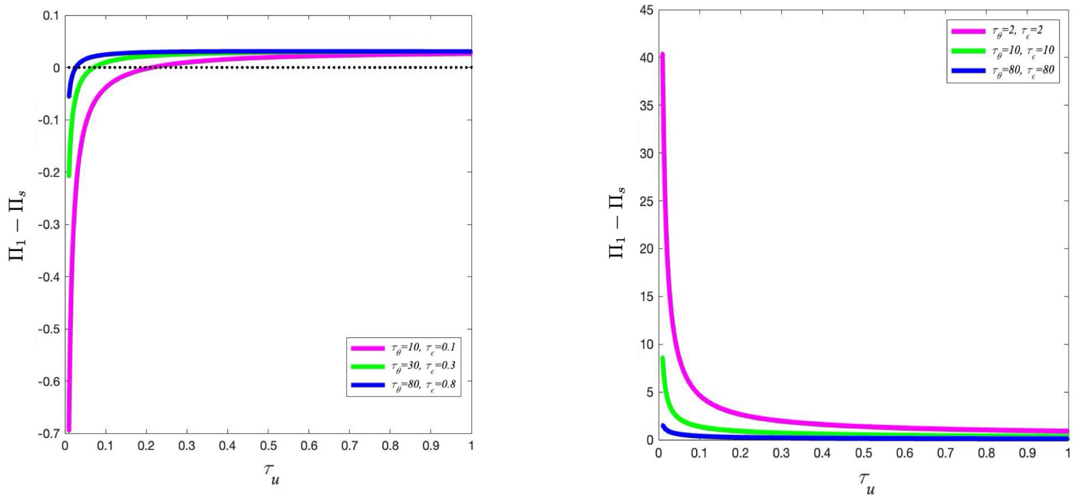


Figure 3: **The effect of noise-trading volume on difference between trading profits**: This figure plots how noise-trading volume affects the difference between trading profits of the institutional investor Π_1 and the naive individual investors Π_s. The left panel shows the results of three sets of parameters $\{\tau_\theta, \tau_\epsilon\}$ that satisfy the condition of the institutional investor underperforming naive individual investors in Proposition 5. The right panel displays the results of three other sets of parameters $\{\tau_\theta, \tau_\epsilon\}$ that do not satisfy the condition in Proposition 5. Here the risk aversion parameter is $\rho = 2$ and the total number of investors in the market is $n = 10$.

The institutional investor would underperform naive individual investors even though he would always beat sophisticated individual investors. This occurs when both the noise-trading volume and noise-to-signal ratio for information are sufficiently large. Intuitively, the institutional investor underperforms when information effect is weak and risk effect is significant. When noise-trading volume is sufficiently large (i.e., τ_u is sufficiently small), the institutional investor's information effect diminishes. This is because both the information efficiency Ψ_1 and Ψ_s approaching to one as indicated by Equations (28) and (29).[20] Furthermore, when noise-to-signal ratio for information is sufficiently large, the naive individual investors obtain imprecise information. The increased payoff uncertainty resulting from a less precise private signal decreases the price sensitivity (as well as the signal sensitivity) of naive individual investors' demands, and this decreased price sensitivity raises the price impact of the institutional investor. Hence, under this scenario, the risk effect is more significant for the institutional investor. We next provide a benchmark for better understanding on Proposition 5.

[20] This is because $\text{Var}(\theta - p) \to \infty$ as $\tau_u \to 0$, and the conditional variance $\text{Var}[\theta|y_1, y_2, ..., y_n]$ and $\text{Var}[\theta|y_1, y_i, p]$ are bounded above by $\text{Var}(\theta)$.

LEMMA **1.** *When 1) neither the institutional investors nor naive investors learn from prices (corresponding to sufficiently large noise-trading volume), and 2) each investor makes decision based only on the prior information about fundamental* $\theta \sim \mathcal{N}(0, 1/\tau_\theta)$ *(corresponding to sufficiently large noise-to-signal ratio for information), there exits a linear Bayesian Nash equilibrium with equilibrium price form:*

$$p = \gamma u,$$

where

$$\gamma = n\left(\frac{k}{\lambda_1 + \xi_1} + \frac{n-k}{\xi_n}\right)^{-1}, \quad \lambda_1 = \left(\frac{k-1}{\lambda_1 + \xi_1} + \frac{n-k}{\xi_n}\right)^{-1}, \quad \xi_1 = \xi_n = \frac{\rho}{\tau_\theta},$$

k *(*$1 \leq k < n$*) denotes the number of institutional investors (investors that take their price impact into consideration when making decisions) and* n *is the total amount of investors in the market. Moreover,*

$$\mathbb{E}[(\theta - p)x_l^*] = \frac{\gamma^2}{(\lambda_1 + \xi_1)\tau_u} < \mathbb{E}[(\theta - p)x_j^*] = \frac{\gamma^2}{\xi_n \tau_u},$$

where x_l^* *and* x_j^* *denotes the equilibrium demand of institutioanl and naive investors respectively. That is, the institutional investors always underperform the naive individual investors.*

The benchmark in Lemma 1 offers a clear intuition for the results. Assume that all investors are naive and equally make decisions based on prior information. Now, an investor becomes sophisticated and takes his price impact into consideration. This, in turn, enhances the benefit not only for the institutional investor but also for other naive individual investors. However, since the naive individual investors have more aggressive trades, the benefits are higher for them, and consequently, the institutional investor underperforms naive individual investors.

OBSERVATION **2.** *The institutional investor outperforms the naive individual investors when* τ_u *or* $\tau_\epsilon/\tau_\theta$ *is relatively large.*

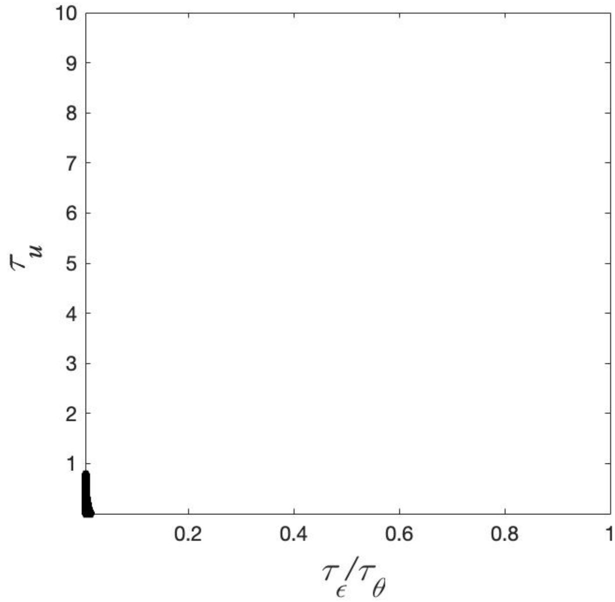


Figure 4: This figure presents the parameter regions (consisting of $\tau_\theta/\tau_\epsilon$ on the x-axis and τ_u on the y-axis) of whether the institutional investor can beat naive individual investors. The black area indicates the set of parameters under which the institutional investor cannot beat naive individual investors, while the blank area indicates the set of parameters under which the institutional investor can beat naive individual investors.

5.2 Why the institutional investor underperforms?

We first illustrate the intuition in Table 1. We simplify the main model to a two-player game (the institutional investor (I) v.s. the individual investor) with two strategies (aggressive (A) or conservative (C) trading strategies). The individual investor can be either sophisticated (S) or naive (N). The payoff matrix on the left represents the real payoffs recognized by investor I and investor S, while the "fictional" payoff matrix on the right reflects the mistakes made by investor N in estimating payoffs as a result of failing to internalize his price impact.

The payoff matrix demonstrates three characteristics in the main model. First, each investor's trade has a price impact. Strategy A influences the equilibrium price and imposes negative externalities on other investors, as $\pi^S(A,A) < \pi^S(C,A)$ and $\pi^S(A,C) < \pi^S(C,C)$ for investor S and $\pi^I(A,A) < \pi^I(A,C)$ and $\pi^I(C,A) < \pi^I(C,C)$ for investor I, because more trading drives the price up more. Second, the institutional investor has an information advantage over the individual investor. Investor I tends to trade more aggressively when investor S trades conservatively, i.e., $\pi^I(A,C) > \pi^I(C,C)$. Conversely, without an information advantage, investor S stays conservative even investor I trades conservatively, i.e., $\pi^S(C,A) < \pi^S(C,C)$.

Combining previous two characteristics, when one of the investors has imposed an aggressive trading strategy already, following up with aggressive trading is not optimal even if the investor has an information advantage (that is, $\pi^I(A,A) < \pi^I(C,A)$ and $\pi^S(A,A) < \pi^S(A,C)$), which will impose too much impact on equilibrium price. Third, the naive individual investor is irrational. Investor N does not internalize the impact of his own trading on the market. This leads to a belief that aggressive trading always yields higher payoffs, regardless of the trading behavior of the other player (investor N believes $\pi^N(A,A) > \pi^N(A,C)$ and $\pi^N(C,A) > \pi^N(C,C)$).

The irrationality of the naive individual investors serves as a commitment mechanism, which renders the institutional investor to shrink his trading aggressiveness. As shown by the left table, when individual investors recognize their price impact, they would know more aggressive trading hurts their profits, especially they do not have information advantage. Hence, the sophisticated individual investors would trade the asset conservatively. Facing this situation, the institutional investor, who obtains information advantage, could explore the profits through aggressive trading. Therefore, the institutional investor has a higher expected trading profit than sophisticated individual investors.

However, when naive individual investors consider themselves to be price-takers, they believe their trading do not impact equilibrium price. Hence, they tend to buy more when receiving good signals and vice versa. This is depicted by the "fictional" payoff matrix in the right table. The institutional investor, who awares the irrationality of naive individual investors, would expect the naive individual investors to trade very aggressively. This, in turn, forces the institutional investor to shrink his trading aggressiveness even though he has information advantage. This is because the price impact is significant now due to aggressive trading by all naive individual investors. The expansion of trading due to more accurate information cannot compensate the decreasing share profits due to sensitive price movement.

Mathematically, we start with the equilibrium described in Proposition 4. Now, assume there is only one naive individual investor who awares his price impact and switches to be a sophisticated individual investor. Fixing other investors' demand schedules and the equilibrium price form, his equilibrium price impact should be

$$\lambda_s = \left[\frac{(n-2)(1-\beta^n)}{\xi_n} + \frac{1}{\lambda_1 + \xi_1}\right]^{-1}.$$

Therefore, the market-implied risk switches from ξ_n to $\xi_n + \lambda_s$. This means that, considering his price impact, the sophisticated individual investor shrinks his demand. In this sense, the irrationality of naive individual investors serves as a commitment mechanism for them to impound aggressive trades into market.[21]

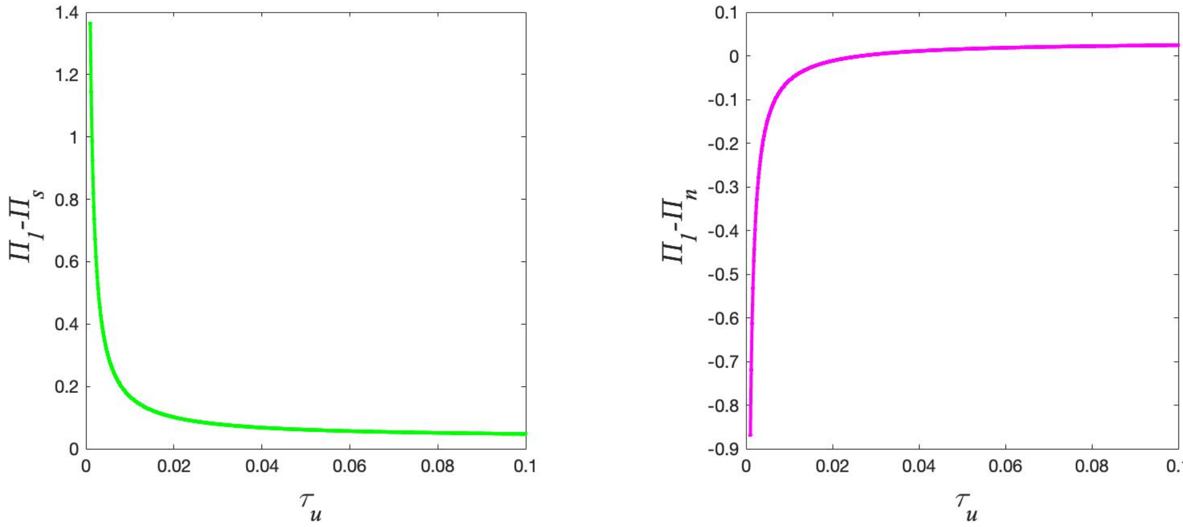
5.3 Generalized equilibrium: Impact of increasing sophistication

This subsection numerically explores the situation when all three types of investors trade at the market (as in Proposition 1) and examines the impact of increasing sophistication of individual investors as indicated by higher m, i.e., more naive individual investors switch to sophisticated individual investors. We have two main findings: i) the main results in Subsection 4.3 (Propositions 3) and Subsection 5.1 (Proposition 5) remain robust; ii) naive individual investors have negative externalities on other market participants.

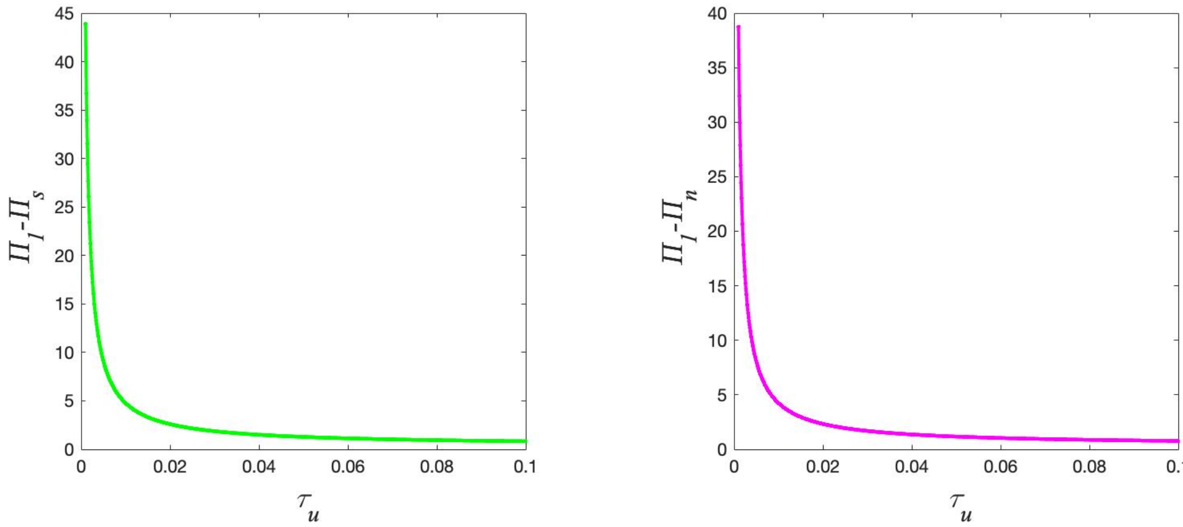
First, Figure 5 shows the effect of τ_u on the trading profits when either $\tau_\epsilon/\tau_\theta$ is sufficiently small (Panel a) or relatively moderate (Panel b). From Figure 5, we find that the conclusions of Proposition 3 and Proposition 5 also apply to the model containing three types of investors at the same time. On the one hand, the institutional investor's trading profit is always higher than that of the sophisticated individual investors as indicated by left figures in Panel a and Panel b. On the other hand, when τ_u is sufficiently small, the naive individual investor's trading profits are higher (lower) than that of the institutional investor when condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ holds (fails) as indicated by the right figure in Panel a (Panel b). Hence, the main results in Subsection 4.3 and Subsection 5.1 remain robust.

Second, when the naive individual investors consider themselves to be price-takers, they believe that their trading does not impact the equilibrium price. Hence, they tend to buy more when receiving good signals and vice versa. This might make the naive individual investors beat the institutional investor; however, the aggressive trading from naive individual investors' irrational trades also imposes negative effect on all investors. This is reflected by Figure 6 that

[21] We finally go back to the discussion on the conditions in Proposition 5. The advantage of the institutional investor is that he has more information while the disadvantage is that naive individual investors commit to an aggressive trading. On the one hand, when the noise-to-signal ratio for information is sufficiently large, the institutional investor's information advantage is sufficiently small and the information advantage cannot compensate the conservatory trade. On the other hand, when noise-trading volume is sufficiently large, the change in price impact becomes more significant and, hence, the disadvantage also becomes more prominent.



(a) Panel a



(b) Panel b

Figure 5: **The effect of noise-trading volume on difference between trading profits**: The green line represents the difference between trading profits of the institutional investor Π_1 and the sophisticated individual investors Π_s, while the pink line displays the difference between trading profits of the institutional investor Π_1 and the naive individual investors Π_n. The parameter values are $m = 4$, $n = 10$, $\rho = 2$ and τ_u ranges from 0.001 to 0.1. For other parameters, we set $\tau_\theta = 100$ and $\tau_\epsilon = 1$ (satisfying the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5) in Panel a and $\tau_\theta = 25$ and $\tau_\epsilon = 5$ (which are used in Han and Yang (2013) and do not satisfy the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5) in Panel b.

all investors' expected trading profits increase with the increasing sophistication.[22] Furthermore, the number of strategic individual investors m can only affect the value of the trading profits of the three types of investors, but not the ordering of the three. In other words, our conclusions about the ordering of trading profits of the three types of investors discussed earlier is robust with respect to m.

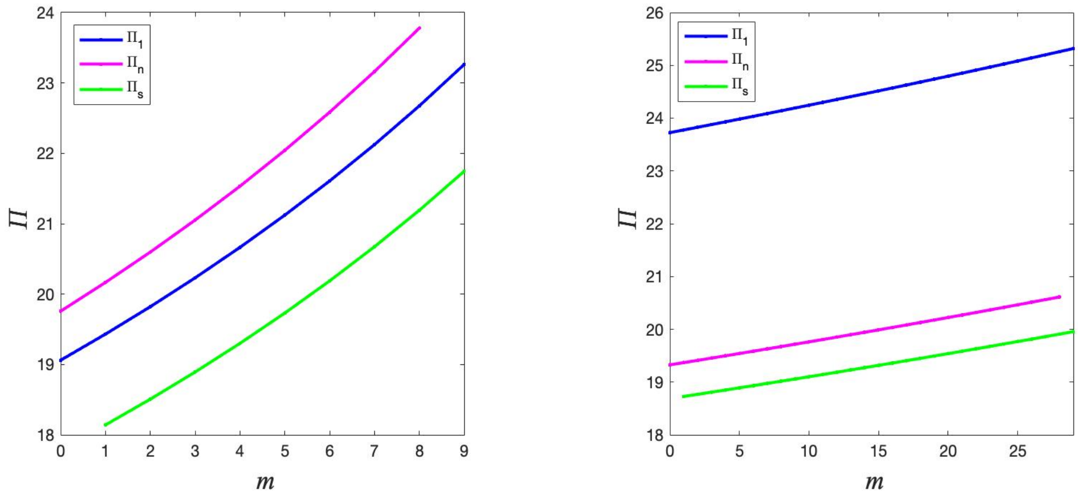


Figure 6: **The effect of the level of sophistication on trading profits**: This figure plots how the sophistication level m affects the trading profits of the institutional investor Π_1, the sophisticated individual investors Π_s, and the naive individual investors Π_n. We set $n = 10$ in left while $n = 30$ in right. The other parameter values are $\tau_\theta = 100$, $\tau_u = 0.001$, $\tau_\epsilon = 1$ and $\rho = 2$. The parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ holds in left and fails in right.

6 Conclusions

We explore a noisy imperfectly competitive market where an institutional investor possesses all the information in the market while individual investors hold only a single piece of asymmetric information. Among these individual investors, some internalize their price impact and are classified as "sophisticated," whereas others view themselves as price-takers and are labeled "naive." We identify the conditions under which the institutional investor can/cannot outperform individual investors in terms of trading profits. Our findings demonstrate that

[22] Figure 6 shows how the trading profits of the three types of investors are affected by the level of sophistication m in the market. Here, we both consider the cases of $n = 10$ and $n = 30$, respectively (as $n = 10$, m takes values from 0 to 9, and as $n = 30$, m takes values from 0 to 29). In Appendix B3, we do more simulations to illustrate that our conclusions are robust.

the institutional investor consistently outperforms sophisticated individual investors across all market conditions. However, when the noise-trading volume and noise-to-signal ratio reach sufficiently high levels, the institutional investor is unable to outperform naive individual investors. This is because naive individual investors, who disregard their price impact, tend to trade aggressively, forcing the institutional investor to curtail his own trading aggressiveness despite holding an information advantage.

Appendix A: Proofs

A1. Proof of Proposition 1

Note that while $\xi_1 = \rho\,\mathrm{Var}[\theta|y_1, y_2, ..., y_n, p] = \rho\,\mathrm{Var}[\theta|y_1, y_2, ..., y_n]$ is a constant depending only on exogenous parameters (see Equation (10)), ξ_s also depends on the endogenous parameter z_s (see Equation (11) and (6) by setting $m = n-1$). To highlight the dependence, in the following proof sometimes we will write $\xi_s = \xi_s(z_s)$.

From (22) and (23),

$$\frac{1}{\lambda_s} = \frac{(n-2)(1-\beta^s)}{\lambda_s + \xi_s} + \frac{1}{\left[\frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}\right]^{-1} + \xi_1},$$

which is equivalent to

$$\begin{aligned}
&[(2n-3)(1-\beta^s)-1]\,\lambda_s^2 \\
&\quad + [(n-2)(1-\beta^s)[(n-1)(1-\beta^s)\xi_1+\xi_s] + (n-1)(1-\beta^s)(\xi_s-\xi_1) - 2\xi_s]\,\lambda_s \\
&\quad - \left[(n-1)(1-\beta^s)\xi_s\xi_1 + \xi_s^2\right] = 0. \qquad (39)
\end{aligned}$$

Note that the discriminant of the quadratic equation (39) is non-negative:

$$\begin{aligned}
&[(n-2)(1-\beta^s)((n-1)(1-\beta^s)\xi_1+\xi_s) + (n-1)(1-\beta^s)(\xi_s-\xi_1) - 2\xi_s]^2 \\
&\qquad + 4\,[(2n-3)(1-\beta^s)-1]\left[(n-1)(1-\beta^s)\xi_s\xi_1+\xi_s^2\right] \\
&= (n-2)^2(1-\beta^s)^2[(n-1)(1-\beta^s)\xi_1+\xi_s]
\end{aligned}$$

$$+ (n-1)(1-\beta^s)(\xi_s-\xi_1)^2 + 4(n-1)^2(1-\beta^s)^2\xi_s\xi_1 \geq 0.$$

We restrict $\beta^s \in (0,1)$ since otherwise, λ_1 will be negative and not well defined. We first claim that (39) has a positive root λ_s if and only if $(2n-3)(1-\beta^s)-1>0$. If $(2n-3)(1-\beta^s)-1>0$, then (39) must have a unique positive root no matter the coefficient on the term λ_s is positive or negative. However, if $(2n-3)(1-\beta^s)-1 \leq 0$, i.e., $1-\beta^s \leq \frac{1}{2n-3}$, the coefficient of λ_s in (39) equals

$$(n-1)(1-\beta^s)[(n-2)(1-\beta^s)-1]\xi_1 + [(2n-3)(1-\beta^s)-2]\xi_s,$$

which is negative, implying that (39) has no positive root. Denote the threshold value

$$\beta^+ := \frac{2n-4}{2n-3}.$$

Hence, to show the equilibrium existence, it suffices to restrict β^s in the interval $(0,\beta^+)$.

The proof outline is that we first express all the variables λ_1, λ_s, ξ_s, z_s, π_s as functions of the variable β^s, and then substitute them into the equation that β^s satisfies, and finally solve the resulting equation involving only the variable β^s.

At the first step, we first try to solve z_s as a function of the variable β^s. Fix a value $\beta^s \in (0,\beta^+)$. We can first uniquely solve $\lambda_s = \lambda_s(\beta^s; z_s)$ by (39) as discussed above (note that $\lambda_s(\beta^s; z_s)$ depends on z_s since ξ_s does), and then

$$\lambda_1 = \lambda_1(\beta^s; z_s) = \left[\frac{(n-1)(1-\beta^s)}{\lambda_s(\beta^s; z_s)+\xi_s(z_s)}\right]^{-1} \tag{40}$$

by (22). Substituting $\lambda_s = \lambda_s(\beta^s; z_s)$ and $\lambda_1 = \lambda_1(\beta^s; z_s)$ into (19), we get

$$\gamma = \gamma(\beta^s; z_s) = n\left[\frac{1}{\lambda_1(\beta^s; z_s)+\xi_1} + \frac{(n-1)(1-\beta^s)}{\lambda_s(\beta^s; z_s)+\xi_s(z_s)}\right]^{-1}. \tag{41}$$

From (21) and (24), using the definition $z_s = \frac{\gamma}{(n-2)\pi_s}$, we have

$$\frac{n}{n-2} = z_s \left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1(\beta^s; z_s) + \xi_1)} + \frac{\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon} + \frac{(n-2)z_s^2}{\tau_u}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{z_s^2}{\tau_u}}}}{\lambda_s(\beta^s; z_s) + \xi_s(z_s)} \right]. \tag{42}$$

Observe that $\{\lambda_s(\beta^s; z_s)\}$ is uniformly bounded over $z_s \in (0, \infty)$ since (42) depends on z_s only through the terms ξ_1 and ξ_s which satisfy the relation

$$\frac{\rho}{\tau_\theta + n\tau_\epsilon} \leq \xi_s(z_s) \leq \frac{\rho}{\tau_\theta + 2\tau_\epsilon}$$

for any $z_s > 0$, see equation (2) and (11). Consequently, $\{\lambda_1(\beta^s; z_s)\}$ is also uniformly bounded over $z_s \in (0, \infty)$. The uniform boundedness implies that the term on the RHS of (42) tends to infinity as $z_s \to \infty$ and to zero as $z_s \to 0$. Thus, there is a solution, denoted as $z_s(\beta^s)$, which is a function of the variable β^s, to the equation (42). We then immediately get the values of $\lambda_1(\beta^s; z_s(\beta^s))$ and $\lambda_s(\beta^s; z_s(\beta^s))$, and then the value of $\gamma(\beta^s, z_s(\beta^s))$ from (41). Also, we get the value of $\pi_s(\beta^s)$ by the relation $\pi_s(\beta^s) = \frac{\gamma(\beta^s, z_s(\beta^s))}{(n-2)z_s(\beta^s)}$; they are functions of the variable β^s. Then substituting $z_s(\beta^s)$ and $\pi_s(\beta^s)$ into the expression of β^s, we get an equation involving only the variable β^s (as well as other exogenous parameters)

$$\beta^s = \frac{\frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{(z_s(\beta^s))^2}{\tau_u}} \frac{1}{(n-2)\pi_s(\beta^s)}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{(z_s(\beta^s))^2}{\tau_u}}}. \tag{43}$$

To show the equilibrium existence, it suffices to show that the equation (43) has a positive root in $(0, \beta^+)$. The proof outline is to apply the intermediate value theorem by showing that the limit of the term on the RHS is larger (respectively smaller) than that of the term on the LHS as $\beta^s \to 0$ (respectively $\beta^s \to \beta^+$). Let us first consider the limit of $\beta^s \to 0$. In this case, (39) reduces to

$$(2n-4)\lambda_s^2 + [(n-2)[(n-1)\xi_1 + \xi_s] + (n-1)(\xi_s - \xi_1) - 2\xi_s]\,\lambda_s - \left[(n-1)\xi_s\xi_1 + \xi_s^2\right] = 0,$$

which implies that λ_s, and then λ_1, γ, z_s and π_s are bounded and far away from zero, by (22), (19), (42) and the relation $\pi_s = \gamma/(z_s(n-2))$, respectively. This implies that the limit inferior of the term on the RHS of (43) is greater than zero as $\beta^s \to 0$. We now consider the limit of $\beta^s \to \beta^+ = \frac{2n-4}{2n-3}$. In this case, we can first show that $\lambda_s \to \infty$ by contradiction. Otherwise, if $\{\lambda_s\}$ is bounded, then on the one hand, from (39)

$$\lambda_s \to \frac{\frac{n-1}{2n-3}\xi_s\xi_1 + \xi_s^2}{\frac{n-2}{2n-3}\left(\frac{n-1}{2n-3}\xi_1 + \xi_s\right) + \frac{n-1}{2n-3}(\xi_s - \xi_1) - 2\xi_s} < 0.$$

On the other hand, as the positive solution to (39), any limit point of λ_s should be non-negative, leading to a contradiction. Hence, $\lambda_s \to \infty$. We can further show that $\lambda_1 \to \infty$ by (22), $\gamma \to \infty$ by (19), and $z_s \to \infty$ by (42). Moreover, from (21), (19) and (22),

$$\begin{aligned}\pi_s &= \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^s}{\lambda_s+\xi_s}}{\frac{1}{\lambda_1+\xi_1} + \frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}} = \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^s}{\lambda_1(n-1)(1-\beta^s)}}{\frac{1}{\lambda_1+\xi_1} + \frac{1}{\lambda_1}} \\ &= \frac{\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^s}{(n-1)(1-\beta^s)}}{\frac{\lambda_1}{\lambda_1+\xi_1} + 1},\end{aligned} \tag{44}$$

which implies that

$$\pi_s \to \frac{1}{2}\left(\frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon} + \frac{\tau_\epsilon}{(\tau_\theta + 2\tau_\epsilon)(n-1)(1-\beta^s)}\right),$$

where we use the limit $\alpha_o^s \to \frac{\tau_\epsilon}{\tau_\theta+2\tau_\epsilon}$. This implies that the term on the RHS of (43) tends to zero as $\beta^s \to \beta^+$. Thus, according to the intermediate value theorem, (43) has a positive root $\beta^s \in (0, \beta^+)$. By the obtained value of β^s, we now also get the values of λ_s, λ_1, γ, z_s, π_s. Finally, from (20),

$$\pi_1 = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1 + \xi_1)} + \frac{(n-1)\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}\frac{\pi_1}{\pi_s}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}}{\lambda_s + \xi_s}\right],$$

from which we get the value of π_1:

$$\pi_1 = \frac{\frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{(n-1)\tau_\epsilon}{(\lambda_s+\xi_s)\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\right)}\right]}{1+\frac{(n-1)(n-2)z_s}{n}\frac{\frac{\frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}}{\lambda_s+\xi_s}}.$$

The proof is completed. □

A2. Proof of Proposition 2

Proof of (i). From (22) and (23),

$$\lambda_s = \left[\frac{n-2}{n-1}\frac{1}{\lambda_1} + \frac{1}{\lambda_1+\xi_1}\right]^{-1},$$

which implies $\lambda_1 > \lambda_s$ if and only if

$$\frac{n-2}{n-1}\frac{1}{\lambda_1} + \frac{1}{\lambda_1+\xi_1} > \frac{1}{\lambda_1}.$$

The preceding inequality is equivalent to

$$(n-2)\lambda_1 > \xi_1.$$

Observe that $(n-2)\lambda_1 > \xi_1$ if and only if

$$\begin{aligned}
&(n-2)(\lambda_s+\xi_s) > (n-1)(1-\beta^s)\xi_1 \\
\Leftrightarrow&(n-2)(\lambda_s/\xi_s+1) > (n-1)(1-\beta^s)\xi_1/\xi_s \\
\Leftrightarrow&\frac{\lambda_s}{\xi_s} > \frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s} - 1,
\end{aligned}$$

which holds if

$$\frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s} - 1 \le 0,$$

or if $\frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s} - 1 > 0$ and (from (39))

$$
\begin{aligned}
&[(2n-3)(1-\beta^s)-1]\left[\frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s}-1\right]^2 \\
&+\left[(n-2)(1-\beta^s)\left((n-1)(1-\beta^s)\frac{\xi_1}{\xi_s}+1\right)+(n-1)(1-\beta^s)\left(1-\frac{\xi_1}{\xi_s}\right)-2\right]\left[\frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s}-1\right] \\
&-\left[(n-1)(1-\beta^s)\frac{\xi_1}{\xi_s}+1\right] < 0.
\end{aligned}
$$

After some simple but tedious calculations, we can find that the preceding inequality is equivalent to

$$(n-1)^3\xi_1^2(1-\beta^s) - \left[(n-1)\xi_1^2 + (n-2)(n-1)^2\xi_s\xi_1 + (n-2)(n-1)\xi_1^2\right] < 0,$$

i.e.,

$$1-\beta^s < \frac{(n-1)\xi_1^2 + (n-1)(n-2)\xi_s\xi_1}{(n-1)^2\xi_1^2} = \frac{\xi_1+(n-2)\xi_s}{(n-1)\xi_1}.$$

As a summary, we show that $\lambda_1 > \lambda_s$ if and only if

$$1-\beta^s < \frac{\xi_1+(n-2)\xi_s}{(n-1)\xi_1} = \frac{1+(n-2)\xi_s/\xi_1}{n-1}.$$

Note that $\xi_1 \le \xi_s$ (see equations (10) and (11)), and consequently, $\lambda_1 > \lambda_s$.

Proof of (ii). From (22),

$$\lambda_1 + \xi_1 = \frac{\lambda_s + \xi_s}{(n-1)(1-\beta^s)} + \xi_1. \tag{45}$$

This implies that when $(n-1)(1-\beta^s) \le 1$, it holds that $\lambda_1 + \xi_1 \ge \lambda_s + \xi_s$. We next suppose $(n-1)(1-\beta^s) > 1$. Then we see that $\lambda_1 + \xi_1 < \lambda_s + \xi_s$ if and only if

$$\lambda_s + \xi_s > \frac{\xi_1}{1-\frac{1}{(n-1)(1-\beta^s)}}. \tag{46}$$

Note that (39) can be alternatively written as

$$[(2n-3)(1-\beta^s)-1](\lambda_s+\xi_s)^2$$

$$
\begin{aligned}
&- \left[(n-1)(1-\beta^s)\xi_1 + (2n-3)(1-\beta^s)\xi_s - (n-1)(n-2)(1-\beta^s)^2\xi_1\right](\lambda_s + \xi_s) \\
&- (n-1)(n-2)(1-\beta^s)^2\xi_s\xi_1 = 0. \qquad (47)
\end{aligned}
$$

From (47), we see that (46) holds if and only if

$$
\begin{aligned}
&[(2n-3)(1-\beta^s)-1]\left(\frac{\xi_1}{1-\frac{1}{(n-1)(1-\beta^s)}}\right)^2 \\
&- \left[(n-1)(1-\beta^s)\xi_1 + (2n-3)(1-\beta^s)\xi_s - (n-1)(n-2)(1-\beta^s)^2\xi_1\right]\frac{\xi_1}{1-\frac{1}{(n-1)(1-\beta^s)}} \\
&- (n-1)(n-2)(1-\beta^s)^2\xi_s\xi_1 < 0.
\end{aligned}
$$

After some calculations, we can find that the preceding inequality is equivalent to

$$
(n-1)(n-2)(1-\beta^s)^2\xi_1 - \left[(n-1)(n-2)(1-\beta^s)^2 - \beta^s\right]\xi_s < 0.
$$

The proof is completed. □

A3. Proof of Proposition 3

Proof of (i). To begin with, from (26), (27), (28) and (29), we see that the institutional investor can beat the sophisticated individual investors if and only if

$$
\frac{\lambda_s + \xi_s}{\lambda_1 + \xi_1} > \frac{\Psi_s}{\Psi_1}. \qquad (48)
$$

From (42),

$$
\begin{aligned}
\frac{n}{n-2} &\le z_s\left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)\xi_1} + \frac{\tau_\epsilon}{(\tau_\theta + 2\tau_\epsilon)\xi_s(z_s)}\right] \\
&\le z_s\left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)\xi_1} + \frac{\tau_\epsilon(\tau_\theta + n\tau_\epsilon)}{(\tau_\theta + 2\tau_\epsilon)\rho}\right], \qquad (49)
\end{aligned}
$$

combines which and expression of α_o^s (see (24)) implies

$$
\alpha_o^s \to \frac{\tau_\epsilon}{\tau_\theta + 2\tau_\epsilon}
$$

as $\tau_u \to 0$. Hence from (21) and (19),

$$\pi_s = \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^s}{\lambda_s+\xi_s}}{\frac{1}{\lambda_1+\xi_1} + \frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}} > \min\left\{\frac{\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}, \frac{\tau_\epsilon}{2(\tau_\theta+2\tau_\epsilon)(n-1)}\right\} > 0$$

for all sufficiently small τ_u. As a result, it follows from the expression of β^s that $\beta^s \to 0$ as $\tau_u \to 0$. Moreover, from (19),

$$\gamma \geq n\left(\frac{1}{\xi_1} + \frac{n-1}{\xi_s}\right)^{-1} \geq n\left[\frac{\tau_\theta+n\tau_\epsilon}{\rho} + \frac{(n-1)(\tau_\theta+n\tau_\epsilon)}{\rho}\right]^{-1}.$$

This implies that $\mathrm{Var}(\theta-p) \geq \gamma^2/\tau_u \to \infty$ as $\tau_u \to 0$. Thus, the term at the RHS of (48) tends to one as $\tau_u \to 0$.

Since we have shown that $\beta^s \to 0$, it follows from (47) and (22) that $\{\lambda_s+\xi_s\}$ and $\{\lambda_1+\xi_1\}$ are bounded and far away from zero. Part (ii) of Proposition 2 tells us that the limit of $(\lambda_s+\xi_s)/(\lambda_1+\xi_1)$ is greater than one as $\frac{1}{\tau_\theta+n\tau_\epsilon} < \frac{1}{\tau_\theta+2\tau_\epsilon}$ by noting that $\xi_s \to \frac{\rho}{\tau_\theta+2\tau_\epsilon}$ as $\tau_u \to 0$ and $\xi_1 = \frac{\rho}{\tau_\theta+n\tau_\epsilon}$. Part (i) thus follows from (48).

Proof of (ii). We consider the term on the LHS of (48). We first show by contradiction that $(2n-3)(1-\beta^s) \to 1$, i.e., $\beta^s \to \beta^+ = \frac{2n-4}{2n-3}$ as $\tau_u \to \infty$. Otherwise, it follows from (39) that $\{\lambda_s\}$ is bounded, and then $\{\lambda_1\}$ is bounded and away from zero by (22). Consequently, $\{z_s\}$ is bounded by (42). As a result, $\alpha_o^s \to 0$, $\beta^s - \frac{\tau_\epsilon}{\pi_s(\tau_\theta+n\tau_\epsilon)} \to 0$ by (24) and (25), so that

$$\beta^s - \frac{\frac{\lambda_1}{\lambda_1+\xi_1}+1}{\frac{\lambda_1}{\lambda_1+\xi_1}} \to 0$$

by (44), arising a contradiction since we have shown that the equilibrium parameter β^s is smaller than one, $\beta^s \in (0, \beta^+)$. Hence, $\beta^s \to \beta^+$ as $\tau_u \to \infty$.

As shown in Proposition 1, we can show by contradiction that $\lambda_s \to \infty$, and $\lambda_1 \to \infty$. Considering these limits, (44), and (43), we can conclude that $(n-2)^2 z_s^2/\tau_u$ converges to a finite positive number, denoted as $\hat{d}$, and $\hat{d}$ satisfies the following equation:

$$\beta^+ = \frac{\frac{1}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}}{\tau_\theta+2\tau_\epsilon+\frac{n-2}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}} \frac{2}{\frac{\tau_\epsilon}{\tau_\theta+n\tau_\epsilon} + \frac{1}{(n-1)(1-\beta^+)}\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}}{\tau_\theta+2\tau_\epsilon+\frac{n-2}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}}},$$

from which and the relation $\beta^+ = \frac{2n-4}{2n-3}$, we get

$$\hat{d} = \frac{1}{\tau_\epsilon} \frac{(n-1)^2(\tau_\theta + n\tau_\epsilon)}{(3n-4)\tau_\theta + (2n^2 - n - 2)\tau_\epsilon}. \tag{50}$$

It follows from (45) that

$$\frac{\lambda_s + \xi_s}{\lambda_1 + \xi_1} = \frac{\lambda_s + \xi_s}{\frac{\lambda_s+\xi_s}{(n-1)(1-\beta^s)} + \xi_1} = \frac{1}{\frac{1}{(n-1)(1-\beta^s)} + \frac{\xi_1}{\lambda_s+\xi_s}} \to (n-1)(1-\beta^+) = \frac{n-1}{2n-3}, \tag{51}$$

since we have shown that $\lambda_s \to \infty$ and $\beta^s \to \beta^+$ as $\tau_u \to \infty$. Hence, given that the limit of the term on the LHS of (48) is a constant independent of any other parameters, we complete the proof by the following two steps.

Step one: Here we show that the limit of the term on the RHS of (48) as $\tau_u \to \infty$ strictly decreases with τ_θ. To this end, we first give the limit of π_s and π_1 as $\tau_u \to \infty$. Note that we have shown that $\beta^s \to \beta^+$, $(n-2)^2 z_s^2/\tau_u \to \hat{d}$ and $\lambda_1 \to \infty$. It follows from (44) and (50) that

$$\pi_s = \frac{\frac{\tau_\epsilon \lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon} + \frac{(n-2)z_s^2}{\tau_u}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{z_s^2}{\tau_u}}}}{(n-1)(1-\beta^s)}}{\frac{\lambda_1}{\lambda_1+\xi_1} + 1} \tag{52}$$

$$\to \frac{1}{2}\left[\frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon} + \frac{2n-3}{n-1} \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon} + \frac{\hat{d}}{(n-2)}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{\hat{d}}{(n-2)^2}}}\right]$$

$$=: \hat{\pi}_o < \infty. \tag{53}$$

We also have

$$\pi_1 \overset{(20),(19)}{=} \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{(n-1)\alpha_1^s}{\lambda_s+\xi_s}}{\frac{1}{\lambda_1+\xi_1} + \frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}} \overset{(22)}{=} \frac{\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_1^s}{1-\beta^s}}{\frac{\lambda_1}{\lambda_1+\xi_1} + 1}$$

$$\overset{(24)}{=} \frac{\frac{\tau_\epsilon \lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}\frac{\pi_1}{\pi_s}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}}{1-\beta^s}}{\frac{\lambda_1}{\lambda_1+\xi_1}+1}, \tag{54}$$

which leads to

$$\pi_1 = \frac{\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)\left(\frac{\lambda_1}{\lambda_1+\xi_1}+1\right)} + \frac{\tau_\epsilon}{(1-\beta^s)\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\right)\left(\frac{\lambda_1}{\lambda_1+\xi_1}+1\right)}}{\frac{1}{\pi_s\left(\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}\right)(1-\beta^s)\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\right)\left(\frac{\lambda_1}{\lambda_1+\xi_1}+1\right)}+1} \tag{55}$$

$$= \frac{\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\right)+\frac{\tau_\epsilon}{1-\beta^s}}{\frac{1}{\pi_s\left(\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}\right)(1-\beta^s)}+\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\right)\left(\frac{\lambda_1}{\lambda_1+\xi_1}+1\right)}$$

$$\to \frac{\frac{\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{\hat{d}}{(n-2)^2}}\right)+\tau_\epsilon(2n-3)}{\frac{2(2n-3)}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}\left[\frac{\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}+\frac{2n-3}{n-1}\frac{\tau_\epsilon-\frac{1}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{\hat{d}}{(n-2)^2}}}\right]^{-1}+2\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{\hat{d}}{(n-2)^2}}\right)}$$

$$=: \hat{\pi}_1 < \infty, \tag{56}$$

where the limit follows from $\beta^s \to \beta^+$, $(n-2)^2 z_s^2/\tau_u \to \hat{d}$, $\lambda_1 \to \infty$, and the limit of π_s given by (53).

Now, we show that the limit of the term on the RHS of (48) as $\tau_u \to \infty$ decreases with τ_θ. Using the limits of π_s and π_1 and $(n-2)^2 z_s^2/\tau_u$ by (53), (56) and (50), we have

$$\begin{aligned}\mathrm{Var}(\theta-p) &= \frac{(\pi_1+(n-1)\pi_s-1)^2}{\tau_\theta}+\frac{\pi_1^2+(n-1)\pi_s^2}{\tau_\epsilon}+\frac{\gamma^2}{\tau_u}\\ &\to \frac{(\hat{\pi}_1+(n-1)\hat{\pi}_s-1)^2}{\tau_\theta}+\frac{\hat{\pi}_1^2+(n-1)\hat{\pi}_s^2}{\tau_\epsilon}+\hat{\pi}_s^2\hat{d}\\ &=: V_1(\tau_\theta),\end{aligned} \tag{57}$$

$$\begin{aligned}\mathrm{Var}[\theta|y_1,y_i,p] &= \frac{1}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}\\ &\to \frac{1}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{\hat{d}}{(n-2)^2}}} =: V_2(\tau_\theta),\end{aligned} \tag{58}$$

$$\mathrm{Var}[\theta|y_1,...,y_n,p]=\mathrm{Var}[\theta|y_1,...,y_n]=\frac{1}{\tau_\theta+n\tau_\epsilon}=:V_3(\tau_\theta), \tag{59}$$

as $\tau_u\to\infty$. Hence, the term on the RHS of (48)

$$\frac{\mathrm{Var}(\theta-p)-\mathrm{Var}[\theta|y_1,y_i,p]}{\mathrm{Var}(\theta-p)-\mathrm{Var}[\theta|y_1,y_2,...,y_n]}\to\frac{V_1(\tau_\theta)-V_2(\tau_\theta)}{V_1(\tau_\theta)-V_3(\tau_\theta)}. \tag{60}$$

Applying (57), (58), (59)and the definition of $\hat{d}$ (see (50)), with some calculations we can see that

$$\frac{\partial\left(\frac{V_1(\tau_\theta)-V_2(\tau_\theta)}{V_1(\tau_\theta)-V_3(\tau_\theta)}\right)}{\partial\tau_\theta}<0$$

is equivalent to

$$-\frac{4\tau_\epsilon(n-1)^3(n-2)^2}{(2n-3)(8\tau_\theta+6\tau_\epsilon-11n\tau_\theta-5n\tau_\epsilon+4n^2\tau_\theta-2n^2\tau_\epsilon+2n^3\tau_\epsilon)^2}<0,$$

which is indeed true. Therefore, we complete the proof of this step.

Step two: Here we show that the limit of the term at the RHS of (48) as $\tau_u\to\infty$ satisfies the relation (48) as $\tau_\theta\to 0$. First, we have

$$\begin{aligned}
&\mathrm{Var}(\theta-p)-\mathrm{Var}[\theta|y_1,y_i,p]=\mathbb{E}[(\theta-p)(\mathbb{E}[\theta|y_1,y_i,p]-p)]\\
&=\mathbb{E}\Bigg\{\Bigg[(1-\pi_1-(n-1)\pi_s)\theta-\pi_1\epsilon_1-\pi_s\sum_{i=2}^{n}\epsilon_i-\gamma u\Bigg]\\
&\qquad\times\Big[(\alpha_o^s+\alpha_1^s-(1-\beta^s)(\pi_1+(n-1)\pi_s))\theta+(\alpha_1^s-(1-\beta^s)\pi_1)\epsilon_1+(\alpha_o^s-(1-\beta^s)\pi_s)\epsilon_i\\
&\qquad\quad-\pi_s(1-\beta^s)\sum_{j\notin\{1,i\}}^{n}\epsilon_j-(1-\beta^s)\gamma u\Big]\Bigg\}\\
&=\left[(1-\pi_1-(n-1)\pi_s)(\alpha_o^s+\alpha_1^s-(1-\beta^s)(\pi_1+(n-1)\pi_s))\right]/\tau_\theta+(1-\beta^s)\gamma^2/\tau_u\\
&\qquad+\left[-\pi_1(\alpha_1^s-(1-\beta^s)\pi_1)-\pi_s(\alpha_o^s-(1-\beta^s)\pi_s)+\pi_s^2(1-\beta^s)(n-2)\right]/\tau_\epsilon,
\end{aligned} \tag{61}$$

and

$$\mathrm{Var}(\theta-p)-\mathrm{Var}[\theta|y_1,y_2,...,y_n]=\mathbb{E}[(\theta-p)(\mathbb{E}[\theta|y_1,...,y_n]-p)]$$

$$
\begin{aligned}
&= \mathbb{E}\Bigg\{ \Bigg[(1-\pi_1-(n-1)\pi_s)\theta - \pi_1\epsilon_1 - \pi_s\sum_{i=2}^{n}\epsilon_i - \gamma u\Bigg] \\
&\qquad \times \Bigg[(nb\tau_\epsilon - (\pi_1+(n-1)\pi_s))\theta + (b\tau_\epsilon - \pi_1)\epsilon_1 + (b\tau_\epsilon - \pi_s)\sum_{i=2}^{n}\epsilon_i - \gamma u\Bigg]\Bigg\} \\
&= \left[(1-\pi_1-(n-1)\pi_s)(nb\tau_\epsilon - (\pi_1+(n-1)\pi_s))\right]/\tau_\theta \\
&\quad - \left[\pi_1(b\tau_\epsilon - \pi_1) + \pi_s(b\tau_\epsilon - \pi_s)(n-1)\right]/\tau_\epsilon + \gamma^2/\tau_u,
\end{aligned} \tag{62}
$$

where $b = \frac{1}{\tau_\theta + n\tau_\epsilon}$. Then, applying (51), (61) and (62),

$$
\frac{\lambda_s+\xi_s}{\lambda_1+\xi_1}\mathbb{E}[(\theta-p)(\mathbb{E}[\theta|y_1,...,y_n,p]-p)] - \mathbb{E}[(\theta-p)(\mathbb{E}[\theta|y_1,y_i,p]-p)]
$$

tends to

$$
\begin{aligned}
&(n-1)(1-\beta^+)\Bigg\{\Big[(1-\hat{\pi}_1-(n-1)\hat{\pi}_s)(nb\tau_\epsilon-(\hat{\pi}_1+(n-1)\hat{\pi}_s))\Big]/\tau_\theta \\
&\qquad - \Big[\hat{\pi}_1(b\tau_\epsilon-\hat{\pi}_1)+\hat{\pi}_s(b\tau_\epsilon-\hat{\pi}_s)(n-1)\Big]/\tau_\epsilon + \hat{d}\hat{\pi}_s^2\Bigg\} \\
&- \Bigg\{\Big[(1-\hat{\pi}_1-(n-1)\hat{\pi}_s)(\hat{\alpha}_o^s+\hat{\alpha}_1^s-(1-\beta^+)(\hat{\pi}_1+(n-1)\hat{\pi}_s))\Big]/\tau_\theta \\
&\quad + \Big[-\hat{\pi}_1(\hat{\alpha}_1^s-(1-\beta^+)\hat{\pi}_1)-\hat{\pi}_s(\hat{\alpha}_o^s-(1-\beta^+)\hat{\pi}_s)+\hat{\pi}_s^2(1-\beta^+)(n-2)\Big]/\tau_\epsilon + (1-\beta^+)\hat{d}\hat{\pi}_s^2\Bigg\} \\
&\propto (1-\hat{\pi}_1-(n-1)\hat{\pi}_s)[n(n-1)(1-\beta^+)b\tau_\epsilon - \hat{\alpha}_1^s - \hat{\alpha}_o^s - (\hat{\pi}_1+(n-1)\hat{\pi}_s)(1-\beta^+)(n-2)]\tau_\epsilon \\
&\quad - [\hat{\pi}_s^2(1-\beta^+)(n-2) + \hat{\pi}_1((n-1)(1-\beta^+)(b\tau_\epsilon-\hat{\pi}_1) - (\hat{\alpha}_1^s-(1-\beta^+)\hat{\pi}_1)) \\
&\qquad + \hat{\pi}_s((n-1)^2(1-\beta^+)(b\tau_\epsilon-\hat{\pi}_s) - (\hat{\alpha}_o^s-(1-\beta^+)\hat{\pi}_s))]\tau_\theta + (1-\beta^+)(n-2)\hat{d}\hat{\pi}_s^2\tau_\epsilon\tau_\theta.
\end{aligned} \tag{63}
$$

We define $\tilde{d}$, $\tilde{\pi}_s$, $\tilde{\pi}_1$, $\tilde{\alpha}_o^s$ and $\tilde{\alpha}_1^s$ as the limit of $\hat{d}$, $\hat{\pi}_s$, $\hat{\pi}_1$, $\hat{\alpha}_0^s$ and $\hat{\alpha}_1^s$ as $\tau_\theta \to 0$. Then, the term in (63) tends to

$$
\begin{aligned}
&\big(n(n-1)(1-\beta^+)b\tau_\epsilon - \tilde{\alpha}_1^s - \tilde{\alpha}_o^s\big) + (\tilde{\pi}_1+(n-1)\tilde{\pi}_s)^2(1-\beta^+)(n-2) \\
&\quad - (\tilde{\pi}_1+(n-1)\tilde{\pi}_s)(n(n-1)(1-\beta^+)b\tau_\epsilon - \tilde{\alpha}_1^s - \tilde{\alpha}_o^s) - (\tilde{\pi}_1+(n-1)\tilde{\pi}_s)(1-\beta^+)(n-2) \\
&= \big(n(n-1)(1-\beta^+)b\tau_\epsilon - \tilde{\alpha}_1^s - \tilde{\alpha}_o^s\big)[1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)] \\
&\quad - (1-\beta^+)(n-2)(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)[1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)].
\end{aligned} \tag{64}
$$

Following from (20), (21), (19) and (22), we have

$$\pi_1 + (n-1)\pi_s = \frac{1}{\frac{\lambda_1}{\lambda_1+\xi_1}+1}\left[\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{(n-1)\alpha_1^s}{(n-1)(1-\beta^s)} + \frac{(n-1)\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{(n-1)\alpha_o^s}{(n-1)(1-\beta^s)}\right],$$

which implies

$$\tilde{\pi}_1 + (n-1)\tilde{\pi}_s = \frac{1}{2}\left(\frac{1}{n} + \frac{\tilde{\alpha}_1^s}{1-\beta^+} + \frac{n-1}{n} + \frac{\tilde{\alpha}_o^s}{1-\beta^+}\right) = \frac{1}{2}\left(1 + \frac{\tilde{\alpha}_o^s + \tilde{\alpha}_1^s}{1-\beta^+}\right).$$

Therefore, (64) can be written as

$$\begin{aligned}
&(1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s))\left[(n-1)(1-\beta^+) - (\tilde{\alpha}_1^s+\tilde{\alpha}_o^s) - \frac{1}{2}(n-2)((1-\beta^+)+(\tilde{\alpha}_1^s+\tilde{\alpha}_o^s))\right] \\
&= (1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s))\left[\frac{n}{2}(1-\beta^+) - \frac{n}{2}(\tilde{\alpha}_1^s+\tilde{\alpha}_o^s)\right] \\
&= n(1-\beta^+)(1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s))^2 \geq 0,
\end{aligned}$$

which implies that the term in (64) is non-negative. This completes the proof. □

A4. Proof of Proposition 4

The proof is similar to that of Proposition 1, and then here we only present the proof outline and omit the detail. To prove the equilibrium existence, we only need to show that the system of equilibrium equations has a positive solution. Similar to (40), (41), (42) and using the relation $z_n = \frac{\gamma}{(n-2)\pi_n}$, we can first express all variables λ_1, γ, z_n and π_n as functions of the variable β^n, and then substitute them into the equation that β^n satisfies (similar to (43)) to obtain a univariate equation of β^n. Next, analyzing the two cases where β^n tends to 0 and β^n tends to 1, respectively, and applying the intermediate value theorem of continuous functions, we can show that the univariate equation of β^n has a positive solution, which in turn gives us the equilibrium values of the other endogenous constants. □

A5. Proof of Proposition 5

Note that the institutional investor cannot beat the naive individual investors if and only if

$$\frac{\xi_n}{\lambda_1+\xi_1} \leq \frac{\Psi_n}{\Psi_1}. \tag{65}$$

Moreover, similar to the arguments given in the proof of Proposition 3, we can show that $\alpha_o^n \to \frac{\tau_\epsilon}{\tau_\theta+2\tau_\epsilon}$ and $\beta^n \to 0$ (see (35) and (36)). Hence, from (33) and (31), we have

$$\pi_n = \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^n}{\xi_n}}{\frac{1}{\lambda_1+\xi_1} + \frac{(n-1)(1-\beta^n)}{\xi_n}} > \min\left\{\frac{\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}, \frac{\tau_\epsilon}{2(\tau_\theta+2\tau_\epsilon)(n-1)}\right\} > 0$$

for all sufficiently small τ_u. As a result, we have that $\beta^n \to 0$ as $\tau_u \to 0$. Moreover, from (31),

$$\gamma \geq n\left(\frac{1}{\xi_1} + \frac{n-1}{\xi_n}\right)^{-1} \geq n\left[\frac{\tau_\theta+n\tau_\epsilon}{\rho} + \frac{(n-1)(\tau_\theta+n\tau_\epsilon)}{\rho}\right]^{-1},$$

as $\tau_u \to 0$. Consequently, Ψ_n and Ψ_1 tend to one as $\tau_u \to 0$.

In terms of the term at the LHS of (65), we have

$$\frac{\xi_n}{\lambda_1+\xi_1} = \frac{\xi_n}{\frac{\xi_n}{(n-1)(1-\beta^n)} + \xi_1} = \frac{1}{\frac{1}{(n-1)(1-\beta^n)} + \frac{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}}{\tau_\theta+n\tau_\epsilon}} \to \frac{1}{\frac{1}{n-1} + \frac{\tau_\theta+2\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}},$$

where the first equality follows from (34), and the limit follows from $\xi_n \to \frac{\rho}{\tau_\theta+2\tau_\epsilon}$, note that similar to (49) $z_n \not\to 0$ as $\tau_u \to 0$, and $\xi_1 = \frac{\rho}{\tau_\theta+n\tau_\epsilon}$. Thus, $\frac{\xi_n}{\lambda_1+\xi_1}$ is smaller than one if

$$\frac{1}{n-1} + \frac{\tau_\theta+2\tau_\epsilon}{\tau_\theta+n\tau_\epsilon} > 1,$$

which is equivalent to

$$(n^2-4n+2)\tau_\epsilon/\tau_\theta < 1.$$

The conclusion follows immediately. This completes the proof. □

A6. Proof of Lemma 1

Since each investor only has prior information about fundamental and does not learn from price, the optimal demands of institutional investor $l = 1, ..., k$ and naive individual investor $j = k+1, ..., n$ are respectively given by

$$x_l^* = \frac{\mathbb{E}(\theta) - p}{\lambda_1 + \xi_1} = -\frac{p}{\lambda_1 + \xi_1}, \tag{66}$$

$$x_j^* = \frac{\mathbb{E}(\theta) - p}{\xi_n} = -\frac{p}{\xi_n}, \tag{67}$$

where $\xi_1 = \xi_n = \rho \operatorname{Var}(\theta) = \rho/\tau_\theta$. Following from (66) and (67), the market clearing condition $\sum_{l=1}^{k} x_l^* + \sum_{j=k+1}^{n} x_j^* + nu = 0$ can be reduced to

$$-\frac{kp}{\lambda_1 + \xi_1} - \frac{(n-k)p}{\xi_n} + nu = 0,$$

which indicates

$$p = n\left(\frac{k}{\lambda_1 + \xi_1} + \frac{n-k}{\xi_n}\right)^{-1} u =: \gamma u.$$

Furthermore, the price impact parameter satisfies

$$\lambda_1 = \left(\frac{k-1}{\lambda_1 + \xi_1} + \frac{n-k}{\xi_n}\right)^{-1}.$$

Then, the expected trading profits of institutional investor $l = 1, ..., k$ are given by

$$\mathbb{E}[(\theta - p)x_l^*] \overset{(66)}{=} \frac{\mathbb{E}[(\theta - p)(\mathbb{E}(\theta) - p)]}{\lambda_1 + \xi_1} = \frac{\mathbb{E}[-(\theta - \gamma u)\gamma u]}{\lambda_1 + \xi_1} = \frac{\gamma^2}{(\lambda_1 + \xi_1)\tau_u},$$

and the expected trading profits of naive individual investor $j = k+1, ..., n$ are given by

$$\mathbb{E}[(\theta - p)x_j^*] \overset{(67)}{=} \frac{\mathbb{E}[(\theta - p)(\mathbb{E}(\theta) - p)]}{\xi_n} = \frac{\mathbb{E}[-(\theta - \gamma u)\gamma u]}{\xi_n} = \frac{\gamma^2}{\xi_n \tau_u}.$$

The completes the proof. □

Appendix B: Further Discussion

B1. Certainty equivalents

In the main content, we compare the trading profits of institutional investor with those of sophisticated or naive individual investors. Here, we provide further discussion about certainty equivalent since it can be treated as a risk-adjusted measure of investor's performance. The certainty equivalent of one strategic investor is given by

$$\begin{aligned} CE &:= -\frac{1}{\rho}\log\left(-\mathbb{E}[-\exp\{-\rho x(\theta-p)\}]\right) \\ &= -\frac{1}{\rho}\log\left(-\mathbb{E}[\mathbb{E}(-\exp\{-\rho x(\theta-p)\}|\mathcal{F})]\right), \end{aligned} \tag{68}$$

where $\mathcal{F}$ denotes the information set of the investor including the price p, $x = \frac{\mathbb{E}[\theta|\mathcal{F}]-p}{\lambda+\xi}$ is the optimal equilibrium demand, $\xi = \rho\,\mathrm{Var}[\theta|\mathcal{F}]$ is the risk-aversion-adjusted conditional variance, and the second equality follows from the law of total expectation. Then it follows from (68) that

$$\begin{aligned} CE &= -\frac{1}{\rho}\log\left(-\mathbb{E}\left[\mathbb{E}\left(-\exp\left\{-\rho\frac{(\theta-p)\mathbb{E}(\theta-p|\mathcal{F})}{\lambda+\xi}\right\}\Big|\mathcal{F}\right)\right]\right) \\ &= -\frac{1}{\rho}\log\left(-\mathbb{E}\left[-\exp\left\{-\rho\frac{\mathbb{E}^2(\theta-p|\mathcal{F})}{\lambda+\xi}+\frac{\rho^2}{2}\frac{\mathbb{E}^2(\theta-p|\mathcal{F})}{(\lambda+\xi)^2}\mathrm{Var}(\theta-p|\mathcal{F})\right\}\right]\right) \\ &= -\frac{1}{\rho}\log\left(-\mathbb{E}\left[-\exp\left\{-\rho\frac{\lambda+\xi/2}{(\lambda+\xi)^2}\mathbb{E}^2(\theta-p|\mathcal{F})\right\}\right]\right) \\ &= -\frac{1}{\rho}\log\left(\frac{1}{\sqrt{1+2\rho\frac{\lambda+\xi/2}{(\lambda+\xi)^2}\mathrm{Var}[\mathbb{E}(\theta-p|\mathcal{F})]}}\right) \\ &= -\frac{1}{\rho}\log\left(\frac{1}{\sqrt{1+2\rho\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta-p)-\mathrm{Var}(\theta|\mathcal{F})]}}\right) \\ &= \frac{1}{2\rho}\log\left(1+2\rho\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta-p)-\mathrm{Var}(\theta|\mathcal{F})]\right), \end{aligned}$$

where the penultimate equality follows from the law of total variance. We can similarly get the certainty equivalent of one price-taking investor by setting $\lambda = 0$ directly. We see that CE is a monotonic transformation of $\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta-p)-\mathrm{Var}(\theta|\mathcal{F})]$, and consequently, it is reasonable to compare the measure $\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta - p) - \mathrm{Var}(\theta|\mathcal{F})]$ for different investors. In fact, we numerically find that all results in the paper also qualitatively hold under the new risk-adjusted measure $\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta - p) - \mathrm{Var}(\theta|\mathcal{F})]$.

B2. Notes for Figure 2

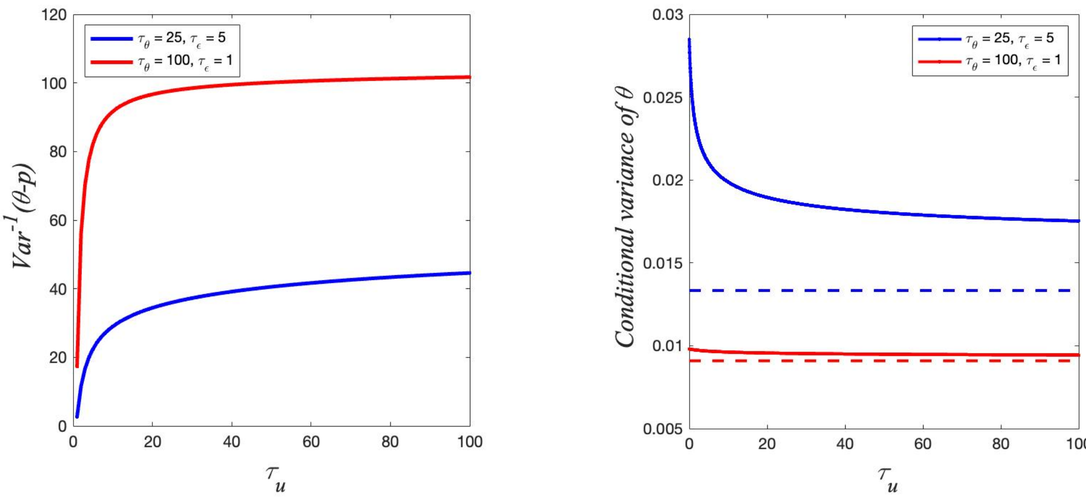
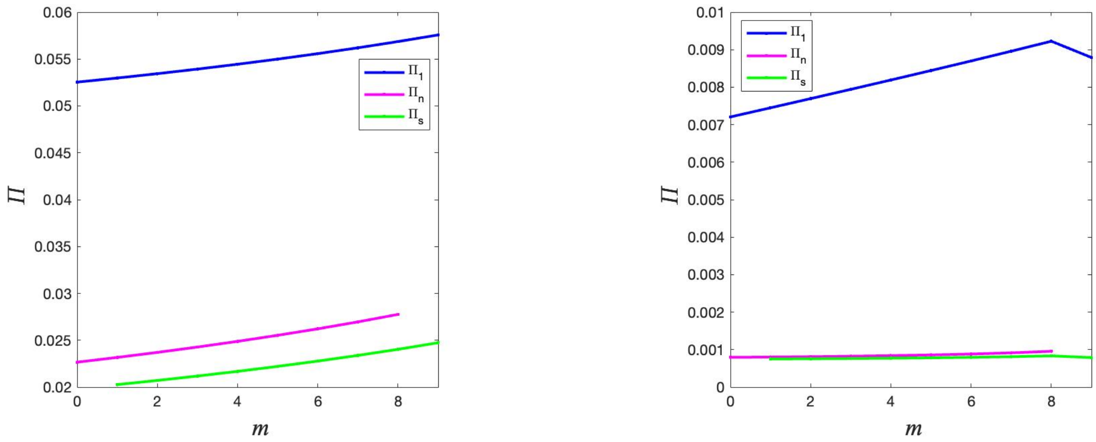


Figure 7: **The impact of noise-trading volume on ratio of $1/\,\mathrm{Var}\,(\theta - p)$ and the conditional variance of θ** . The left panel plots how noise-trading volume τ_u (ranging from 0.01 to 100) affects $1/\,\mathrm{Var}(\theta - p)$, while solid lines in right panel display how it affects investor i's conditional variance $\mathrm{Var}[\theta|y_1, y_i, p]$ and dotted lines represent investor 1's conditional variance $\mathrm{Var}[\theta|y_1, y_2, ..., y_n]$. In each panel, red lines represent the results under the setting that $\tau_\theta = 25$ and $\tau_\epsilon = 5$, and blue lines refer to the results when setting $\tau_\theta = 100$ and $\tau_\epsilon = 1$. The other parameters are $\rho = 2$ and $n = 10$.

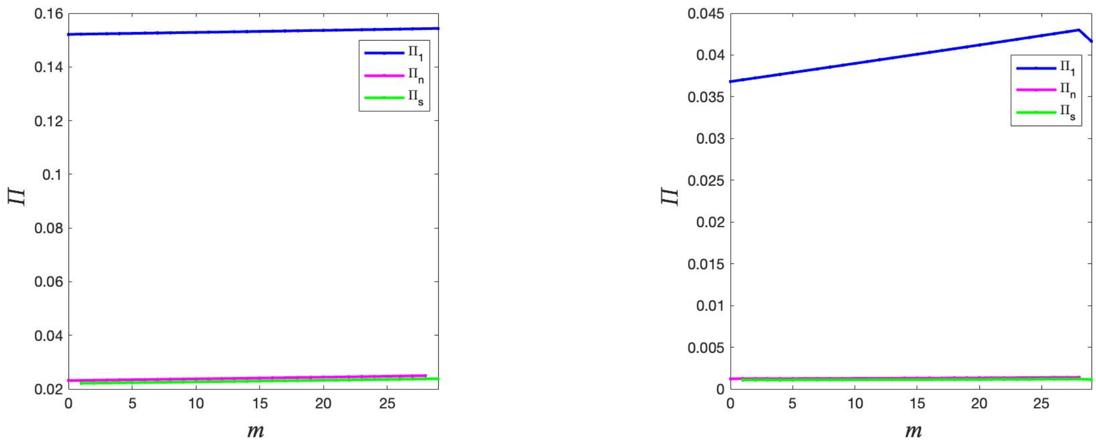
In Figure 2, due to the information advantage, the institutional investor always has a higher information effect as indicated by the fact that the value of Ψ_s/Ψ_1 is always lower than 1. However, the relative strength of institutional investor's information effect amplifies as τ_u increases. Intuitively, as τ_u increases, price information becomes more informative. However, here the information effect is characterized as information efficiency. With relatively low τ_u, the price becomes too noisy. To sum up, as shown in Figure 7, $\mathrm{Var}(\theta - p) \to \infty$ and $\mathrm{Var}[\theta|y_1, y_i, p]$ is bounded as $\tau_u \to 0$. Given that $\mathrm{Var}[\theta|y_1, y_2, ..., y_n]$ is a constant, both $\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta|y_1, y_2, ..., y_n]$ and $\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta|y_1, y_i, p]$ are dominated by $\mathrm{Var}(\theta - p)$ as $\tau_u \to 0$, and thus $\Psi_s/\Psi_1 \to 1$. For the case $\tau_u \to \infty$, we know that $\mathrm{Var}(\theta - p)$, $\mathrm{Var}[\theta|y_1, y_2, ..., y_n]$ and $\mathrm{Var}[\theta|y_1, y_i, p]$ are bounded, and note $\mathrm{Var}[\theta|y_1, y_2, ..., y_n] < \mathrm{Var}[\theta|y_1, y_i, p]$, we can see that Ψ_s/Ψ_1 is bounded above by one.

B3. Robustness check

Here we show the robustness of the conclusions in the main body. From Figures 8 and 9, we can see that our conclusions about the ordering of trading profits of the three types of investors discussed earlier is robust with respect to m in the general model with the coexistence of three types of investors.



(a) Panel a

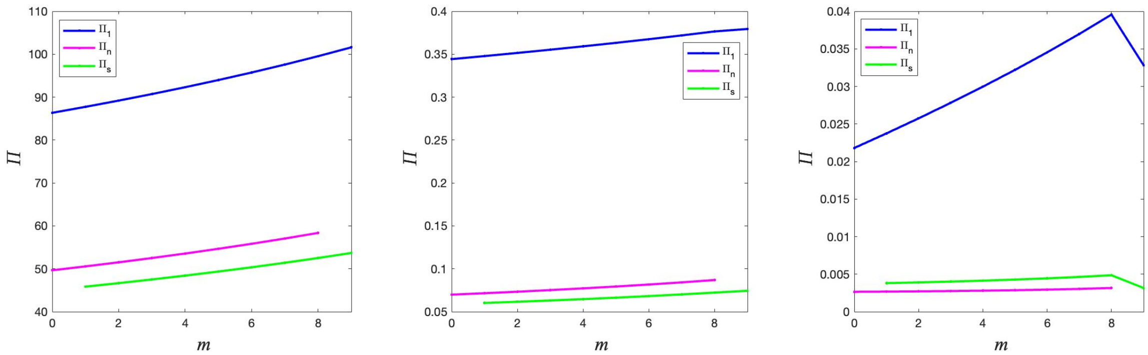


(b) Panel b

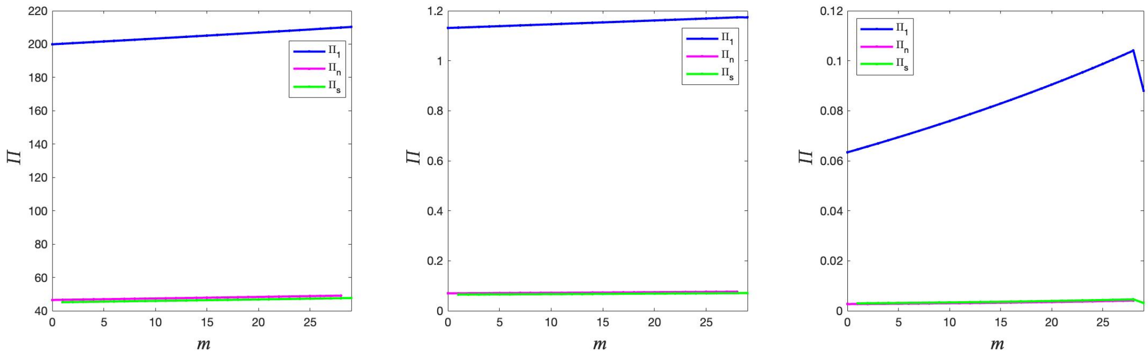
Figure 8: Panel a show the effect of m on the trading profits of the three types of investors when $n = 10$ and the value of τ_u from left to right are 1 and 100. Panel b display the effect of m on the trading profits of the three types of investors when $n = 30$ and the values of τ_u from left to right are 1 and 100. The other parameter values are $\tau_\theta = 100, \tau_\epsilon = 1$ and $\rho = 2$.

Figures 8 and 9 both show how the trading profits of the three types of investors are affected by the level of sophistication m in the market. In the two figures, we both consider the cases $n = 10$ and $n = 30$, respectively (as $n = 10$, m takes values from 0 to 9, and as $n = 30$, m

takes values from 0 to 29), and the two cases of $\tau_u = 1, 100$ for Figure 8 and three cases of $\tau_u = 0.001, 1, 100$ for Figure 9, respectively (see panel a and panel b in Figure 8 and 9 from left to right, where $n = 10$ in panel a and $n = 30$ in panel b). The only difference between Figure 8 and 9 is that in Figure 8 we let $\tau_\theta = 100$ and $\tau_\epsilon = 1$, but in Figure 9 we let $\tau_\theta = 25$ and $\tau_\epsilon = 5$. Combining Figures 8 and 9, we find that the level of sophistication m can only affect the value of the trading profits of the three types of investors, but not the ordering of the trading profits of the three. In other words, our conclusions about the ordering of trading profits of the three types of investors discussed earlier is robust with respect to m.



(a) Panel a



(b) Panel b

Figure 9: Panel a show the effect of m on the trading profits of the three types of investors when $n = 10$ and the value of τ_u from left to right are $0.001, 1$ and 100. Panel b display the effect of m on the trading profits of the three types of investors when $n = 30$ and the values of τ_u from left to right are $0.001, 1$ and 100. The other parameter values are $\tau_\theta = 25, \tau_\epsilon = 5$ and $\rho = 2$.

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