Can institutional investors always beat individual investors?

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Abstract

In an imperfectly competitive market, we find that an institutional investor with an information advantage consistently earns higher expected trading profits than sophisticated individual investors who internalize their price impact. However, when noise-trading volume and the noise-to-signal ratio are sufficiently high, the institutional investor underperforms naive individual investors who act as price-takers. The aggressive trading behavior of naive investors, driven by their failure to account for price impact, forces the institutional investor to reduce his trading aggressiveness. Our findings highlight that, under certain conditions, the irrationality of naive traders can erode the advantages of information-driven trading strategies.

Keywords: Information advantage, price impact, institutional investor, trading profits

JEL Classification: D82; D85; G11; G14

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1 Introduction

On the demand side of financial markets, investors are typically categorized into institutional investors, who possess the ability to produce information, and individual investors, who tend to be less informed. In classical perfectly competitive markets (Grossman and Stiglitz 1980), investors with an information advantage (institutional investors) consistently achieve higher expected trading profits relative to their less-informed counterparts (individual investors). However, substantial evidence indicates that large financial institutions exert significant market influence. Moreover, with algorithms now becoming an essential feature of institutional order executions, individual traders' order flow may even exhibit a larger average trade size than other flows (Boehmer et al. 2021).[1] In light of these developments, this paper investigates whether an institutional investor, endowed with an information advantage, can still outperform less-informed individual investors in an imperfectly competitive market.

We consider a financial market in which a single risky asset is traded by a finite number of investors, who differ in their information advantages and levels of rationality, alongside noise traders. There are three types of investors. The first is an *institutional* investor, who possesses complete information about the market. The remaining investors have access to a single piece of asymmetric information alongside a public signal, yet they differ in how they leverage this information. Among them, *sophisticated* individual investors act strategically, internalizing the impact of their demand on asset prices when formulating optimal demand schedules. In contrast, *naive* individual investors are unaware of their price impact and perceive themselves as price-takers, assuming their trades have no influence on market prices.[2] The interaction among the demand schedules of all investors, combined with the presence of noise trading, determines the endogenous equilibrium price. This price reflects the aggregation of all market information

[1]Previous studies typically treat individual investors as small competitive traders (Kacperczyk et al. 2023) and many researchers use trade size as a proxy for retail order flow (Campbell et al. 2009). However, with the rise of algorithmic trading in the early 2000s, institutional investors have started to split their trades. As a result, trade-size partitioning has become significantly less effective as a proxy for retail order flow. Furthermore, evidence indicates that retail investors tend to have a meaningful influence on the returns of stocks with small market capitalization (Kumar and Lee 2006), and it is well known that the market for such stocks is imperfectly competitive.

[2]The institutional investor tends to be more experienced and can accurately assess their price impact. In contrast, individual investors may fail to recognize that their trading activity affects asset prices, or even if they do, they often lack the ability to calculate it correctly due to limitations such as a lack of investment experience and understanding of the market environment.

and noise.

We analyze the expected trading profits of different investors, focusing on the interplay between two key effects: the positive information effect, reflected in information efficiency (Rahi and Zigrand 2018; Lou and Rahi 2023), and the negative risk effect, characterized by market-implied risk aversion—defined as the sum of price impact and conditional uncertainty regarding the asset payoff. The institutional investor, endowed with superior access to information, naturally benefits from a stronger information effect. However, the magnitude of the risk effect varies with the rationality of individual investors and market conditions, particularly the volume of noise trading and the noise-to-signal ratio. Our analysis reveals that while the institutional investor consistently outperforms sophisticated individual investors across all market conditions, there exist scenarios—specifically when noise-trading volume and the noise-to-signal ratio are sufficiently high—where the institutional investor may underperform naive individual investors.

When all individual investors are sophisticated, our analytical and numerical analysis demonstrates that the institutional investor consistently outperforms sophisticated individual investors. In markets with sufficiently large noise-trading volume, excessive noise becomes embedded in prices, reducing their informativeness in forecasting fundamentals. Consequently, sophisticated individual investors place less weight on the informational content of prices and trade more aggressively against price movements. This increased trading activity enhances liquidity for the institutional investor, thereby reducing his price impact and weakening the associated risk effect. As noise-trading volume gradually decreases, the institutional investor begins to experience a more pronounced negative risk effect. Nevertheless, the presence of imperfect competition ensures that the institutional investor's information advantage persists. Consequently, the positive information effect consistently dominates the trade-off, enabling the institutional investor to outperform sophisticated individual investors.

The results differ significantly when all individual investors are naive and act as price-takers. Our analysis demonstrates that the institutional investor cannot outperform naive individual investors when both noise-trading volume and the noise-to-signal ratio are sufficiently high. The intuition is illustrated in Table 1. Naive individual investors, perceiving themselves as price-takers, mistakenly believe that their trading does not affect the equilibrium price. Consequently, they tend to trade more aggressively—buying heavily on positive signals and selling on negative

Table 1: **Intuition for results on expected trading profits.** We simplify the main model to a two-player game (the institutional investor (I) v.s. the individual investor) with two strategies (aggressive (A) or conservative (C) trading strategies). The individual investor can be either sophisticated (S) or naive (N). The payoff matrix on the left represents the real payoffs recognized by investor I and investor S, while the "fictional" payoff matrix on the right reflects the mistakes made by investor N in estimating payoffs resulting from failing to internalize his price impact. The payoff matrix demonstrates three key characteristics in the main model. First, each investor's trade exerts a price impact. Strategy A influences the equilibrium price and imposes negative externalities on other investors, as $\pi^S(A,A) < \pi^S(C,A)$ and $\pi^S(A,C) < \pi^S(C,C)$ for investor S and $\pi^I(A,A) < \pi^I(A,C)$ and $\pi^I(C,A) < \pi^I(C,C)$ for investor I, because more trading drives the price up more. Second, the institutional investor has an information advantage over the individual investors. Investor I tends to trade more aggressively when investor S trades conservatively, i.e., $\pi^I(A,C) > \pi^I(C,C)$. Conversely, without an information advantage, investor S stays conservative even investor I trades conservatively, i.e., $\pi^S(C,A) < \pi^S(C,C)$. Combining previous two characteristics, when one of the investors has imposed an aggressive trading strategy already, following up with aggressive trading is not optimal even if the investor has an information advantage (i.e., $\pi^I(A,A) < \pi^I(C,A)$ and $\pi^S(A,A) < \pi^S(A,C)$), which will impose too much impact on equilibrium price. Third, the naive individual investor is irrational. Investor N does not internalize the impact of his own trading on the market. This leads to a belief that aggressive trading always yields higher payoffs, regardless of the trading behavior of the other player (investor N believes $\pi^N(A,A) > \pi^N(A,C)$ and $\pi^N(C,A) > \pi^N(C,C)$).

		S	
		A	C
I	A	1 2	2 7
	C	4 3	5 6

		N	
		A	C
I	A	'3' 2	2 7
	C	'6' 3	5 6

ones. This behavior is captured by the "fictional" payoff matrix in the right table, where the payoffs for naive investors ($\pi^N(A,A) > \pi^N(A,C)$ and $\pi^N(C,A) > \pi^N(C,C)$) reflect their belief that their trades have no price impact. Recognizing the irrationality of naive investors, the institutional investor anticipates their aggressive trading behavior and is compelled to reduce his own trading aggressiveness, despite possessing an informational advantage. The institutional investor's potential gains from additional information are outweighed by the diminished profit share. In essence, the irrationality of naive individual investors serves as a commitment device, embedding their aggressive trading into the market dynamics.

The institutional investor's advantage stems from more information, but this is offset by the disadvantage arising from naive individual investors' commitment to aggressive trading. When the noise-trading volume is sufficiently large, the equilibrium price variance also rises. This leads to a decline in the institutional investor's information effect because the information efficiency measures for both institutional and individual investors approach one. Additionally, when the noise-to-signal ratio is sufficiently large, naive individual investors receive imprecise information. The resulting increase in payoff uncertainty, due to less precise private signals, reduces the price sensitivity of naive individual investors' demands. This reduction in price sensitivity, in return, amplifies the price impact of the institutional investor. Consequently, in this scenario, the risk effect becomes more pronounced for the institutional investor. Hence, naive individual investors outperform the institutional investor.

Our findings for the two special cases also extend to the general model, where both sophisticated and naive individual investors participate in the market. However, the aggressive trading behavior of naive investors, driven by their failure to internalize price impact, imposes negative externalities on all market participants. Specifically, we find that the expected trading profits of all investors increase as more individual investors transition from naive to sophisticated. Furthermore, we demonstrate the robustness of our results by examining two extensions: (i) a setting where the private signals of sophisticated and naive individual investors are heterogeneous, and (ii) a scenario in which naive investors exhibit partial awareness of their price impact.

Related Literature. Our imperfectly competitive market equilibrium is based on the seminal framework of Kyle (1989) (see Zhou (2022), Kacperczyk et al. (2023), Glebkin et al. (2023), and Anthropelos and Robertson (2024) for recent extensions) and is most related to Nezafat and Schroder (2023), which theoretically establish the existence of a zero-precision symmetric equilibrium in an imperfectly competitive market.[3] Two key distinctions differentiate

[3]The model in Nezafat and Schroder (2023) encompasses two stages: an information-acquisition stage and a trading stage. In the information-acquisition stage, each investor chooses a signal precision to maximize his expected utility at the trading stage accounting for the price impact in the trading stage and the influence of his precision choice on the trading strategies of other market participants. The reduction in payoff uncertainty resulting from a more precise private signal enhances the price sensitivity (as well as the signal sensitivity) of the deviating investor's demand, thereby reducing the price impact of the conforming investors. Moreover, the decline in conforming investors' price impact increases their demand-function price sensitivities, further reducing price impact. Lower price impact (i.e., more liquid markets) generated by the deviating investor's improved signal induces all rational investors to trade more aggressively (i.e., increase the absolute size of their trades), thereby reducing the stock's equilibrium absolute risk premium. A zero-precision equilibrium arises when the utility cost of the lower risk premium exceeds the utility benefit from more precise private information.

our study from Nezafat and Schroder (2023). First, while all traders in Nezafat and Schroder (2023) are rational, we introduce irrationality for some individual investors. Specifically, naive individual investors are unaware of their price impact and perceive themselves as price-takers. Second, while Nezafat and Schroder (2023) focus on the existence of a zero-precision equilibrium with cost-free signals, our study investigates whether unconsciousness can strictly dominate rationality and survive in the long run.[4] Since unconsciousness acting likes a commitment device in a standard Cournot model, institutional investors underperform naive individual investors who consider themselves to be price-takers when noise-trading volume and the noise-to-signal ratio for information are sufficiently large.

We further contribute to the emerging literature on behavioral rational expectations equilibrium (REE). Banerjee et al. (2009) and Banerjee (2011) integrate REE with disagreement frameworks, allowing investors to underestimate the precision of other investors' private information. Basak and Buffa (2019) examine the decision-making of financial institutions in the presence of novel implementation frictions that generate operational risk. A more sophisticated model produces a more informative signal about investment opportunities by leveraging advanced IT infrastructure and data analytics. However, the use of these technologies also increases susceptibility to operational errors. Mondria et al. (2022) propose an optimal inattention–style variant of partial cursedness, where traders observe prices but employ noisy signals to infer the underlying information and can pay a cost to reduce the noise. They endogenize traders' sophistication levels, demonstrating that sophistication acquisition can exhibit complementarities. Eyster et al. (2019) model a financial market in which some traders of a risky asset fail to fully appreciate what prices convey about others' private information. Malikov and Pasquariello (2022) characterize quantitative investing as myopic due to its reliance on backtested trading strategies; that is, quantitative investors are unaware that other investors are aware of their existence. While we share the feature that some traders irrationally neglect rational elements in financial markets, our focus differs.

Regarding the economic mechanism, our study is closely related to Kyle and Wang (1997),

[4]Furthermore, Proposition 2 in Nezafat and Schroder (2023) shows that when noise-trading volume is sufficiently large, a deviating trader benefits from a positive-precision signal only when the absolute expected noise trading is small (i.e., the mean of the noise trading is small). This contrasts with our Proposition 3, which does not depend on the mean of noise trading (i.e., our main results remain valid even for a high mean of noise trading).

who show that overconfidence may strictly dominate rationality and survive in the long run because overconfidence functions as a commitment device in a standard Cournot duopoly model. Xiong and Yang (2024) also incorporate a commitment mechanism, albeit in a different context. They show that corporate social responsibility toward consumers can facilitate a commitment to lower product prices, which helps resolve the coordination problem among consumers and increases firm profits, thereby supporting the notion of "doing well by doing good." In our study, this commitment device arises from naive investors' unconsciousness of their price impact. Similarly, commitment can be used as a device to enhance profits. However, while the commitment in Kyle and Wang (1997) stems from overconfidence, it originates from unconsciousness in our framework.

2 The model

2.1 Model setup

Assets: We consider a Kyle (1989)-type economy with imperfect competition. The financial market consists of a risk-free asset, with a normalized price and payoff of 1, and a risky asset with price p and a random payoff $\theta \sim \mathcal{N}(0, 1/\tau_\theta)$, $\tau_\theta > 0$. To prevent prices from being fully revealing, there is also per-capita random demand by noise traders $u \sim \mathcal{N}(0, 1/\tau_u)$, $\tau_u = 1/\sigma_u^2$, where u is independent of other random variables.[5]

Preference: There are $n \geq 3$ investors,[6] who are divided into three groups: an institutional investor, m sophisticated individual investors, and $n - m - 1$ naive individual investors, as detailed later. The utility of investor i, who buys $x_i \in \mathbb{R}$ units of the risky asset at price $p \in \mathbb{R}$, is given by

$$-\exp\{-\rho x_i(\theta - p)\},$$

where ρ is the constant absolute risk aversion (CARA) parameter. Without loss of generality, we assume that all investors have zero initial wealth due to the CARA assumption, which

[5]For simplicity, we assume zero means for the random variables θ and u. However, our main results continue to hold even in more general cases with nonzero means.

[6]When $n = 2$, a linear equilibrium does not exist, see Equation (40) in the Appendix. This aligns with Proposition 5.1 in Kyle (1989), which states that in the absence of uninformed investors, a linear equilibrium exists only when the number of informed investors is greater than or equal to three.

abstracts away wealth effects.

Institutional investors: We assume that there is only one investor ($i = 1$) who possesses all the information in the economy.[7] Throughout this paper, we refer to this investor with an information advantage as the institutional investor. Specifically, each individual investor $i = 2, ..., n$ observes a private signal $y_i = \theta + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, 1/\tau_\epsilon)$ and $\tau_\epsilon > 0$. Additionally, there is a public signal $y_1 = \theta + \epsilon_1$, where $\epsilon_1 \sim \mathcal{N}(0, 1/\tau_\epsilon)$. The idiosyncratic noise terms $\{\epsilon_1, ..., \epsilon_n\}$ are mutually independent and independent of other random variables in the model. The institutional investor possesses all the information in the market; that is, his information set is given by $\{y_1, y_2, ..., y_n\}$. The institutional investor behaves strategically since he tends to be experienced in financial markets and can accurately calculate and estimate his price impact. Following a similar analysis to Kyle (1989), the institutional investor's optimal demand is given by

$$x_1^* = \frac{\mathbb{E}[\theta|y_1, y_2, ..., y_n, p] - p}{\lambda_1 + \xi_1}, \quad \xi_1 = \rho \operatorname{Var}[\theta|y_1, y_2, ..., y_n, p], \tag{1}$$

where $\lambda_1 > 0$, which will be generated endogenously, denotes the institutional investor's price impact. The institutional investor realizes that his demand has an impact on the equilibrium price, and incorporates this impact when optimizing his demand schedule.

Sophisticated individual investors: The individual investors are assumed to be either strategic or price-takers. We refer to strategic individual investors as *sophisticated*, and price-taking individual investors as *naive*. The information set of individual investor $i = 2, ..., n$ is $\{y_1, y_i\}$.[8] The optimal demand of sophisticated individual investor $i = 2, ..., m + 1$ is given by

$$x_i^* = \frac{\mathbb{E}[\theta|y_1, y_i, p] - p}{\lambda_s + \xi_s}, \quad \xi_s = \rho \operatorname{Var}[\theta|y_1, y_i, p], \tag{2}$$

where $\lambda_s > 0$, which will be generated endogenously, denotes the price impact of sophisticated

[7]Our results extend to a more general setting where multiple investors have access to all the information in the market.

[8]The model can also be nested within the framework of information sharing (Colla and Antonio 2010; Ozsoylev and Walden 2011; Han and Yang 2013; Lou and Yang 2023), where the information network is represented by a star structure. In this structure, investor 1 is the central node who initially has access to a private signal y_1, while the other investor $i = 2, ..., n$ are non-central nodes, each initially possessing an individual private signal y_i.

individual investors $2, ..., m+1$. Previous studies typically treat individual investors as small competitive traders (Kacperczyk et al. 2023), and trade size is often used as a proxy for retail order flow (Campbell et al. 2009). However, with the rise of algorithmic trading in the early 2000s, trade-size partitioning has become significantly less effective as a proxy for retail order flow. In fact, the retail order flow may exhibit a larger average trade size compared to other flows (Boehmer et al. 2021).

Naive individual investors: A key feature of the model is that some individual investors perceive themselves as price-takers. Naive individual investors fail to realize that their demands have an impact on asset prices, or even if they do, they are unable to calculate this impact correctly due to limitations such as limited investment experience or limited understanding of the market environment, etc. The optimal demand of naive individual investor $j = m+2, ..., n$ is given by[9]

$$x_j^* = \frac{\mathbb{E}[\theta|y_1, y_j, p] - p}{\xi_n}, \quad \xi_n = \rho \, \mathrm{Var}[\theta|y_1, y_j, p]. \tag{3}$$

Even though naive investors consider themselves to be price-takers, their trading actually impacts the equilibrium price, as indicated by the market-clearing condition (5). In this regard, we are related to the emerging literature on behavioral rational expectations equilibrium such as Eyster et al. (2019), Mondria et al. (2022), and Malikov and Pasquariello (2022).

2.2 Discussion of assumptions

We summarize the main characteristics of the model setup in Table 2 and discuss its underlying assumptions. The model is designed to clearly differentiate between two key features: information advantage and rationality.

Why do sophisticated individual investors lack an information advantage? The framework deliberately differentiates between two key features: rationality (the ability to internalize price impact) and information asymmetry (the possession of superior information). The model categorizes investors into three types: institutional, sophisticated individual, and naive individual. Among these, the institutional investor is the only participant endowed with an informational

[9] Due to symmetry, $\lambda_s + \xi_s$ and ξ_n do not depend on the specific indices i and j, respectively.

Table 2: Assumptions and Extensions

	Institutional	Sophisticated individual	Naive individual
Information advantage	✓		
Rationality	✓	✓	
Information set	$\{y_1, y_2, ..., y_n, p\}$	$\{y_1, y_i, p\}$	$\{y_1, y_j, p\}$
Demand function	Equation (1)	Equation (2)	Equation (3)
Extension I Appendix A.1	Certainty equivalent/Standardized expected profits		
Extension II Appendix A.4.1	Partial information asymmetry		
Extension III Appendix A.4.2			Partial awareness

advantage, as they observe all private signals available in the market. This design focuses on isolating and analyzing the interplay between information asymmetry and price impact in shaping trading outcomes. Sophisticated individual investors, while rational in internalizing their price impact and acting strategically, have access to a more limited information set compared to institutional investors. Thus, sophisticated individual investors can be conceptualized as high-profile retail investors who exhibit rationality and strategic behavior but lack the informational advantage of institutional investors. In Appendix A.4.1, we extend the model to consider an alternative specification where the information precision of sophisticated individual investors lies between that of institutional and naive individual investors, offering further insights into the role of *partial information asymmetry*.

Why cannot naive individual investors internalize price impact? The assumption that naive individual investors are oblivious to their price impact simplifies the equilibrium characterization, enabling the model to focus on the role of irrational trading behavior. This assumption highlights how the aggressive trading strategies of naive individual investors, driven by their perceived inability to influence prices, can inadvertently affect institutional investors. Naive individual investors are modeled as price-takers, assuming their trades do not affect market prices. The model emphasizes that the irrationality of naive investors functions as a commitment mechanism, compelling them to trade more aggressively in response to their private signals. In practice, many naive individual investors may exhibit some degree of awareness of their price impact, even if they do not fully internalize it in their decision-making. Incorporating *partial awareness* into the model would enhance its realism and bridge the gap between naive and sophisticated investors. In Appendix A.4.2, we extend the model to explore this scenario.

Why do individual investors have a significant price impact? The significant price impact attributed to individual investors in the model may appear counterintuitive, given their characterization as "small" and "retail-like." However, this outcome stems from the specific market structure and modeling assumptions. In an imperfectly competitive market, the price impact of a trader is determined not only by the size of their trades but also by the elasticity of demand exhibited by other market participants. Empirical evidence supports this notion, particularly in markets with low capitalization and liquidity. In such markets, even modest trading volumes—common among retail investors—can trigger significant price movements. With algorithms becoming an essential feature of institutional order executions, individual traders' order flow may exhibit a larger average trade size than other flows (Boehmer et al. 2021). The model attributes significant price impact to naive individual investors because their aggressive, uncoordinated, and collective trading behavior disrupts equilibrium, amplifying market effects in an imperfectly competitive environment.

2.3 Equilibrium definition

As is standard in the literature, we focus on linear equilibria in this paper. Specifically, the price of the risky asset is a linear function of investors' signals and noise demand. Suppose that the equilibrium price takes the following linear form:[10]

$$p = \pi_1 y_1 + \pi_s \sum_{i=2}^{m+1} y_i + \pi_n \sum_{j=m+2}^{n} y_j + \gamma u. \tag{4}$$

A *linear Bayesian Nash equilibrium* $\{p, x_1^*, x_i^*, x_j^*\}$ is defined as a linear price function p, together with the demands x_1^* for the institutional investor, $x_i^*, i = 2, ..., m+1$ for sophisticated individual investors, and $x_j^*, j = m + 2, ..., n$ for naive individual investors, such that

[10] Here we postulate that the coefficients on the signals $y_2, ..., y_{m+1}$ ($y_{m+2}, ..., y_n$) in the conjectured linear equilibrium price p are identical because sophisticated (naive) individual investors are symmetric in the model. Additionally, since we assume that all random variables have zero mean without loss of generality, there is no intercept in the conjectured price function p.

(i) **Trading strategy:** Institutional, sophisticated individual, and naive individual investors submit orders that maximize their expected utility, as defined in Equations (1), (2), and (3);

(ii) **Market clearance:** the market-clearing condition

$$x_1^* + \sum_{i=2}^{m+1} x_i^* + \sum_{j=m+2}^{n} x_j^* + nu = 0 \tag{5}$$

holds almost surely.[11]

Equation (5) indicates that the naive individual investors also exert price impact. In this paper, we are interested in the question of whether the institutional investor can beat individual investors in the sense that the institutional investor has higher trading profits than individual investors.[12]

3 Equilibrium characterization

In this section, we establish the existence of linear Bayesian Nash equilibria using the *first-conjecture-then-verification* approach, which is widely used in the literature. Since sophisticated and naive individual investors differ in their awareness of their price impact on equilibrium price, their demand sensitivities to price information vary, leading to different price coefficients π_s and π_n in (4). Based on the equilibrium price form (4), from the perspective of a sophisticated individual investor who observes $\{y_1, y_i, p\}$, the price information p is equivalent to

$$\begin{aligned} s_{p,i}^s &= \frac{p - \pi_1 y_1 - \pi_s y_i}{\pi_s(m-1) + \pi_n(n-m-1)} \\ &=: \frac{1}{\pi_s(m-1) + \pi_n(n-m-1)} \left[\pi_s \sum_{r=2, r\neq i}^{m+1} y_r + \pi_n \sum_{j=m+2}^{n} y_j + \gamma u \right] \end{aligned}$$

[11] Alternatively, we can interpret $-u$ as the per-capita random supply in the market.

[12] In Appendix A.1, we discuss both certainty equivalent and standardized expected trading profit (adjusted for information precision). Certainty equivalent can be treated as a risk-adjusted measure of investor performance. Given that information is costly and signal precision varies across investors, we further compare standardized trading profits between investors in Appendix A.1.

$$=: \theta + v_s \sum_{r=2, r\neq i}^{m+1} \epsilon_r + q_s \sum_{j=m+2}^{n} \epsilon_j + z_s u,$$

where

$$v_s = \frac{\pi_s}{\pi_s(m-1) + \pi_n(n-m-1)}, \tag{6}$$

$$q_s = \frac{\pi_n}{\pi_s(m-1) + \pi_n(n-m-1)}, \tag{7}$$

$$z_s = \frac{\gamma}{\pi_s(m-1) + \pi_n(n-m-1)}. \tag{8}$$

Applying the projection theorem for normal random variables, we obtain

$$\mathbb{E}[\theta|y_1, y_i, p] = \frac{\tau_\epsilon(y_1 + y_i) + \Theta^s s^s_{p,i}}{\tau_\theta + 2\tau_\epsilon + \Theta^s} =: \alpha^s_1 y_1 + \alpha^s_o y_i + \beta^s p,$$

where

$$\begin{aligned}
\Theta^s &= \left[(v_s^2(m-1) + q_s^2(n-m-1))/\tau_\epsilon + z_s^2/\tau_u\right]^{-1}, \qquad (9)\\
\alpha^s_1 &= \frac{\tau_\epsilon - \Theta^s \frac{\pi_1}{\pi_s(m-1)+\pi_n(n-m-1)}}{\tau_\theta + 2\tau_\epsilon + \Theta^s},\\
\alpha^s_o &= \frac{\tau_\epsilon - \Theta^s v_s}{\tau_\theta + 2\tau_\epsilon + \Theta^s},\\
\beta^s &= \frac{\Theta^s}{\tau_\theta + 2\tau_\epsilon + \Theta^s} \frac{1}{\pi_s(m-1) + \pi_n(n-m-1)}.
\end{aligned}$$

The parameters α^s_1, α^s_o, and β^s measure sophisticated individual investors' expectation sensitivity to public information, private information, and price, respectively.

Similarly, based on the equilibrium price form (4), from the perspective of a naive individual investor who observes $\{y_1, y_j, p\}$, the price information is equivalent to

$$\begin{aligned}
s^n_{p,j} &= \frac{p - \pi_1 y_1 - \pi_n y_j}{\pi_s m + \pi_n(n-m-2)}\\
&=: \frac{1}{\pi_s m + \pi_n(n-m-2)} \left[\pi_s \sum_{i=2}^{m+1} y_i + \pi_n \sum_{r=m+2, r\neq j}^{n} y_r + \gamma u\right]\\
&=: \theta + v_n \sum_{i=2}^{m+1} \epsilon_i + q_n \sum_{r=m+2, r\neq j}^{n} \epsilon_r + z_n u,
\end{aligned}$$

where

$$
\begin{aligned}
v_n &= \frac{\pi_s}{\pi_s m + \pi_n(n-m-2)}, && (10)\\
q_n &= \frac{\pi_n}{\pi_s m + \pi_n(n-m-2)}, && (11)\\
z_n &= \frac{\gamma}{\pi_s m + \pi_n(n-m-2)}. && (12)
\end{aligned}
$$

Applying the projection theorem for normal random variables, we obtain

$$
\mathbb{E}[\theta|y_1, y_j, p] = \frac{\tau_\epsilon(y_1 + y_j) + \Theta^n s^n_{p,j}}{\tau_\theta + 2\tau_\epsilon + \Theta^n} =: \alpha^n_1 y_1 + \alpha^n_o y_j + \beta^n p,
$$

where

$$
\begin{aligned}
\Theta^n &= \left[(v_n^2 m + q_n^2(n-m-2))/\tau_\epsilon + z_n^2/\tau_u\right]^{-1}, && (13)\\
\alpha^n_1 &= \frac{\tau_\epsilon - \Theta^n \frac{\pi_1}{\pi_s m + \pi_n(n-m-2)}}{\tau_\theta + 2\tau_\epsilon + \Theta^n},\\
\alpha^n_o &= \frac{\tau_\epsilon - \Theta^n q_n}{\tau_\theta + 2\tau_\epsilon + \Theta^n},\\
\beta^n &= \frac{\Theta^n}{\tau_\theta + 2\tau_\epsilon + \Theta^n}\frac{1}{\pi_s m + \pi_n(n-m-2)}.
\end{aligned}
$$

The parameters α^n_1, α^n_o, and β^n measure naive individual investors' expectation sensitivity to public information, private information, and price, respectively.

For the institutional investor who observes $\{y_1, y_2, ..., y_n, p\}$, based on the equilibrium price form (4) and applying the projection theorem for normal random variables, we have

$$
\mathbb{E}[\theta|y_1, y_2, ...y_n, p] = \mathbb{E}[\theta|y_1, y_2, ...y_n] = \frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon}\sum_{i=1}^{n} y_i,
$$

where we use the fact that the price p is redundant for the institutional investor, given his access to full market information.

Furthermore, the conditional uncertainty about the fundamental for the institutional investor, strategic individual investors, and naive individual investors is respectively given by

$$\mathrm{Var}[\theta|y_1, ..., y_n, p] = \mathrm{Var}[\theta|y_1, ..., y_n] = \frac{1}{\tau_\theta + n\tau_\epsilon}, \tag{14}$$

$$\mathrm{Var}[\theta|y_1, y_i, p] = \frac{1}{\tau_\theta + 2\tau_\epsilon + \Theta^s}, \tag{15}$$

$$\mathrm{Var}[\theta|y_1, y_j, p] = \frac{1}{\tau_\theta + 2\tau_\epsilon + \Theta^n}.$$

The market-clearing condition (5) indicates

$$\frac{\frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon}\sum_{r=1}^n y_r - p}{\lambda_1 + \xi_1} + \sum_{i=2}^{m+1} \frac{\alpha_1^s y_1 + \alpha_o^s y_i + \beta^s p - p}{\lambda_s + \xi_s} + \sum_{j=m+2}^{n} \frac{\alpha_1^n y_1 + \alpha_o^n y_j + \beta^n p - p}{\xi_n} + nu = 0,$$

which implies

$$p = \left[\frac{1}{\lambda_1 + \xi_1} + \frac{m(1-\beta^s)}{\lambda_s + \xi_s} + \frac{(n-m-1)(1-\beta^n)}{\xi_n}\right]^{-1} \times \left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1 + \xi_1)}\sum_{r=1}^n y_r + \sum_{i=2}^{m+1} \frac{\alpha_1^s y_1 + \alpha_o^s y_i}{\lambda_s + \xi_s} + \sum_{j=m+2}^{n} \frac{\alpha_1^n y_1 + \alpha_o^n y_j}{\xi_n} + nu\right].$$

Matching coefficients over both right-hand sides of the preceding price function and the conjectured price function (4) leads to

$$\gamma = n\left[\frac{1}{\lambda_1 + \xi_1} + \frac{m(1-\beta^s)}{\lambda_s + \xi_s} + \frac{(n-m-1)(1-\beta^n)}{\xi_n}\right]^{-1}, \tag{16}$$

$$\pi_1 = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1 + \xi_1)} + \frac{m\alpha_1^s}{\lambda_s + \xi_s} + \frac{(n-m-1)\alpha_1^n}{\xi_n}\right],$$

$$\pi_s = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1 + \xi_1)} + \frac{\alpha_o^s}{\lambda_s + \xi_s}\right], \tag{17}$$

$$\pi_n = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1 + \xi_1)} + \frac{\alpha_o^n}{\xi_n}\right]. \tag{18}$$

Moreover, the price impact parameters should satisfy

$$\lambda_1 = \left[\frac{m(1-\beta^s)}{\lambda_s + \xi_s} + \frac{(n-m-1)(1-\beta^n)}{\xi_n}\right]^{-1}, \tag{19}$$

$$\lambda_s = \left[\frac{1}{\lambda_1 + \xi_1} + \frac{(m-1)(1-\beta^s)}{\lambda_s + \xi_s} + \frac{(n-m-1)(1-\beta^n)}{\xi_n}\right]^{-1}. \tag{20}$$

Similar to Kyle (1989), Equations (19) and (20) indicate that each investor's price impact equals the reciprocal of the sum of the price sensitivities of all other investors. We now get a system of equilibrium equations (16)-(20) with variables π_s, π_n, π_1, γ, λ_1, and λ_s.[13] Furthermore, although naive individual investors perceive themselves as price-takers, they indeed exert price impact. Table 3 summarizes the key variables required to establish the equilibrium:

Table 3: Notations

Symbol		Definition
Exogenous		
θ		Random payoff of the risky asset, $\theta \sim N(0, 1/\tau_\theta)$
u		Per-capita random demand of noise traders, $u \sim N(0, 1/\tau_u)$
ρ		Risk aversion parameter
n		Total number of investors
m		Number of sophisticated individual investors
y_1		Public signal, $y_1 = \theta + \epsilon_1$
$y_r, r = 2, ..., n$		Private signal of investor r, $y_r = \theta + \epsilon_r$, $\epsilon_r \sim N(0, 1/\tau_\epsilon)$
Endogenous		
p	π_1	Price sensitivity to public information
	π_s	Price sensitivity to private information of sophisticated individual investors
	π_n	Price sensitivity to private information of naive individual investors
	γ	Price sensitivity to demand of noise traders
x_r	λ_1	Price impact of the institutional investor
	λ_s	Price impact of sophisticated individual investor
	ξ_1	Risk aversion adjusted conditional variance for institutional investor
	ξ_s	Risk aversion adjusted conditional variance for sophisticated individual investors
	ξ_n	Risk aversion adjusted conditional variance for naive individual investors
	α_1^s	Sophisticated individual investor's expectation sensitivity to public information
	α_1^n	Naive individual investor's expectation sensitivity to public information
	α_o^s	Sophisticated individual investor's expectation sensitivity to private information
	α_o^n	Naive individual investor's expectation sensitivity to private information
	β^s	Sophisticated individual investor's expectation sensitivity to price
	β^n	Naive individual investor's expectation sensitivity to price

Section 4 and Section 5 examine two special cases, respectively: one in which all individual investors behave strategically, and the other in which all individual investors act as price-takers. Section 6 analyzes the more general case of the coexistence of all three types of investors.

[13] In Appendix A.4, we extend the model to incorporate heterogeneity in the signal precision of the two types of individual investors and accounts for naive individual investors' partial awareness of their price impact.

4 Institutional investors and sophisticated individuals

This section examines the case where all individual investors behave strategically, accounting for the impact of their demands on the asset price (i.e., $m = n - 1$), and investigates whether the institutional investor can beat sophisticated individual investors.

4.1 Equilibrium

We first establish a linear Bayesian Nash equilibrium and then calculate the expected trading profits.

Proposition 1. *Suppose that all individual investors are sophisticated. Then there exists a linear Bayesian Nash equilibrium determined by the following system of equations*

$$\gamma = n\left[\frac{1}{\lambda_1+\xi_1}+\frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}\right]^{-1}, \tag{21}$$

$$\pi_1 = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{(n-1)\alpha_1^s}{\lambda_s+\xi_s}\right], \tag{22}$$

$$\pi_s = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{\alpha_o^s}{\lambda_s+\xi_s}\right], \tag{23}$$

$$\lambda_1 = \left[\frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}\right]^{-1}, \tag{24}$$

$$\lambda_s = \left[\frac{(n-2)(1-\beta^s)}{\lambda_s+\xi_s}+\frac{1}{\lambda_1+\xi_1}\right]^{-1}, \tag{25}$$

where

$$\alpha_1^s = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}\frac{\pi_1}{\pi_s}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}, \quad \alpha_o^s = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}, \tag{26}$$

$$\beta^s = \frac{\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\frac{1}{(n-2)\pi_s}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}, \quad z_s = \frac{\gamma}{(n-2)\pi_s}. \tag{27}$$

Proposition 1 establishes the financial market equilibrium in terms of the equilibrium price coefficients γ, π_1, and π_s, as well as traders' price impacts λ_1 and λ_s. The equilibrium system (21)-(25) can be directly derived from the system (16)-(20) by setting $m = n - 1$ and omitting equation (18), which corresponds to naive individual investors. Consistent with Kyle (1989), Equations (24) and (25) show that each investor's price impact equals the reciprocal of the sum of the price sensitivities of all other investors.

The expected trading profits of the institutional investor (investor 1) are given by

$$\Pi_1 := \mathbb{E}[(\theta - p)x_1^*] = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_2, ..., y_n, p]}{\lambda_1 + \xi_1}, \tag{28}$$

where we utilize the relation in (1) and the law of total variance. Similarly, from (2), the (expected) trading profits of sophisticated individual investor $i = 2, ..., n$ are given by

$$\Pi_s := \mathbb{E}[(\theta - p)x_i^*] = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_i, p]}{\lambda_s + \xi_s}, i = 2, ..., n. \tag{29}$$

Following the definition of informational efficiency in the literature (Rahi and Zigrand 2018; Lou and Rahi 2023), we analogously define a measure of informational efficiency for predicting the asset return $\theta - p$:

$$\Psi_1 := \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_2, ..., y_n, p]}{\mathrm{Var}(\theta - p)} = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta|y_1, y_2, ..., y_n]}{\mathrm{Var}(\theta - p)}. \tag{30}$$

Similarly, we also define

$$\Psi_s := \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_i, p]}{\mathrm{Var}(\theta - p)} = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta|y_1, y_i, p]}{\mathrm{Var}(\theta - p)}, i = 2, ..., n. \tag{31}$$

Equations (28) and (29) indicate that investors' trading profits are determined by two key factors: the information effect, captured by information efficiency, and the risk effect, characterized by market-implied risk aversion (i.e., the sum of price impact λ and the risk-aversion-adjusted conditional variance about the fundamental ξ).

Since the institutional investor possesses more information, the information efficiency is higher, i.e., $\Psi_1 > \Psi_s$. Therefore, in a competitive setting, it is clear that the institutional investor can always beat individual investors.[14] The analysis becomes more complex in an im-

[14] When all investors act as price-takers, the trading profits of the institutional investor and individual investors are given by Equations (28) and (29) with $\lambda_1 = 0$ and $\lambda_s = 0$, respectively. It is evident that the institutional investor faces lower conditional uncertainty than individual investors, i.e., $\xi_1 < \xi_s$. Additionally, due to the institutional investor's information advantage, he is able to achieve higher trading profits than

perfectly competitive setting, as illustrated below. In such environments, price impact becomes significant and plays a crucial role. In addition to the information advantage, if the institutional investor also has lower market-implied risk aversion, he will achieve higher trading profits. However, if the institutional investor's market-implied risk aversion is sufficiently high, whether he generates higher trading profits depends on which of the two conflicting effects—information advantage or illiquidity—dominates. In the following, we focus on the risk effect by examining the market-implied risk aversion for both institutional and sophisticated individual investors, before discussing the expected trading profits.

4.2 Market-implied risk aversion: Risk effect

Price impact refers to the change in asset prices resulting from investors' trades. A higher price impact means that a demand shock will drive the price higher, potentially reducing investors' trading profits. In the linearity framework studied, the price impact of one investor is determined by the reciprocal of the price sensitivity of the demand functions of his market counterparties (Kyle 1989). When his counterparties have less price sensitivity, or in other words, exhibit more inelastic demand, any deviation from the investor's equilibrium demand at any given price requires a larger price adjustment for the market to absorb the change. As a result, the investor experiences a higher price impact. Intuitively, less elastic demand from counterparties implies that his counterparties are less willing to provide liquidity to the investor, leaving the investor to face a more illiquid market.

In addition to the price impact parameter, the risk-aversion-adjusted conditional variance also has an effect on investors' demand. Specifically, both a higher price impact and a larger risk-aversion-adjusted conditional variance generally lead to a reduction in investors' demand. Throughout this paper, we refer to the sum of the price impact and the risk-aversion-adjusted conditional variance about the asset payoff as the *market-implied* risk aversion.

PROPOSITION **2.** *Suppose that all individual investors are sophisticated. Then we have*

(i) Price impact: $\lambda_1 > \lambda_s$.

individual investors in such a competitive setting.

(ii) *Risk effect:* $\lambda_1 + \xi_1 \geq \lambda_s + \xi_s$ *if* $(n-1)(1-\beta^s) \leq 1$. *Furthermore, if* $(n-1)(1-\beta^s) > 1$, *then* $\lambda_1 + \xi_1 < \lambda_s + \xi_s$ *if and only if*

$$\xi_1 < \left[1 - \frac{\beta^s}{(n-1)(n-2)(1-\beta^s)^2}\right]\xi_s, \tag{32}$$

and $\lambda_1 + \xi_1 > \lambda_s + \xi_s$ *otherwise.*

Part (i) of Proposition 2 demonstrates that the institutional investor consistently faces a higher price impact compared to sophisticated individual investors. This discrepancy stems from the fact that risk-averse sophisticated individual investors trade less aggressively in response to price movements. Intuitively, two key factors drive this behavior. First, sophisticated individual investors possess less precise information, which amplifies their exposure to fundamental risk and consequently, reduces their trade aggressiveness. Second, their trading decisions incorporate information inferred from the equilibrium price—a consideration that the institutional investor does not need to account for—further diminishing their willingness to provide liquidity in response to price changes. As a result, sophisticated individual investors exhibit a reduced willingness to provide liquidity, effectively rendering the market more illiquid for the institutional investor. Consequently, the institutional investor experiences a larger price impact relative to individual investors.

Part (ii) of Proposition 2 indicates that the risk effect for the institutional investor is relatively weaker only when sophisticated individual investors place less reliance on price information to infer fundamentals. In markets with sufficiently large noise-trading volume, excessive noise becomes embedded in prices, reducing their informativeness in forecasting fundamentals. Consequently, sophisticated individual investors place less weight on the informational content of prices and trade more aggressively against price movements. This increased trading activity enhances liquidity for the institutional investor, thereby reducing his price impact and weakening the associated risk effect.

More explicitly, in the extreme case where sophisticated individual investors entirely disregard the informational content embedded in prices, the price impacts are characterized as follows

$$\lambda_1 = \left(\frac{n-1}{\lambda_s + \xi_s}\right)^{-1}, \quad \lambda_s = \left(\frac{n-2}{\lambda_s + \xi_s} + \frac{1}{\lambda_1 + \xi_1}\right)^{-1}.$$

According to Part (i) of Proposition 2, the institutional investor should experience a weaker risk effect. Otherwise, if sophisticated individual investors have lower market-implied risk aversion $\lambda_s+\xi_s$, they become more willing to provide liquidity to the institutional investor. This, in turn, would lead to a lower price impact λ_1 for the institutional investor, contradicting the result established in Part (i) of Proposition 2. Therefore, when sophisticated individual investors disregard the informational content of prices, the more informed institutional investor faces a relatively weaker negative risk effect. The above arguments also apply to the setting where the individual investors learn from the price but the inference sensitivity β^s is small.

4.3 Expected trading profits: information effect vs. risk effect

This subsection examines the impact of noise-trading volume. The parameter τ_u^{-1} represents the uncertainty associated with noise trading, and is used to measure the trading volume by noise traders (Kovalenkov and Vives 2014; Nezafat and Schroder 2023).[15] Proposition 3 discusses, and Figure 1 numerically illustrates, how noise-trading volume affects the relative profits of the institutional and sophisticated individual investors.

PROPOSITION **3.** *Suppose that all individual investors are sophisticated. Then when the noise-trading volume τ_u^{-1} is either sufficiently large or sufficiently small, the institutional investor consistently outperforms sophisticated individual investors.*

When the noise-trading volume is sufficiently large (i.e., τ_u is sufficiently small), excessive noise is incorporated into the price. In this case, the price becomes less informative for predicting the fundamental value, and individual investors will disregard the information contained in the price when making optimal demand schedules, i.e., $\beta^s \to 0$ (Eyster et al. 2019). As explained earlier in (32), due to the lower conditional variance faced by the institutional investor, the market-implied risk aversion for the institutional investor in this scenario is lower than that for sophisticated individual investors. As a result, the institutional investor experiences a relatively weaker negative risk effect. Therefore, the institutional investor, who also benefits from

[15] Noise-trading volume is typically defined as the expected absolute value of the amount traded by noise traders. We can observe that $\mathbb{E}|nu| = n\sigma_u\sqrt{2/\pi}$ and consequently, it is reasonable to interpret σ_u as the noise-trading volume.

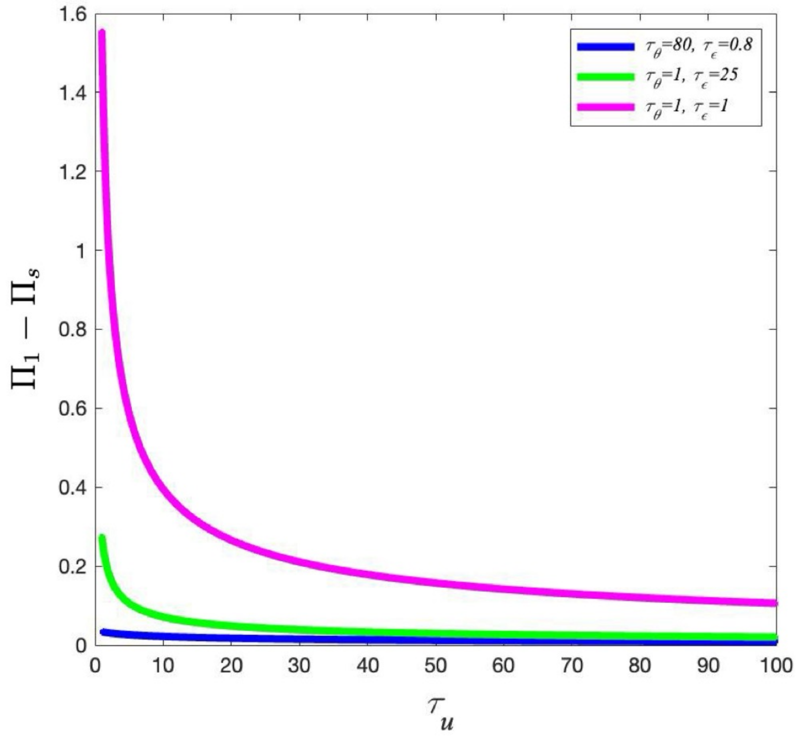


Figure 1: **The effect of noise-trading volume on difference between trading profits.** This figure illustrates the impact of noise-trading volume on the difference between the trading profits of the institutional investor (Π_1) and those of the sophisticated individual investors (Π_s) under three distinct parameter sets $\{\tau_\theta, \tau_\epsilon\}$. The risk aversion parameter is set to $\rho = 2$ and the total number of investors in the market is $n = 10$.

a stronger information effect, faces a larger trading opportunity and achieves higher trading profits than sophisticated individual investors.

When the noise-trading volume is sufficiently low (i.e., τ_u is sufficiently large), sophisticated individual investors rely heavily on price to infer fundamental information, causing the inference sensitivity β^s to approach its upper bound. As a result, the institutional investor experiences a higher market-implied risk aversion and a relatively stronger risk effect (Part (ii) of Proposition 2). However, in contrast to perfectly competitive markets, the price does not fully reveal all the information in the market due to imperfect competition,[16] meaning that the information advantage of the institutional investor survives. Furthermore, more informed signals directly influence the information effect, while indirectly affecting the risk effect through the interaction between the institutional and individual investors. As a result, the information advantage effect

[16]Proposition 7.2 in Kyle (1989) indicates that the price in his model never reveals more than half of the private precision of informed speculators.

of the institutional investor dominates the risk effect driven by higher market-implied risk aversion, leading to higher trading profits for the institutional investor compared to sophisticated individual investors.

Observation **1.** *The institutional investor consistently outperforms sophisticated individual investors for intermediate values of* τ_u.

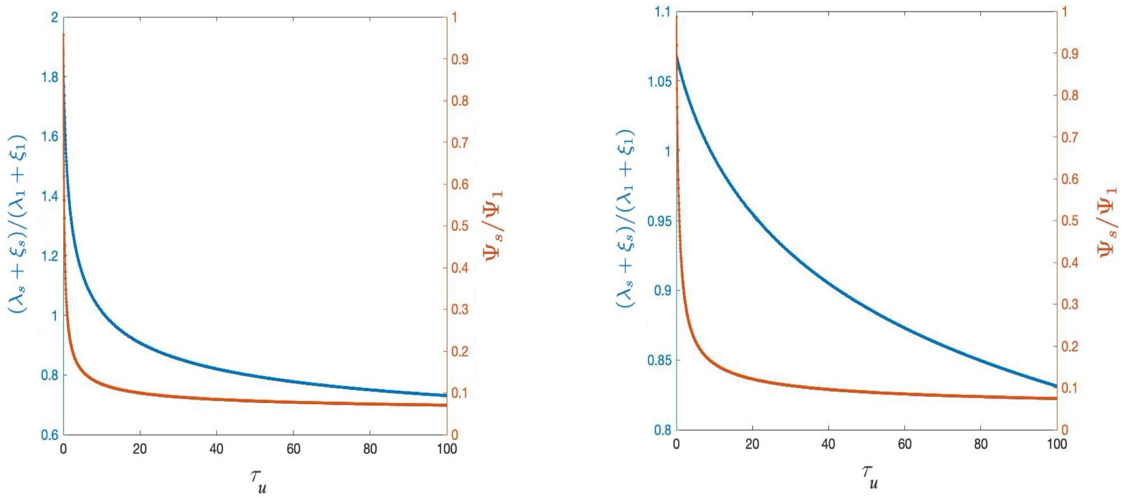


Figure 2: **The impact of noise-trading volume on ratio of market-implied risk aversion and ratio of informational efficiency.** This figure plots how noise-trading volume affects the ratio of market-implied risk aversion $(\lambda_s+\xi_s)/(\lambda_1+\xi_1)$ and the ratio of informational efficiency Ψ_s/Ψ_1. Each panel features two y-axes: the left y-axis (displayed in blue) corresponds to the ratio of market-implied risk aversion, while the right y-axis (displayed in orange) represents the ratio of informational efficiency. The x-axis spans the range of τ_u from 0.01 to 100. In the left panel, $\tau_\theta = 25$ and $\tau_\epsilon = 5$, whereas in the right panel, $\tau_\theta = 100$ and $\tau_\epsilon = 1$. The other parameters are set to $\rho = 2$ and $n = 10$.

While Proposition 3 demonstrates that the institutional investor can beat sophisticated individual investors in the two extreme cases of sufficiently large and small noise trading, Figure 1 illustrates that the results in Proposition 3 also hold for intermediate values of τ_u. In other words, the information advantage of the institutional investor always dominates the risk effect, ensuring that the institutional investor consistently outperforms sophisticated individual investors. We further decompose the two components, i.e., the information effect and risk effect in Figure 2. First, as shown in Figure 2, the relative risk effect of the institutional investor compared to sophisticated individual investors increases with the precision of noise trading τ_u. When noise-trading volume is large (small) (i.e., τ_u is small (large)), the institutional investor experiences a relatively weaker (stronger) negative risk effect than that of sophisticated individual investors. Second, due to the information advantage, the institutional investor always experiences a higher information effect, as evidenced by the fact that the value of Ψ_s/Ψ_1 is always lower than 1. Third, as τ_u increases, on the one hand, sophisticated individual investors infer more information from price (i.e., $\mathrm{Var}[\theta|y_1, y_i, p]$ decreases). On the other hand, the variance of the asset return decreases as well (i.e., $\mathrm{Var}(\theta - p)$ decreases), see Appendix A.2 for more detailed illustrations. As a result, the relative strength of the institutional investor's information effect further amplifies, ensuring that the institutional investor always outperforms sophisticated individual investors.

5 Institutional investors and naive individuals

This section examines the case where all individual investors are naive, being unaware of their price impact and considering themselves price-takers (i.e., $m = 0$), and investigates whether the institutional investor can beat naive individual investors.

5.1 Equilibrium

We first establish a linear Bayesian Nash equilibrium and then calculate the expected trading profits.

Proposition 4. *Suppose that all individual investors are naive. Then there exists a linear Bayesian Nash equilibrium determined by the following system of equations*

$$\gamma = n\left[\frac{1}{\lambda_1+\xi_1} + \frac{(n-1)(1-\beta^n)}{\xi_n}\right]^{-1}, \tag{33}$$

$$\pi_1 = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{(n-1)\alpha_1^n}{\xi_n}\right],$$

$$\pi_n = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^n}{\xi_n}\right], \tag{34}$$

$$\lambda_1 = \left[\frac{(n-1)(1-\beta^n)}{\xi_n}\right]^{-1}, \tag{35}$$

where

$$\alpha_1^n = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_n^2}{\tau_u}}\frac{\pi_1}{\pi_n}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}}, \quad \alpha_o^n = \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_n^2}{\tau_u}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}}, \tag{36}$$

$$\beta^n = \frac{\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}\frac{1}{(n-2)\pi_n}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}}, \quad z_n = \frac{\gamma}{(n-2)\pi_n}. \tag{37}$$

Proposition 4 establishes the financial market equilibrium in terms of equilibrium price coefficients π_1 and π_n and traders' price impacts λ_1 and λ_n. The equilibrium system (33)-(35) can be directly obtained from the system (16)-(20) by setting $m = 0$ and eliminating equation (17), which corresponds to sophisticated individual investors. As in Kyle (1989), Equation (35) reveals that the institutional investor's price impact equals the reciprocal of the sum of the price sensitivities of all naive individual investors. Additionally, since naive individual investors neglect their own price impact, equation (17) is omitted from the system.

The (expected) trading profits of naive individual investors are given by

$$\Pi_n := \mathbb{E}[(\theta - p)x_j^*] = \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_j, p]}{\xi_n}, j = 2, ..., n.$$

We also define informational efficiency for predicting the asset return $\theta - p$ for naive individual investors:

$$\Psi_n := \frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta - p|y_1, y_j, p]}{\mathrm{Var}(\theta - p)}, j = 2, ..., n. \tag{38}$$

Similar to Equations (28) and (29), naive individual investors' trading profits are determined by two key factors: the information effect, captured by information efficiency, and the risk effect, which is solely characterized by the risk-aversion-adjusted conditional variance of the fundamental. Proposition 5 establishes the condition under which the institutional investor underperforms relative to naive individual investors.

Proposition 5. *Suppose that all individual investors are naive. Then when τ_u is sufficiently small, the institutional investor underperforms naive individual investors if the following condition holds: $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$.*

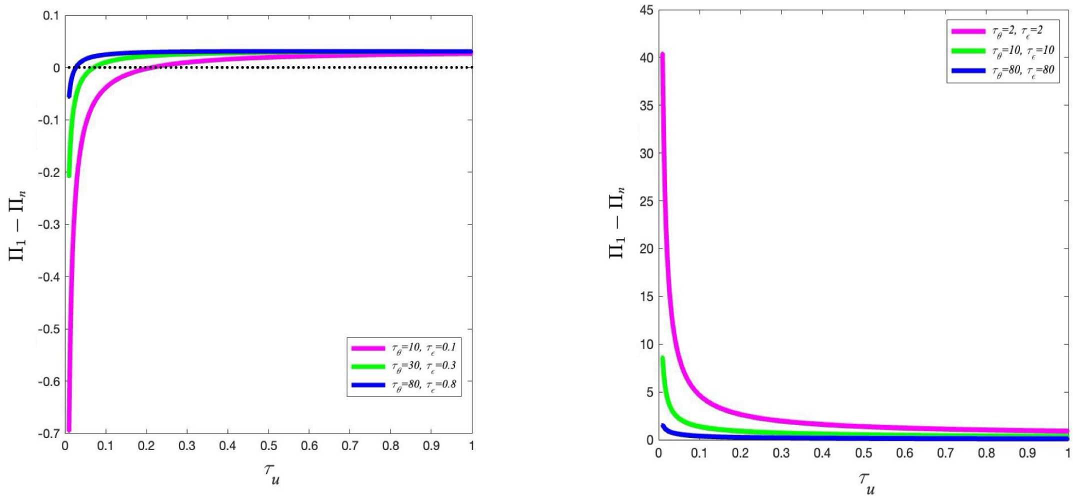


Figure 3: **The effect of noise-trading volume on difference between trading profits**: This figure illustrates how noise-trading volume affects the difference between the trading profits of the institutional investor (Π_1) and those of the naive individual investors (Π_n). The left panel presents the results for three sets of parameters $\{\tau_\theta, \tau_\epsilon\}$ that satisfy the condition of the institutional investor underperforming naive individual investors in Proposition 5. The right panel displays the results for three additional sets of parameters $\{\tau_\theta, \tau_\epsilon\}$ that do not meet the condition specified in Proposition 5. The risk aversion parameter is set to $\rho = 2$, and the total number of investors in the market is $n = 10$.

The institutional investor may underperform naive individual investors, even though he can always beat sophisticated individual investors. This occurs under conditions where both the noise-trading volume (i.e., $\operatorname{Var}(u)$) and the noise-to-signal ratio (i.e., $\operatorname{Var}(\epsilon_i)/\operatorname{Var}(\theta)$) are sufficiently large. Intuitively, the institutional investor underperforms when the information effect is weak and the risk effect is significant. When the noise-trading volume is sufficiently large (i.e., τ_u is sufficiently small), the institutional investor's information effect diminishes. This is because both the information efficiency measures Ψ_1 and Ψ_n approach one, as indicated by Equations (30) and (38).[17] Additionally, when the noise-to-signal ratio is sufficiently large, naive individual investors receive imprecise information. The resulting increase in payoff uncertainty, due to less precise private signals, reduces the price sensitivity of naive individual investors' demands. This reduction in price sensitivity, in return, amplifies the price impact of the institutional investor. Consequently, in this scenario, the risk effect becomes more pronounced for the institutional investor. To facilitate a clearer understanding of Proposition 5,

[17] This is because $\operatorname{Var}(\theta - p) \to \infty$ as $\tau_u \to 0$ (the coefficient γ is bounded away from zero for all sufficiently small τ_u, see the proof of Proposition 5), while the conditional variances $\operatorname{Var}[\theta|y_1, y_2, ..., y_n]$ and $\operatorname{Var}[\theta|y_1, y_i, p]$ are bounded above by $\operatorname{Var}(\theta)$.

we next provide a benchmark analysis.

LEMMA **1.** *When (1) neither institutional investors nor naive investors learn from prices (corresponding to a sufficiently large noise-trading volume), and (2) each investor makes decision based only on the prior information about fundamental $\theta \sim \mathcal{N}(0, 1/\tau_\theta)$ (corresponding to a sufficiently large noise-to-signal ratio), a linear Bayesian Nash equilibrium exists with the equilibrium price given by*

$$p = \gamma u,$$

where

$$\gamma = n\left(\frac{k}{\lambda_1+\xi_1}+\frac{n-k}{\xi_n}\right)^{-1}, \quad \lambda_1 = \left(\frac{k-1}{\lambda_1+\xi_1}+\frac{n-k}{\xi_n}\right)^{-1}, \quad \xi_1 = \xi_n = \frac{\rho}{\tau_\theta},$$

k ($1 \le k < n$) represents the number of institutional investors—those who account for their price impact when making decisions—while n is the total number of investors in the market. Furthermore, the expected trading profits satisfy

$$\mathbb{E}[(\theta - p)x_l^*] = \frac{\gamma^2}{(\lambda_1+\xi_1)\tau_u} < \mathbb{E}[(\theta - p)x_j^*] = \frac{\gamma^2}{\xi_n \tau_u},$$

where x_l^* and x_j^* denote the equilibrium demands of institutional and naive investors, respectively. That is, institutional investors always underperform naive individual investors.

The benchmark in Lemma 1 offers a clear intuition for the results. Consider the scenario that all investors are naive and make decisions based solely on their prior information. Suppose now one investor becomes sophisticated (i.e., the institutional investor in Lemma 1) and begins to account for his price impact, this change enhances the benefit not only for himself but also for other naive investors. However, because naive individual investors trade more aggressively, the benefits are higher for them. Consequently, the institutional investor underperforms relative to naive individual investors.

OBSERVATION **2.** *The institutional investor outperforms the naive individual investors when τ_u or $\tau_\epsilon/\tau_\theta$ is relatively large.*

Figure 3 further shows numerical simulations under three representative parameter sets for cases where the condition in Proposition 5 is violated and satisfied, respectively. And in

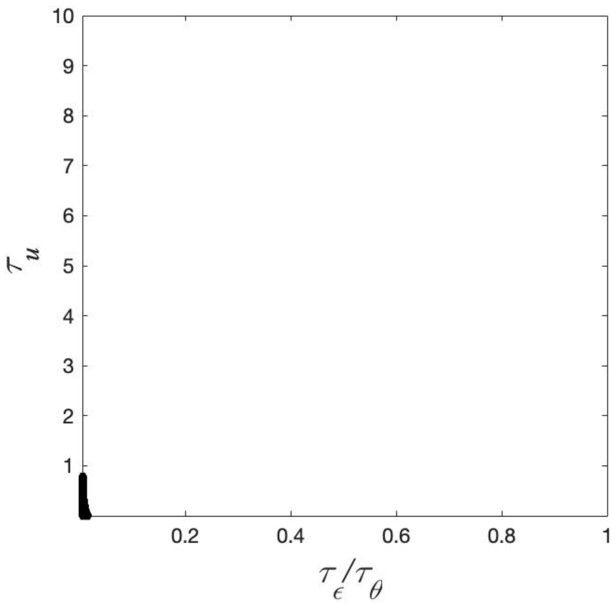


Figure 4: This figure presents the parameter regions—defined by $\tau_\epsilon/\tau_\theta$ on the x-axis and τ_u on the y-axis—that determine whether the institutional investor can beat naive individual investors. The black area represents the set of parameters under which the institutional investor cannot outperform naive individual investors, while the blank area corresponds to the set of parameters under which the institutional investor can beat naive individual investors.

Figure 4, we present the parameter conditions, regarding τ_u and $\tau_\epsilon/\tau_\theta$, that is necessary for the institutional investor to achieve superior performance compared to naive individual investors, providing evidence for Observation 2.

5.2 Why does the institutional investor underperform?

We first illustrate the intuition in Table 1. We simplify the main model to a two-player game (the institutional investor (I) versus the individual investor) with two strategies (aggressive (A) or conservative (C) trading strategies). The individual investor can be either sophisticated (S) or naive (N). The payoff matrix on the left represents the real payoffs recognized by investor I and investor S, while the "fictional" payoff matrix on the right reflects the mistakes made by investor N in estimating payoffs as a result of failing to internalize his price impact.

The payoff matrix demonstrates three key characteristics in the main model. First, each investor's trade exerts a price impact. Adopting strategy A influences the equilibrium price and imposes negative externalities on other investors, as $\pi^S(A, A) < \pi^S(C, A)$ and $\pi^S(A, C) <$

$\pi^S(C,C)$ for investor S and $\pi^I(A,A) < \pi^I(A,C)$ and $\pi^I(C,A) < \pi^I(C,C)$ for investor I, because more trading drives the price up more. Second, the institutional investor possesses an information advantage over the individual investor. Investor I tends to trade more aggressively when investor S adopts a conservative strategy, i.e., $\pi^I(A,C) > \pi^I(C,C)$. In contrast, lacking such an information advantage, investor S remains conservative even if investor I trades conservatively, i.e., $\pi^S(C,A) < \pi^S(C,C)$. Combining these two characteristics, when one of the investors has already adopted an aggressive trading strategy, following with aggressive trading is suboptimal, even for an investor with an information advantage (i.e., $\pi^I(A,A) < \pi^I(C,A)$ and $\pi^S(A,A) < \pi^S(A,C)$), which will impose too much impact on equilibrium price. Third, the naive individual investor behaves irrationally. Investor N fails to internalize the impact of his own trading on the market. This leads to a belief that aggressive trading always yields higher payoffs, regardless of the other player's strategy (i.e., investor N believes $\pi^N(A,A) > \pi^N(A,C)$ and $\pi^N(C,A) > \pi^N(C,C)$).

The irrationality of naive individual investors acts as a commitment mechanism, which renders the institutional investor to shrink his trading aggressiveness. As shown by the left table in Table 1, when individual investors recognize their price impact, they understand that more aggressive trading diminishes their profits, particularly since they lack an information advantage. Consequently, sophisticated individual investors would trade the asset conservatively. Facing this situation, the institutional investor, who possesses an information advantage, can exploit profits through aggressive trading. Therefore, the institutional investor achieves a higher expected trading profit than sophisticated individual investors.

However, when naive individual investors consider themselves to be price-takers, they believe their tradings do not impact equilibrium price. As a result, they tend to buy more upon receiving good signals and sell more upon receiving negative signals. This behavior is depicted by the "fictional" payoff matrix in the right table in Table 1. The institutional investor, aware of the irrationality of naive individual investors, anticipates that these investors will trade aggressively. This, in turn, forces the institutional investor to shrink his trading aggressiveness, despite having an information advantage. This is because the price impact is significant now due to the aggressive trading by all naive individual investors. The expansion of trading due to more accurate information cannot compensate for the declining share profits due to sensitive price movement.

Mathematically, we start with the equilibrium described in Proposition 4. Now, suppose there is only one naive individual investor who becomes aware of his price impact and switches to be a sophisticated individual investor. Holding other investors' demand schedules and the equilibrium price form constant, his equilibrium price impact should be

$$\lambda_s = \left[\frac{(n-2)(1-\beta^n)}{\xi_n} + \frac{1}{\lambda_1 + \xi_1} \right]^{-1}.$$

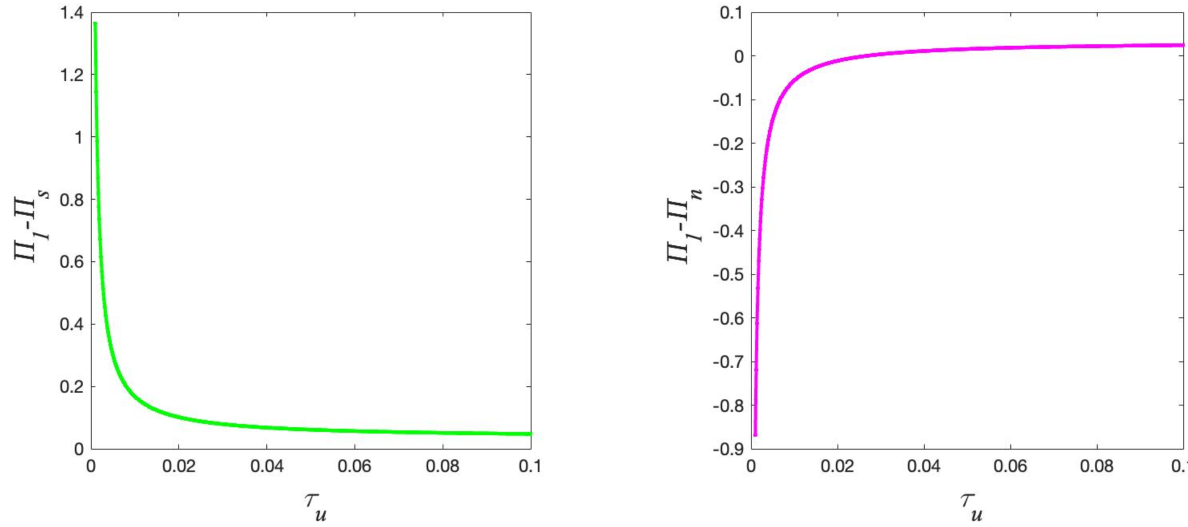
Consequently, the market-implied risk shifts from ξ_n to $\xi_n + \lambda_s$. This implies that, by considering his price impact, the sophisticated individual investor reduces his demand. In this sense, the irrationality of naive individual investors acts as a commitment mechanism, enabling them to impound aggressive trades into the market.[18]

6 Coexistence of institutional, sophisticated, and naive investors

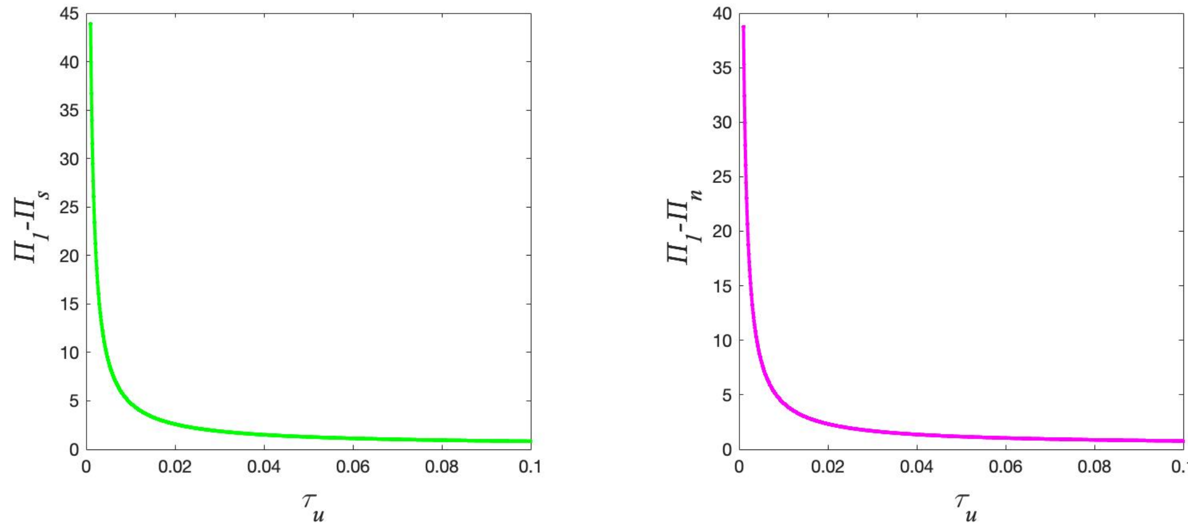
This section numerically investigates the scenario in which all three types of investors participate in the market. Specifically, we examine the effects of increasing the sophistication level of individual investors, represented by a higher value of m, which corresponds to more naive individual investors switching to sophisticated individual investors. We have two main findings: (i) the main results in Subsection 4.3 (Propositions 3) and Subsection 5.1 (Proposition 5) remain robust under this setting; (ii) naive individual investors impose negative externalities on other market participants.

First, Figure 5 illustrates the effect of τ_u on trading profits under two scenarios: when $\tau_\epsilon/\tau_\theta$ is sufficiently small (Panel a) and when it is relatively moderate (Panel b). From Figure 5, we observe that the conclusions of Proposition 3 and Proposition 5 extend to the model

[18] We finally go back to the discussion on the conditions outlined in Proposition 5. The institutional investor's advantage lies in possessing superior information, while the disadvantage stems from naive individual investors committing to aggressive trading. On the one hand, when the noise-to-signal ratio is sufficiently large, the institutional investor's information advantage becomes negligible, and the benefits of this advantage cannot offset the conservative trading behavior. On the other hand, when the noise-trading volume is sufficiently large, the price impact becomes more pronounced, further amplifying the disadvantage.



(a) Panel a



(b) Panel b

Figure 5: **The effect of noise-trading volume on difference between trading profits**: The green line represents the difference between trading profits of the institutional investor (Π_1) and the sophisticated individual investors (Π_s), while the pink line displays the difference between trading profits of the institutional investor (Π_1) and the naive individual investors (Π_n). The parameter values are set to $m = 4$, $n = 10$, $\rho = 2$, and τ_u ranges from 0.001 to 0.1. For other parameters, we set $\tau_\theta = 100$ and $\tau_\epsilon = 1$ (satisfying the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5) in Panel a and $\tau_\theta = 25$ and $\tau_\epsilon = 5$ (which are used in Han and Yang (2013) and do not meet the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5) in Panel b.

incorporating all three types of investors simultaneously. On the one hand, the institutional investor consistently achieves higher trading profits than the sophisticated individual investors, as demonstrated by the left figures in both Panel a and Panel b. On the other hand, when τ_u is sufficiently small, the naive individual investor's trading profits exceed (or fall below) those of the institutional investor depending on whether the condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ holds (or fails), as shown in the right figure of Panel a (Panel b). These findings confirm that the main results in Subsection 4.3 and Subsection 5.1 remain robust.

Second, when naive individual investors perceive themselves as price-takers, they believe that their trading does not influence the equilibrium price. As a result, they tend to buy more upon receiving positive signals and sell more upon receiving negative signals. While this behavior may enable naive individual investors to outperform the institutional investor in certain scenarios, their aggressive trading—driven by irrationality—imposes negative externalities on all market participants. This is evidenced by Figure 6, which shows that the expected trading profits of all investors increase as the level of sophistication m rises.[19] Furthermore, while the number of sophisticated individual investors m affects the magnitude of the trading profits for all three types of investors, it does not alter their relative ordering. In other words, the conclusions regarding the ranking of trading profits among the three types of investors remain robust with respect to changes in m. In Appendix A.3, we do more simulations to test the robustness.

7 Discussions

7.1 Empirical evidences on individual investors with price impact

Algorithmic trading has reshaped the execution dynamics of individual investor order flow. With the widespread adoption of algorithmic execution strategies, the average trade size of individual investors may, in fact, exceed that of other market participants (Boehmer et al. 2021). Furthermore, research indicates that individual investors exhibit herding behavior, resulting in correlated buying and selling activities that can significantly affect asset prices. Barber et al. (2009) document that trades executed by individual investors at a major discount broker and

[19]Figure 6 illustrates how the trading profits of the three types of investors vary with the level of sophistication m in the market. Here, we consider two cases: $n = 10$ (where m ranges from 0 to 9) and $n = 30$ (where m ranges from 0 to 29). In Appendix A.1, we do more simulations to illustrate that our conclusions are robust.

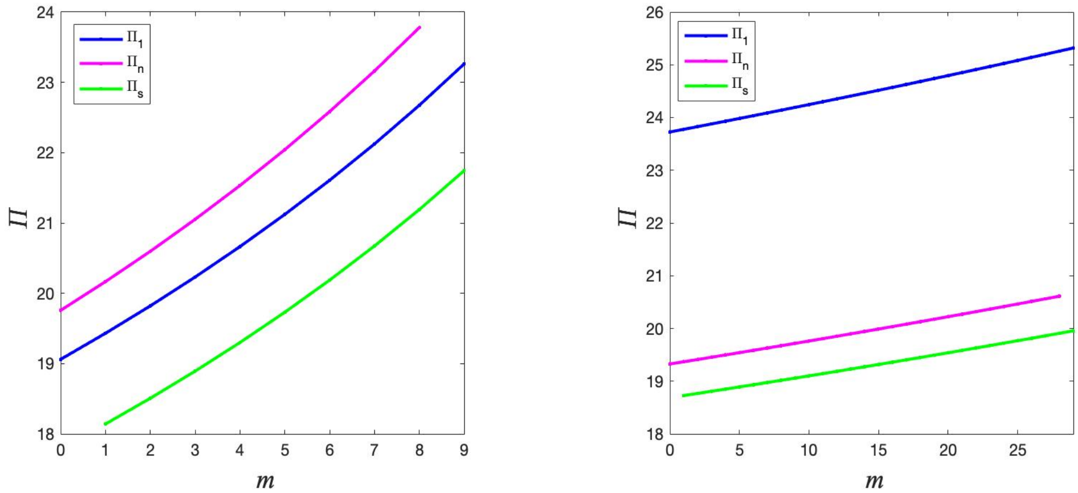


Figure 6: **The effect of the level of sophistication on trading profits**: This figure plots how the sophistication level m affects the trading profits of the institutional investor (Π_1), the sophisticated individual investors (Π_s), and the naive individual investors (Π_n). We set $n = 10$ in the left panel, while $n = 30$ in the right panel. The other parameter values are set to $\tau_\theta = 100$, $\tau_u = 0.001$, $\tau_\epsilon = 1$, and $\rho = 2$. The parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ holds in the left panel but fails in the right panel.

a large retail brokerage are systematically correlated, with their net monthly purchases and sales displaying persistence over time. This pattern suggests that individual investors tend to remain net buyers (or net sellers) of the same stocks in subsequent months. Jackson (2003) provides further evidence using Australian data from 1991–2002, showing that the trading behavior of individual investors was coordinated rather than independent. Expanding on these findings, Barber et al. (2008) analyze eighteen years of high-frequency U.S. stock transaction data and demonstrate that retail trading influences prices over both short and long horizons, particularly for smaller stocks. More recently, studies have focused on the surge in retail investor participation. Eaton et al. (2022) examine Robinhood, an online retail brokerage, and identify a substantial cohort of momentum traders whose herding behavior exacerbates inventory risks and impairs liquidity in stocks with high retail investor interest. A growing body of literature further supports the notion that retail trades exert a substantial price impact (Kaniel et al. 2008; Hvidkjaer 2008; Foucault et al. 2011; Kelley and Tetlock 2013; Peress and Schmidt 2020).

7.2 Empirical relevance for the main theoretical results

Our paper offers three main sets of testable predictions. The first prediction concerns the irrational behavior of investors who are unable to accurately estimate the impact of their trades on market prices. In Section 5.2, we argue that naive individual investors, due to their inability to accurately assess their price impact, irrationally increase their trading volumes, leading to overtrading. Existing literature primarily attributes overtrading to investor overconfidence (Odean 1999; Abramov and Brown 1997; Chuang and Susmel 2011). However, it is important to note that several factors prevent individual investors from accurately estimating the price impact of their trading activities. These include cognitive biases such as overconfidence, as well as a lack of understanding of market depth due to insufficient trading experience or financial knowledge, or being influenced by high-frequency trading and algorithmic strategies. Specifically, ordinary investors often lack the ability to discern the market behavior of high-frequency and algorithmic traders, whose presence tends to amplify price volatility.

The second prediction concerns the performance of institutional investors and sophisticated individual investors. Section 4.3 demonstrates that the institutional investor can consistently outperform sophisticated individual investors. This result provides a new perspective to explain the phenomena identified by Shapira and Venezia (2001) and Hu et al. (2024). The former find that, compared to independent accounts (which correspond to sophisticated individual investors in our study, who make independent investment decisions), professionally managed accounts (managed by professional fund managers, corresponding to institutional investors in our paper) tend to exhibit slightly higher profitability. The latter find that institutions employing more complex strategies (reflecting the information advantage in our study) generally outperform retail investors relying on simpler strategies. In our paper, we categorize the factors influencing investor performance into informational effects and risk effects. We find that the institutional investor's informational advantage consistently dominates the risk effects, leading to his superior performance over rational individual investors.

Our main findings regarding the performance comparison between institutional investors and naive individual investors yield a third prediction. Proposition 5 demonstrates that, under certain market conditions, naive individual investors can outperform institutional investors despite the latter's information advantage. This result offers a potential explanation for the findings of Zhong (2022), which suggests that institutional investors do not consistently outperform the market. An implication of our model is that this underperformance may stem not from a lack of skill but rather from high levels of market noise and the relatively low precision of private signals.

7.3 Implications on overtrading policies

Current financial trading regulations primarily aim to enhance investor protection by mitigating risks associated with improper financial advice, information asymmetry, and hidden fees. For instance, FINRA rules explicitly prohibit brokers from generating commissions through excessive or unnecessary trading, a practice commonly referred to as "churning." Similarly, MiFID II has strengthened regulatory requirements for investment advisors, mandating that they account for an investor's risk tolerance, investment objectives, and financial situation when providing advice.

A key insight from our findings is that irrational overtrading by investors imposes negative externalities on all market participants, as discussed in Section 6. Therefore, regulatory efforts should not only focus on protecting investors from harmful advice but also address the broader market consequence of excessive trading. This suggests that existing investor protection and market stability regulations should expand beyond financial advisory practices to incorporate direct measures aimed at curbing irrational trading behavior, thereby mitigating systemic risks and promoting overall market efficiency.

8 Conclusions

We explore a noisy imperfectly competitive market in which an institutional investor possesses all the information in the market, while individual investors hold only a single piece of asymmetric information alongside a public signal. Among these individual investors, some internalize their price impact and are categorized as "sophisticated," whereas others perceive themselves as price-takers and are labeled "naive." We establish the conditions under which the institutional investor can or cannot outperform individual investors in terms of trading profits. Our findings reveal that the institutional investor consistently outperforms sophisticated individual investors under all market conditions. However, when both the noise-trading volume and the noise-to-signal ratio reach sufficiently high levels, the institutional investor fails to outperform naive individual investors. This occurs because naive individual investors, who neglect their price impact, tend to trade aggressively, compelling the institutional investor to reduce his own trading aggressiveness.

Appendix

A Further discussions

This appendix provides additional discussions on alternative performance measures beyond expected trading profits, as well as an examination of the robustness of the results presented in the main body of this paper.

A.1 Other measures for performance

In the main text, we compare the trading profits of the institutional investor with those of sophisticated and naive individual investors. Here, we extend the discussion by examining two additional performance measures: certainty equivalent and standardized expected trading profit (adjusted for information precision).

We begin by analyzing the certainty equivalent, which serves as a risk-adjusted measure of investor performance. The certainty equivalent for a strategic investor is given by

$$\begin{aligned} CE &:= -\frac{1}{\rho}\log\left(-\mathbb{E}[-\exp\{-\rho x(\theta-p)\}]\right) \\ &= -\frac{1}{\rho}\log\left(-\mathbb{E}[\mathbb{E}(-\exp\{-\rho x(\theta-p)\}|\mathcal{F})]\right), \end{aligned} \tag{39}$$

where $\mathcal{F}$ denotes the information set of the investor including the price p, $x = \frac{\mathbb{E}[\theta|\mathcal{F}]-p}{\lambda+\xi}$ represents the optimal equilibrium demand, $\xi = \rho\,\mathrm{Var}[\theta|\mathcal{F}]$ is the risk-aversion-adjusted conditional variance, and the second equality follows from the law of total expectation. Then it follows from (39) that

$$
\begin{aligned}
CE &= -\frac{1}{\rho}\log\left(-\mathbb{E}\left[\mathbb{E}\left(-\exp\left\{-\rho\frac{(\theta-p)\mathbb{E}(\theta-p|\mathcal{F})}{\lambda+\xi}\right\}\middle|\mathcal{F}\right)\right]\right)\\
&= -\frac{1}{\rho}\log\left(-\mathbb{E}\left[-\exp\left\{-\rho\frac{\mathbb{E}^2(\theta-p|\mathcal{F})}{\lambda+\xi}+\frac{\rho^2}{2}\frac{\mathbb{E}^2(\theta-p|\mathcal{F})}{(\lambda+\xi)^2}\mathrm{Var}(\theta-p|\mathcal{F})\right\}\right]\right)\\
&= -\frac{1}{\rho}\log\left(-\mathbb{E}\left[-\exp\left\{-\rho\frac{\lambda+\xi/2}{(\lambda+\xi)^2}\mathbb{E}^2(\theta-p|\mathcal{F})\right\}\right]\right)\\
&= -\frac{1}{\rho}\log\left(\frac{1}{\sqrt{1+2\rho\frac{\lambda+\xi/2}{(\lambda+\xi)^2}\mathrm{Var}[\mathbb{E}(\theta-p|\mathcal{F})]}}\right)\\
&= -\frac{1}{\rho}\log\left(\frac{1}{\sqrt{1+2\rho\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta-p)-\mathrm{Var}(\theta|\mathcal{F})]}}\right)\\
&= \frac{1}{2\rho}\log\left(1+2\rho\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta-p)-\mathrm{Var}(\theta|\mathcal{F})]\right),
\end{aligned}
$$

where the penultimate equality follows from the law of total variance. The certainty equivalent of a price-taking investor can be similarly derived by directly setting $\lambda = 0$.

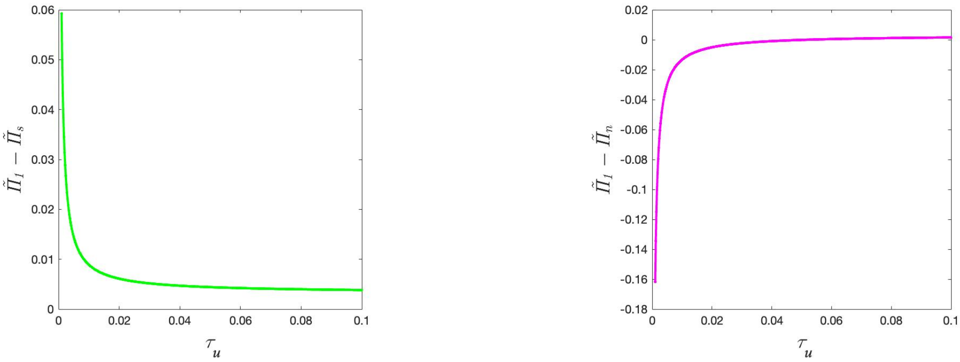
We observe that CE is a monotonic transformation of $\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta-p)-\mathrm{Var}(\theta|\mathcal{F})]$. Consequently, it is reasonable to compare the measure $\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta-p)-\mathrm{Var}(\theta|\mathcal{F})]$ across different investors. Indeed, our numerical analysis confirms that all results in the paper qualitatively hold under this new risk-adjusted measure $\frac{\lambda+\xi/2}{(\lambda+\xi)^2}[\mathrm{Var}(\theta-p)-\mathrm{Var}(\theta|\mathcal{F})]$.[20]

Furthermore, recognizing that information acquisition is costly and that signal precision varies across investors, we standardize the expected trading profit by information precision. Specifically, we define the standardized trading profits as

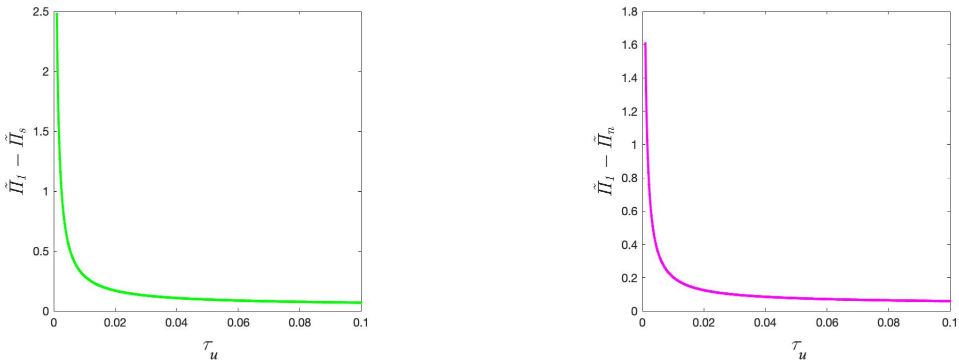
$$
\tilde{\Pi}_1 := \Pi_1/\sqrt{\tau_\theta+n\tau_\epsilon},\quad \tilde{\Pi}_s := \Pi_s/\sqrt{\tau_\theta+2\tau_\epsilon+\Theta_s},\quad \tilde{\Pi}_n := \Pi_n/\sqrt{\tau_\theta+2\tau_\epsilon+\Theta_n},
$$

replacing the original trading profits Π_1, Π_s, and Π_n. Our analysis demonstrates that the conclusions derived from expected trading profits in the baseline model remain robust even when this standardized measure, which accounts for information acquisition costs, is applied. This is evidenced by the results shown in Figures 7, 8, and 9.

[20]The numerical results are available upon request from the authors.

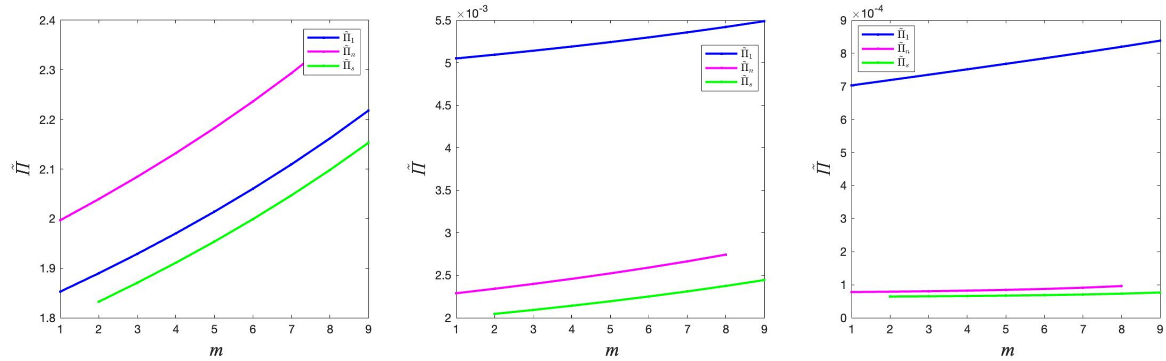


(a) Panel a

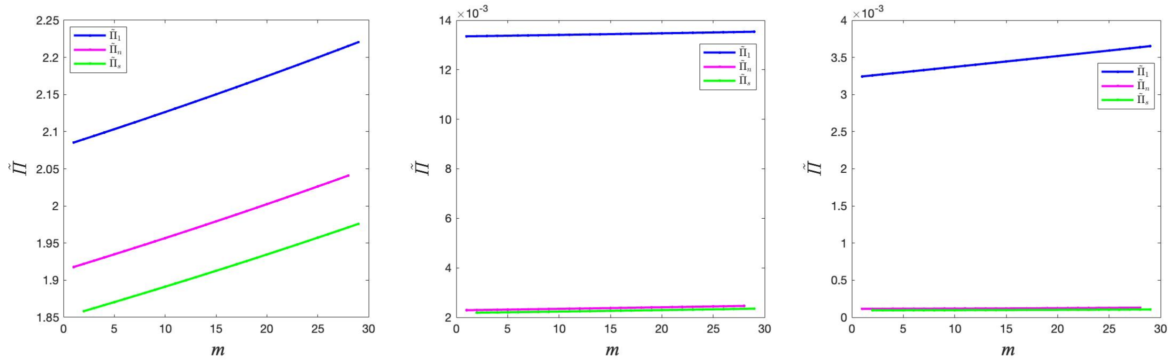


(b) Panel b

Figure 7: **Robustness check for other measures for performance.** Corresponding to Figures 5, the green line represents the difference between the standardized trading profits of the institutional investor ($\tilde{\Pi}_1$) and those of the sophisticated individual investors ($\tilde{\Pi}_s$) in general model, while the pink line displays the difference between the standardized trading profits of the institutional investor ($\tilde{\Pi}_1$) and those of the naive individual investors ($\tilde{\Pi}_n$). The parameter values are set as $m = 4$, $n = 10$, $\rho = 2$, and τ_u ranges from 0.001 to 0.1. For other parameters, in Panel a, we set $\tau_\theta = 100$, and $\tau_\epsilon = 1$, which satisfies the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5. In Panel b, we set $\tau_\theta = 25$ and $\tau_\epsilon = 5$, which does not satisfy the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5.

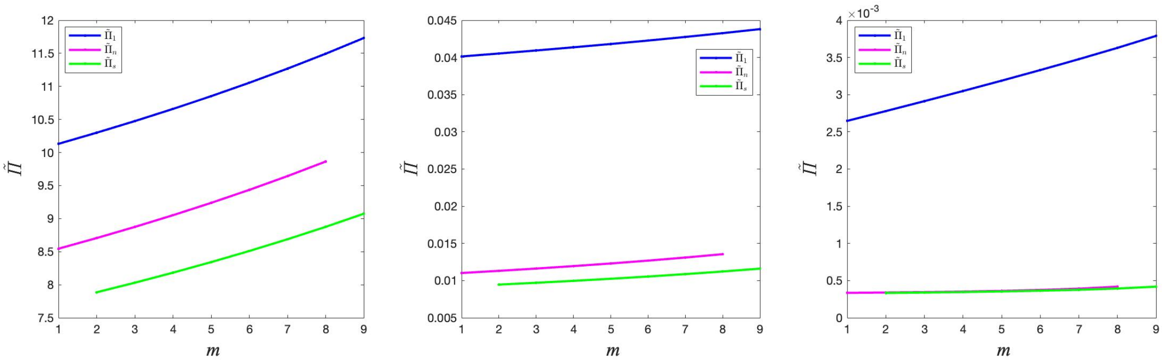


(a) Panel a

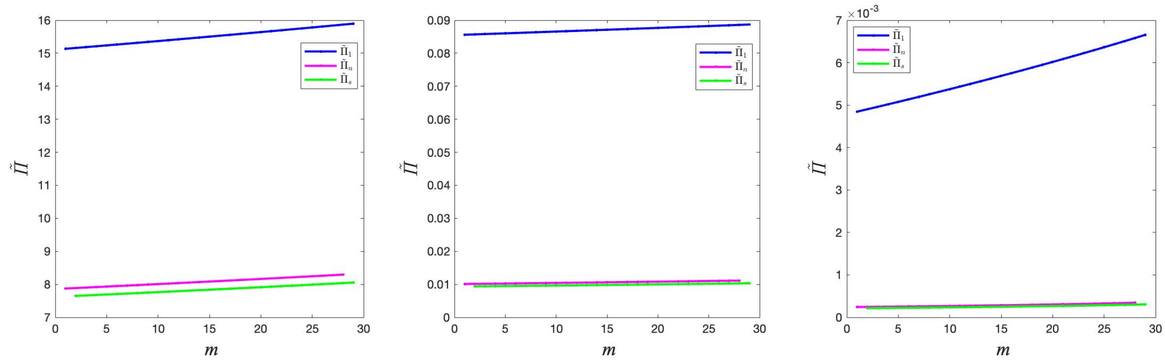


(b) Panel b

Figure 8: **Robustness check for other measures for performance.** Corresponding to Figures 6 and 11, Panel a shows the effect of m on the standardized trading profits of the three types of investors for $n = 10$, and the values of τ_u from left to right are set as $0.001, 1$, and 100. Panel b displays the effect of m on the standardized trading profits of the three types of investors for $n = 30$, and the values of τ_u from left to right are set as $0.001, 1$, and 100 in the general model. The remaining parameter values are $\tau_\theta = 100, \tau_\epsilon = 1$, and $\rho = 2$.



(a) Panel a



(b) Panel b

Figure 9: **Robustness check for other measures for performance.** Corresponding to Figures 12, Panel a shows the effect of m on the standardized trading profits of the three types of investors for $n = 10$, and the values of τ_u from left to right are set as 0.001, 1, and 100. Panel b displays the effect of m on the standardized trading profits of the three types of investors for $n = 30$, and the values of τ_u from left to right are set as $0.001, 1$, and 100 in the general model. The remaining parameter values are $\tau_\theta = 25$, $\tau_\epsilon = 5$, and $\rho = 2$.

A.2 Notes for Figure 2

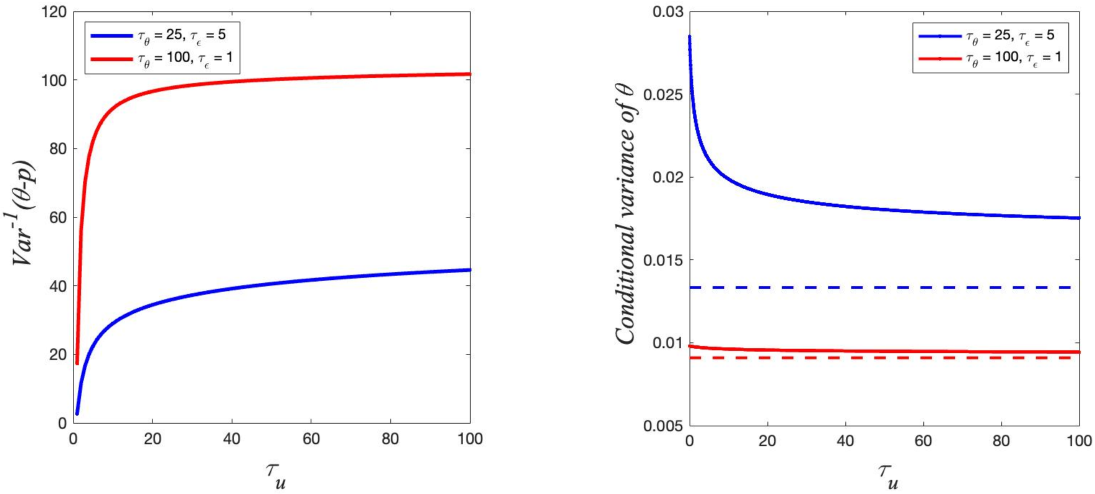
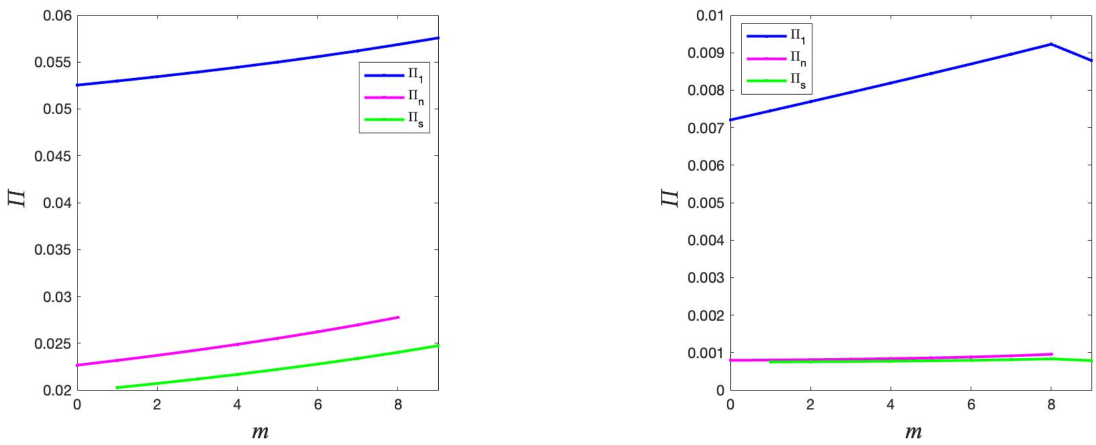


Figure 10: **The impact of noise-trading volume on ratio of $1/\operatorname{Var}(\theta - p)$ and the conditional variance of θ** . The left panel illustrates how noise-trading volume τ_u (ranging from 0.01 to 100) affects $1/\operatorname{Var}(\theta-p)$, while solid lines in the right panel display how it affects investor i's conditional variance $\operatorname{Var}[\theta|y_1, y_i, p]$ and dotted lines represent investor 1's conditional variance $\operatorname{Var}[\theta|y_1, y_2, ..., y_n]$. In each panel, red lines correspond to the results under the parameter setting $\tau_\theta = 25$ and $\tau_\epsilon = 5$, while blue lines represent the results under the setting $\tau_\theta = 100$ and $\tau_\epsilon = 1$. The remaining parameters are fixed at $\rho = 2$ and $n = 10$.

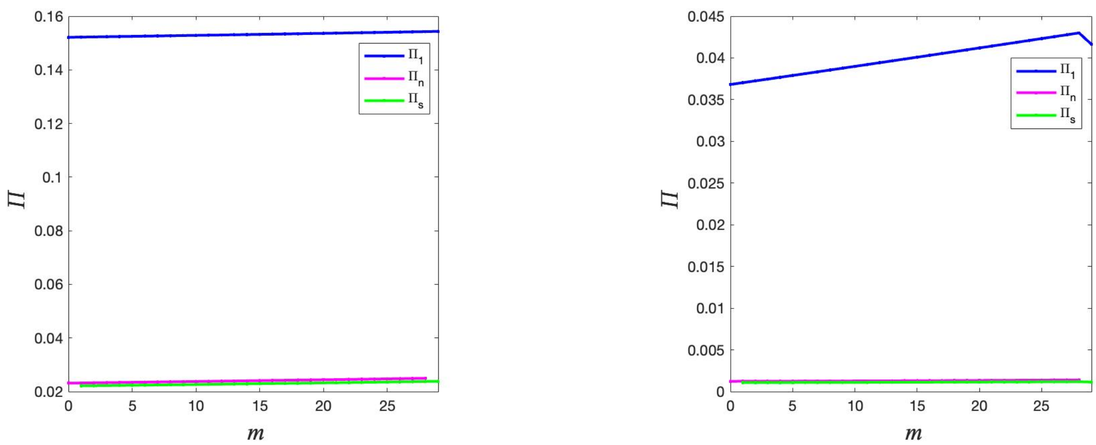
In Figure 2, the institutional investor consistently exhibits a stronger information effect due to his information advantage, as reflected by the fact that the ratio Ψ_s/Ψ_1 is always lower than 1. Moreover, the relative strength of the institutional investor's information effect intensifies as τ_u increases. Intuitively, higher τ_u makes price information more informative. However, since the information effect is characterized by information efficiency, when τ_u is relatively low, the price becomes excessively noisy. To summarize, as illustrated in Figure 10, $\operatorname{Var}(\theta - p) \to \infty$ and $\operatorname{Var}[\theta|y_1, y_i, p]$ remains bounded as $\tau_u \to 0$. Given that $\operatorname{Var}[\theta|y_1, y_2, ..., y_n]$ is a constant, both $\operatorname{Var}(\theta - p) - \operatorname{Var}[\theta|y_1, y_2, ..., y_n]$ and $\operatorname{Var}(\theta - p) - \operatorname{Var}[\theta|y_1, y_i, p]$ are dominated by $\operatorname{Var}(\theta - p)$ as $\tau_u \to 0$, leading to $\Psi_s/\Psi_1 \to 1$ as $\tau_u \to 0$. For the case $\tau_u \to \infty$, we know that $\operatorname{Var}(\theta - p)$, $\operatorname{Var}[\theta|y_1, y_2, ..., y_n]$ and $\operatorname{Var}[\theta|y_1, y_i, p]$ are all bounded. Combining this with the relation $\operatorname{Var}[\theta|y_1, y_2, ..., y_n] < \operatorname{Var}[\theta|y_1, y_i, p]$, we conclude that Ψ_s/Ψ_1 is bounded above by one as $\tau_u \to \infty$.

A.3 Robustness check

Here, we demonstrate the robustness of the conclusions presented in the main body of the paper. As shown in Figures 11 and 12, our findings regarding the ranking of trading profits among the three types of investors remain consistent and robust with respect to variations in m in the model where all three types of investors coexist.



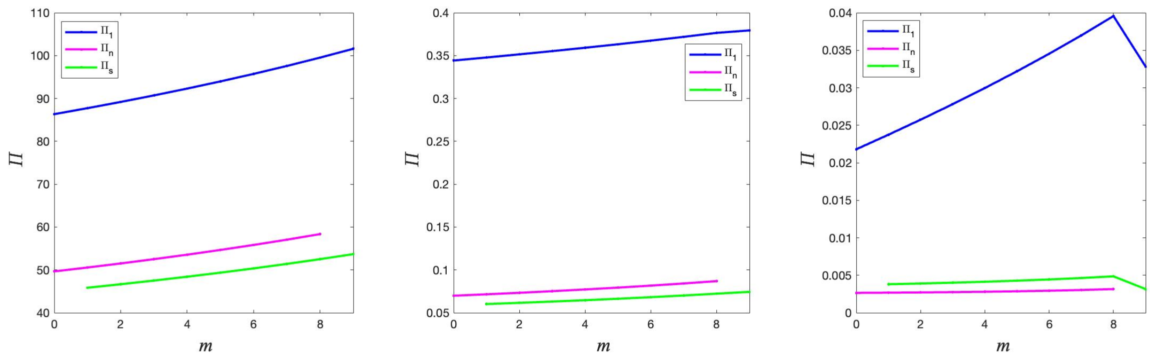
(a) Panel a



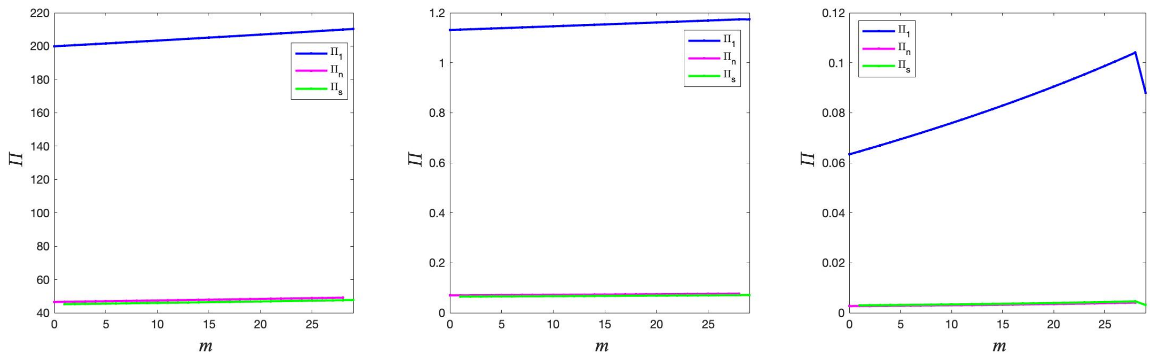
(b) Panel b

Figure 11: **Robustness check.** Panel a illustrates the effect of m on the trading profits of the three types of investors for $n = 10$, and the value of τ_u from left to right are 1 and 100. Panel b demonstrates the effect of m on the trading profits of the three types of investors for $n = 30$, and the values of τ_u from left to right are 1 and 100. The remaining parameter values are $\tau_\theta = 100, \tau_\epsilon = 1$, and $\rho = 2$.

Figures 11 and 12 illustrate how the trading profits of the three types of investors are influenced by the level of sophistication m in the market. In both figures, we examine two scenarios: $n = 10$ and $n = 30$. For $n = 10$, m ranges from 0 to 9, and for $n = 30$, m ranges from 0 to 29. Additionally, Figure 11 considers two cases for τ_u (1 and 100), while Figure 12 considers three cases for τ_u (0.001, 1, 100). These cases are presented in Panels a and b of Figures 11 and 12, with Panel a corresponding to $n = 10$ and Panel b to $n = 30$. The only difference between Figure 11 and 12 lies in the parameter values: we set $\tau_\theta = 100$ and $\tau_\epsilon = 1$ in Figure 11, whereas we use $\tau_\theta = 25$ and $\tau_\epsilon = 5$ in Figure 12. By comparing the two figures, we observe that the level of sophistication m affects only the magnitude of the trading profits for the three types of investors, not their relative ordering. In other words, our conclusions regarding the ranking of trading profits among the three types of investors remain robust with respect to changes in m.



(a) Panel a



(b) Panel b

Figure 12: **Robustness check.** Panel a illustrates the effect of m on the trading profits of the three types of investors for $n = 10$, and the value of τ_u from left to right are 0.001, 1, and 100. Panel b demonstrates the effect of m on the trading profits of the three types of investors for $n = 30$, and the values of τ_u from left to right are 0.001, 1 and 100. The remaining parameter values are $\tau_\theta = 25, \tau_\epsilon = 5$, and $\rho = 2$.

A.4 Two extensions: heterogeneous signal precisions and partial awareness of price impact

In this section, we consider a more general model in which (i) the private signal precision of the two types of individual investors are heterogeneous, and (ii) naive individual investors exhibit partial awareness of their own price impact.

Formally, the private signal for sophisticated individual investor i is given by $y_i = \theta + \epsilon_i$, where $\epsilon_i \sim N(0, 1/\tau_\epsilon^s)$, while the private signal for naive individual investors j is given by $y_j = \theta + \epsilon_j$, where $\epsilon_j \sim N(0, 1/\tau_\epsilon^n)$. Here, the signal precisions satisfy $\tau_\epsilon^s \geq \tau_\epsilon^n \geq \tau_\epsilon^1 > 0$. We define the price impact of naive individual investors as $\lambda_n = \kappa\lambda_s$, where $\kappa \in [0,1]$ represents the degree of naive investors' awareness of their price impact. Specifically, the further κ deviates from 1, the less adequately these investors perceive their influence on prices. Notably, when $\tau_\epsilon^s = \tau_\epsilon^n = \tau_\epsilon^1$ and $\kappa = 0$, the general model reduces to the model presented in Section 6.

Applying the same method as in the system (16)-(20), we can get the following equilibrium system of the general model comprising all three types of investors:

$$
\begin{aligned}
\gamma &= n\left[\frac{1}{\lambda_1+\xi_1} + \frac{m(1-\beta^s)}{\lambda_s+\xi_s} + \frac{(n-m-1)(1-\beta^n)}{\lambda_n+\xi_n}\right]^{-1},\\
\pi_1 &= \frac{\gamma}{n}\left[\frac{\tau_\epsilon^1}{(\tau_\theta+\tau_\epsilon^1+m\tau_\epsilon^s+(n-m-1)\tau_\epsilon^n)(\lambda_1+\xi_1)} + \frac{m\alpha_1^s}{\lambda_s+\xi_s} + \frac{(n-m-1)\alpha_1^n}{\lambda_n+\xi_n}\right],\\
\pi_s &= \frac{\gamma}{n}\left[\frac{\tau_\epsilon^s}{(\tau_\theta+\tau_\epsilon^1+m\tau_\epsilon^s+(n-m-1)\tau_\epsilon^n)(\lambda_1+\xi_1)} + \frac{\alpha_o^s}{\lambda_s+\xi_s}\right],\\
\pi_n &= \frac{\gamma}{n}\left[\frac{\tau_\epsilon^n}{(\tau_\theta+\tau_\epsilon^1+m\tau_\epsilon^s+(n-m-1)\tau_\epsilon^n)(\lambda_1+\xi_1)} + \frac{\alpha_o^n}{\lambda_n+\xi_n}\right],\\
\lambda_1 &= \left[\frac{m(1-\beta^s)}{\lambda_s+\xi_s} + \frac{(n-m-1)(1-\beta^n)}{\lambda_n+\xi_n}\right]^{-1},\\
\lambda_s &= \left[\frac{1}{\lambda_1+\xi_1} + \frac{(m-1)(1-\beta^s)}{\lambda_s+\xi_s} + \frac{(n-m-1)(1-\beta^n)}{\lambda_n+\xi_n}\right]^{-1},\\
\lambda_n &= \kappa\lambda_s,
\end{aligned}
$$

where

$$
\alpha_1^s = \frac{\tau_\epsilon^1 - \Theta^s\frac{\pi_1}{\pi_s(m-1)+\pi_n(n-m-1)}}{\tau_\theta+\tau_\epsilon^1+\tau_\epsilon^s+\Theta^s},
$$

$$
\begin{aligned}
\alpha_o^s &= \frac{\tau_\epsilon^s - \Theta^s v_s}{\tau_\theta + \tau_\epsilon^1 + \tau_\epsilon^s + \Theta^s},\\
\beta^s &= \frac{\Theta^s}{\tau_\theta + \tau_\epsilon^1 + \tau_\epsilon^s + \Theta^s} \frac{1}{\pi_s(m-1) + \pi_n(n-m-1)},\\
\alpha_1^n &= \frac{\tau_\epsilon^1 - \Theta^n \frac{\pi_1}{\pi_s m + \pi_n(n-m-2)}}{\tau_\theta + \tau_\epsilon^1 + \tau_\epsilon^n + \Theta^n},\\
\alpha_o^n &= \frac{\tau_\epsilon^n - \Theta^n q_n}{\tau_\theta + \tau_\epsilon^1 + \tau_\epsilon^n + \Theta^n},\\
\beta^n &= \frac{\Theta^n}{\tau_\theta + \tau_\epsilon^1 + \tau_\epsilon^n + \Theta^n} \frac{1}{\pi_s m + \pi_n(n-m-2)},
\end{aligned}
$$

and v_s, q_s, z_s, Θ^s, v_n, q_n, z_n and Θ^n is defined by (6), (7), (8), (9), (10), (11), (12) and (13), respectively.

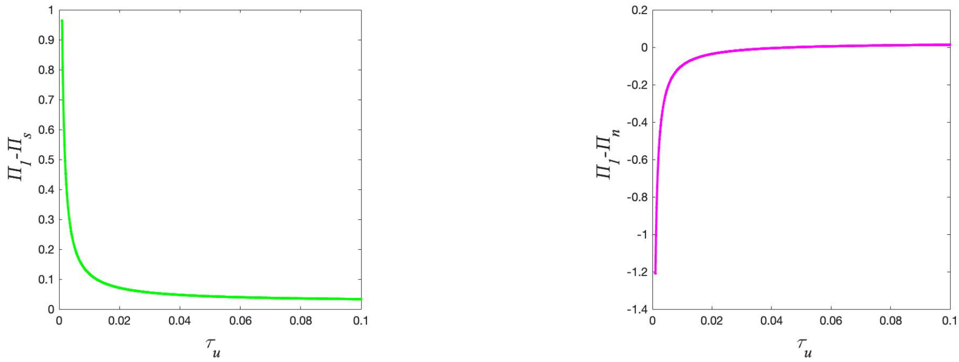
Next, we examine two special cases – individual investors' private signal is partially asymmetric and the naive individual investors have partial awareness of their price impact – under the general model comprising all three types of investors to show the robustness of our main theoretical results.

A.4.1 Partial information asymmetry

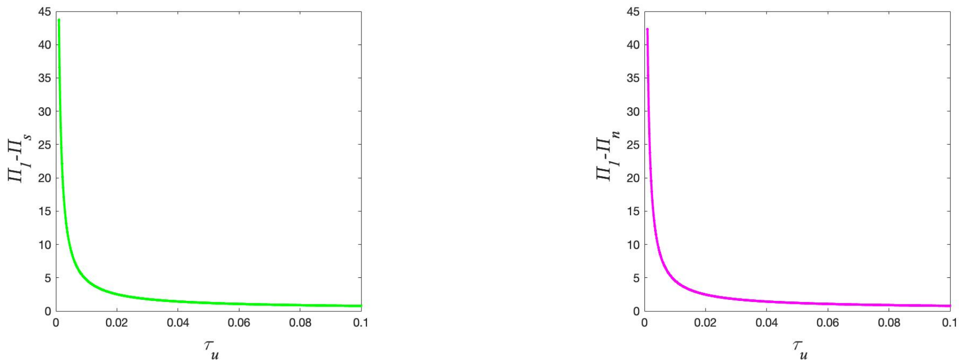
We first examine the case where the private signals of individual investors are asymmetric, i.e., we set $\tau_\epsilon^s > \tau_\epsilon^n > \tau_\epsilon^1$ and $\kappa = 0$ in the equilibrium system of the general model. Figures 13, 14, and 15 numerically illustrate the results, from which we can conclude that the main findings in Section 6 are robust for this extension.

A.4.2 Partial awareness of price impact

To verify the main findings in Section 6 regarding the partial awareness of naive individual investors about their own price impact, we set $\tau_\epsilon^s = \tau_\epsilon^n = \tau_\epsilon^1$ and $\kappa > 0$ in the equilibrium system of the general model, and present the simulation results in Figures 16, 17, and 18.

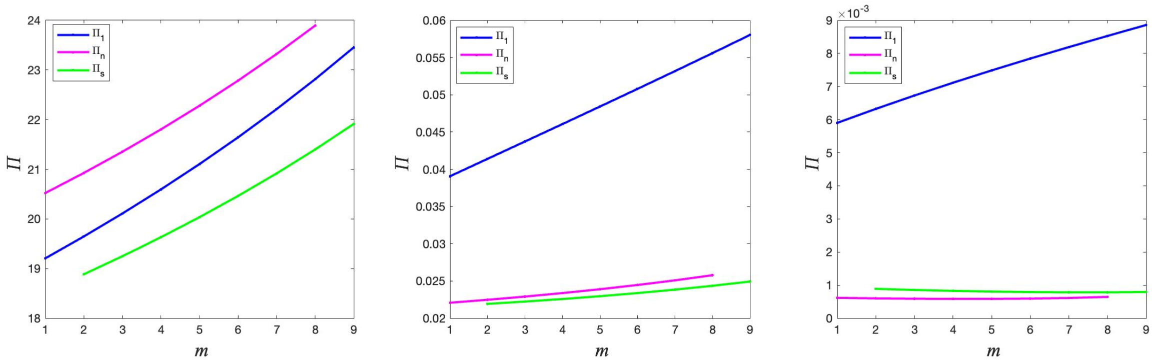


(a) Panel a

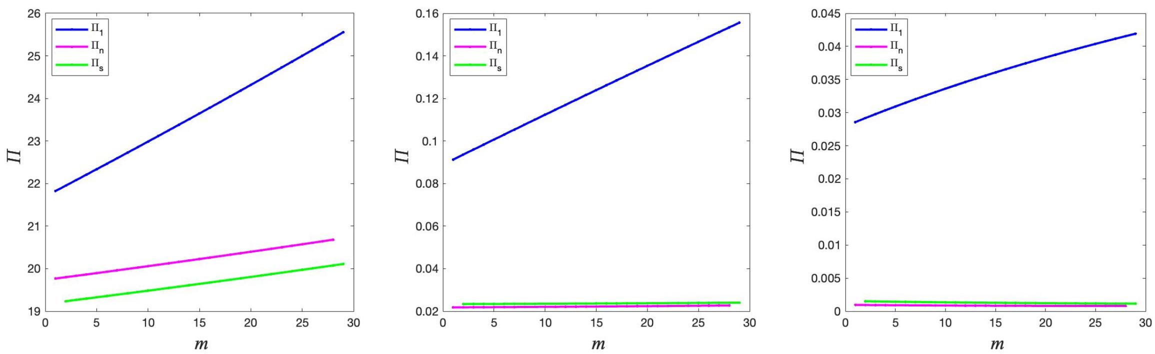


(b) Panel b

Figure 13: **Robustness check for heterogeneous signal precision.** Corresponding to Figure 5, the green line represents, when information is asymmetric, the difference between the trading profits of the institutional investor (Π_1) and those of the sophisticated individual investors (Π_s), while the pink line depicts the difference between the trading profits of the institutional investor (Π_1) and those of the naive individual investors (Π_n). The parameter values are set as $m = 4$, $n = 10$, $\kappa = 0$, $\rho = 2$, and τ_u ranges from 0.001 to 0.1. For other parameters, in Panel a, we set $\tau_\theta = 100$, $\tau_\epsilon^s = 1$, $\tau_\epsilon^n = 0.5$, and $\tau_\epsilon^1 = 0.2$, which approaches the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5. In Panel b, we set $\tau_\theta = 25$, $\tau_\epsilon^s = 5$, $\tau_\epsilon^n = 2.5$, and $\tau_\epsilon^1 = 0.5$, which deviates from the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5.

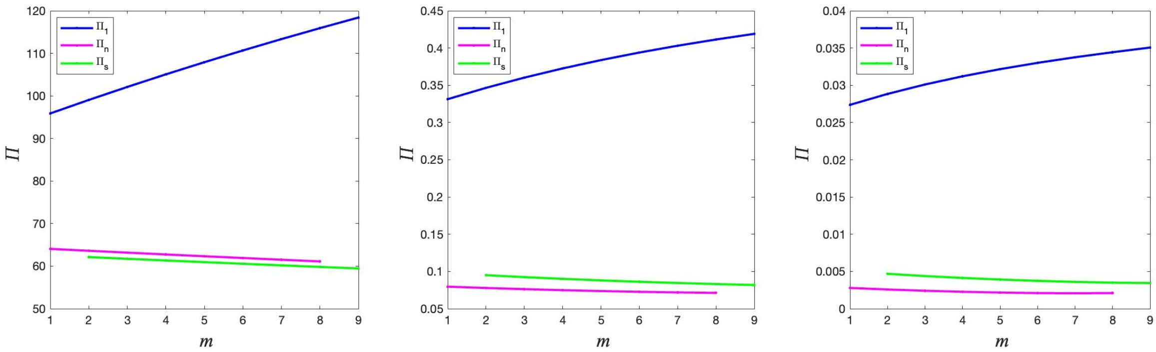


(a) Panel a

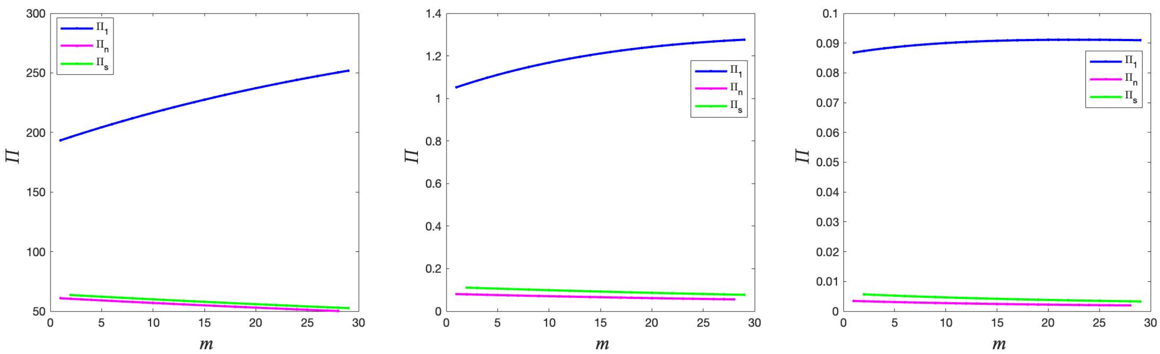


(b) Panel b

Figure 14: **Robustness check for heterogeneous signal precision.** Corresponding to Figures 6 and 11, Panel a illustrates, when information is asymmetric, the effect of m on the trading profits of the three types of investors for $n = 10$, and the values of τ_u from left to right are $0.001, 1$, and 100. Panel b demonstrates the effect of m on the trading profits of the three types of investors for $n = 30$, and the values of τ_u from left to right are 0.001, 1, and 100. The remaining parameter values are $\tau_\theta = 100, \tau_\epsilon^s = 1$, $\tau_\epsilon^n = 0.5$, $\tau_\epsilon^1 = 0.2$, $\kappa = 0$, and $\rho = 2$.

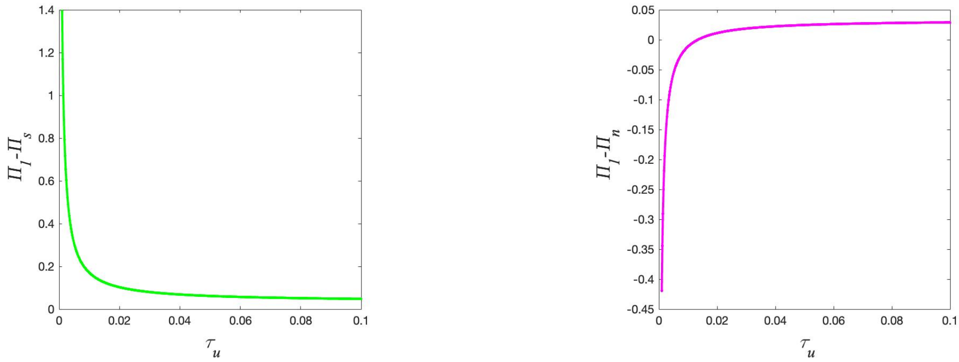


(a) Panel a

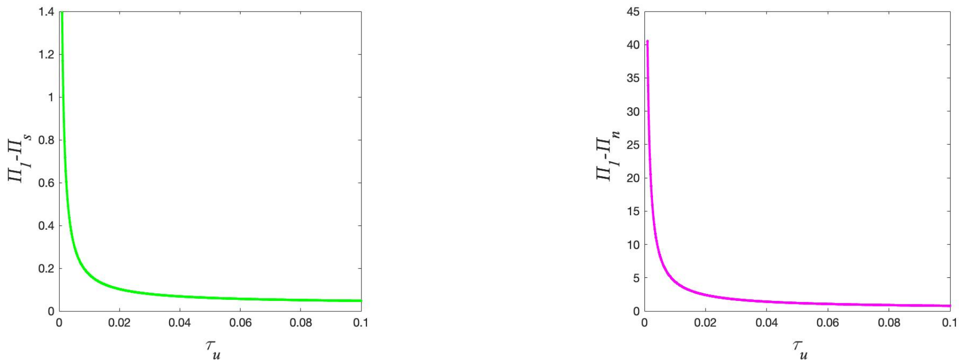


(b) Panel b

Figure 15: **Robustness check for heterogeneous signal precision.** Corresponding to Figures 12, Panel a illustrates, when information is asymmetric, the effect of m on the trading profits of the three types of investors for $n = 10$, and the values of τ_u from left to right are $0.001, 1$, and 100. Panel b demonstrates the effect of m on the trading profits of the three types of investors for $n = 30$, and the values of τ_u from left to right are $0.001, 1$, and 100. The remaining parameter values are $\tau_\theta = 25$, $\tau_\epsilon^s = 5$, $\tau_\epsilon^n = 2.5$, $\tau_\epsilon^1 = 0.5$, $\kappa = 0$, and $\rho = 2$.

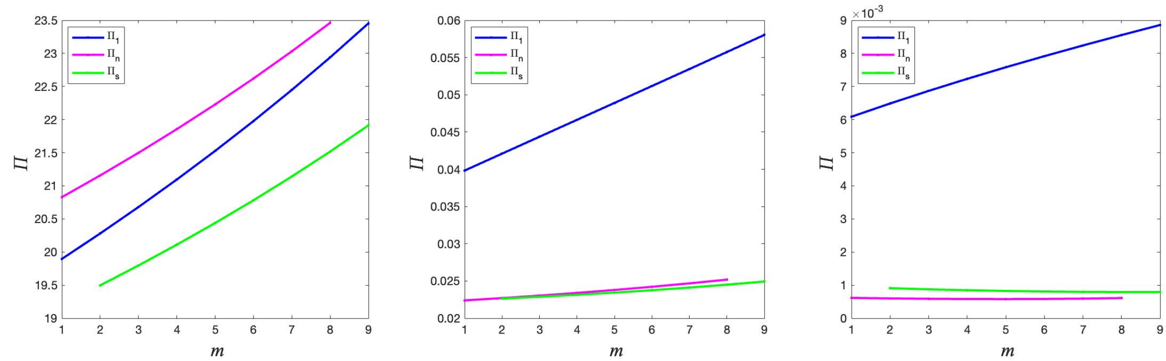


(a) Panel a

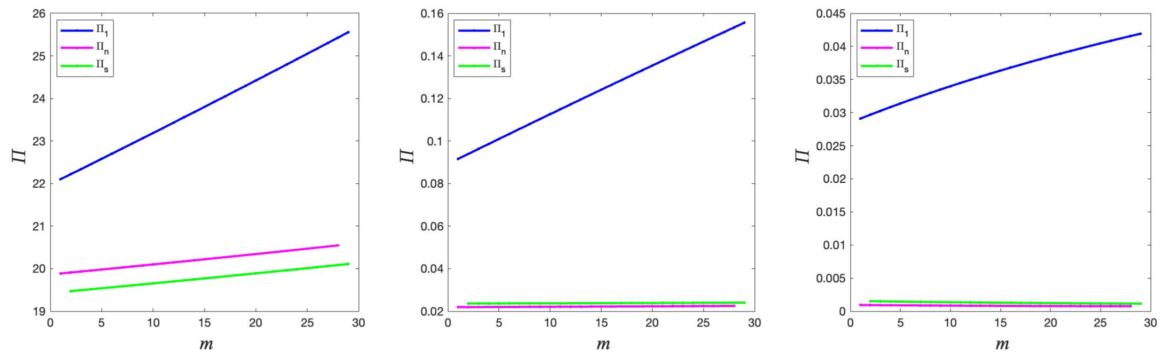


(b) Panel b

Figure 16: **Robustness check for partial awareness of price impact.** Corresponding to Figure 5, the green line represents, when naive investors have partial awareness of price impact, the difference between trading profits of the institutional investor (Π_1) and those of the sophisticated individual investors (Π_s), while the pink line depicts the difference between the trading profits of the institutional investor (Π_1) and those of the naive individual investors (Π_n). The parameter values are set as $m = 4$, $n = 10$, $\kappa = 0.2$, $\rho = 2$, and τ_u ranges from 0.001 to 0.1. For other parameters, in Panel a, we set $\tau_\theta = 100$, $\tau_\epsilon^s = \tau_\epsilon^n = \tau_\epsilon^1 = 1$, which approaches the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5. In Panel b, we set $\tau_\theta = 25$ and $\tau_\epsilon^s = \tau_\epsilon^n = \tau_\epsilon^1 = 5$, which deviates from the parameter condition $(n^2 - 4n + 2)\tau_\epsilon/\tau_\theta < 1$ in Proposition 5.

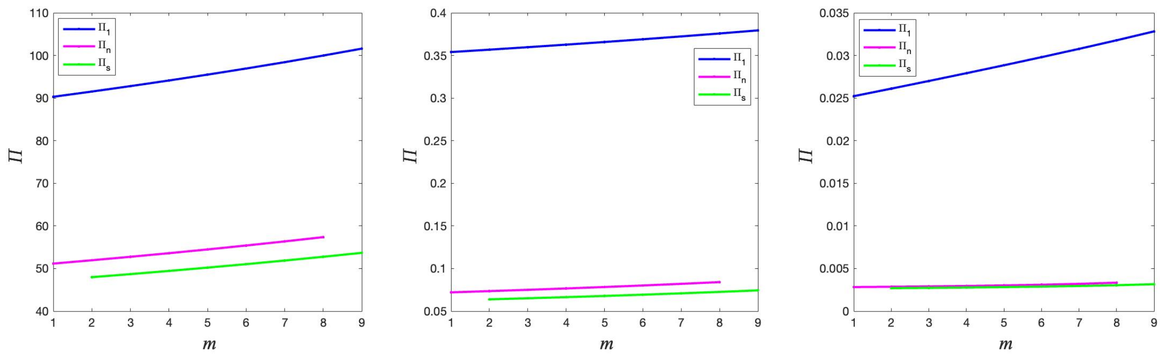


(a) Panel a

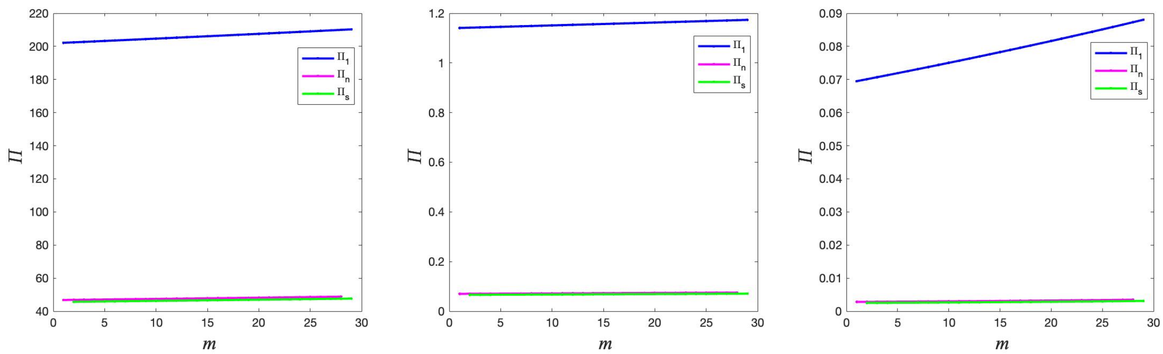


(b) Panel b

Figure 17: **Robustness check for partial awareness of price impact.** Corresponding to Figures 6 and 11, Panel a illustrates, when naive investors have partial awareness of price impact, the effect of m on the trading profits of the three types of investors for $n = 10$, and the values of τ_u from left to right are 0.001, 1, and 100. Panel b demonstrates the effect of m on the trading profits of the three types of investors for $n = 30$, and the values of τ_u from left to right are $0.001, 1$, and 100. The remaining parameter values are $\tau_\theta = 100, \tau_\epsilon^s = \tau_\epsilon^n = \tau_\epsilon^1 = 1$, $\kappa = 0.2$, and $\rho = 2$.



(a) Panel a



(b) Panel b

Figure 18: **Robustness check for partial awareness of price impact.** Corresponding to Figure 12, Panel a illustrates, when naive investors have partial awareness of price impact, the effect of m on the trading profits of the three types of investors for $n = 10$, and the values of τ_u from left to right are 0.001, 1, and 100. Panel b demonstrates the effect of m on the trading profits of the three types of investors for $n = 30$, and the values of τ_u from left to right are 0.001, 1, and 100. The remaining parameter values are $\tau_\theta = 25, \tau_\epsilon^s = \tau_\epsilon^n = \tau_\epsilon^1 = 5$, $\kappa = 0.2$, and $\rho = 2$.

B Proofs

Proof of Proposition 1

Note that while $\xi_1 = \rho\,\mathrm{Var}[\theta|y_1, y_2, ..., y_n, p] = \rho\,\mathrm{Var}[\theta|y_1, y_2, ..., y_n]$ is a constant determined solely by exogenous parameters (see (14)), ξ_s additionally depends on the endogenous parameter z_s (see (15) and (9) by setting $m = n-1$). To highlight this dependence, in the following proof, we will occasionally express ξ_s as $\xi_s(z_s)$.

From (24) and (25), we have

$$\frac{1}{\lambda_s} = \frac{(n-2)(1-\beta^s)}{\lambda_s + \xi_s} + \frac{1}{\left[\frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}\right]^{-1} + \xi_1},$$

which is equivalent to

$$\begin{aligned}
&[(2n-3)(1-\beta^s)-1]\,\lambda_s^2 \\
&\quad + [(n-2)(1-\beta^s)[(n-1)(1-\beta^s)\xi_1+\xi_s] + (n-1)(1-\beta^s)(\xi_s-\xi_1) - 2\xi_s]\,\lambda_s \\
&\quad - \left[(n-1)(1-\beta^s)\xi_s\xi_1 + \xi_s^2\right] = 0. \qquad (40)
\end{aligned}$$

Note that the discriminant of the quadratic equation (40) is non-negative:

$$\begin{aligned}
&[(n-2)(1-\beta^s)((n-1)(1-\beta^s)\xi_1+\xi_s) + (n-1)(1-\beta^s)(\xi_s-\xi_1) - 2\xi_s]^2 \\
&\qquad + 4\,[(2n-3)(1-\beta^s)-1]\left[(n-1)(1-\beta^s)\xi_s\xi_1+\xi_s^2\right] \\
&= [(n-2)(1-\beta^s)((n-1)(1-\beta^s)\xi_1+\xi_s) \\
&\qquad + (n-1)(1-\beta^s)(\xi_s-\xi_1)]^2 + 4(n-1)^2(1-\beta^s)^2\xi_s\xi_1 \geq 0.
\end{aligned}$$

We restrict $\beta^s \in (0,1)$ to ensure that λ_1 remains positive and well-defined. We first claim that (40) has a positive root λ_s if and only if $(2n-3)(1-\beta^s)-1 > 0$. When $(2n-3)(1-\beta^s)-1 > 0$, (40) has a unique positive root, whether the coefficient of λ_s is positive or negative. Conversely, if $(2n-3)(1-\beta^s)-1 \leq 0$, i.e., $1-\beta^s \leq \frac{1}{2n-3}$, the coefficient of λ_s in (40) equals

$$(n-1)(1-\beta^s)[(n-2)(1-\beta^s)-1]\xi_1 + [(2n-3)(1-\beta^s)-2]\xi_s,$$

which is negative, implying that (40) has no positive root. Denote the threshold value

$$\beta^+ := \frac{2n-4}{2n-3}.$$

Hence, to establish the existence of equilibrium, it suffices to restrict β^s to the interval $(0, \beta^+)$.

The proof proceeds as follows: we first express all relevant variables $-\lambda_1$, λ_s, ξ_s, z_s, and π_s- as functions of the variable β^s. These expressions are then substituted into the equation that β^s satisfies. Finally, we solve the resulting equation, which depends solely on the variable β^s, to demonstrate the existence of the equilibrium.

In the first step, we aim to express z_s as a function of the variable β^s. For a fixed value $\beta^s \in (0, \beta^+)$, we can uniquely determine $\lambda_s = \lambda_s(\beta^s; z_s)$ using (40), as discussed above. Note that $\lambda_s(\beta^s; z_s)$ depends on z_s because ξ_s itself is a function of z_s. Subsequently, we can get

$$\lambda_1 = \lambda_1(\beta^s; z_s) = \left[\frac{(n-1)(1-\beta^s)}{\lambda_s(\beta^s; z_s) + \xi_s(z_s)}\right]^{-1} \tag{41}$$

by (24). Substituting $\lambda_s = \lambda_s(\beta^s; z_s)$ and $\lambda_1 = \lambda_1(\beta^s; z_s)$ into (21), we obtain

$$\gamma = \gamma(\beta^s; z_s) = n\left[\frac{1}{\lambda_1(\beta^s; z_s) + \xi_1} + \frac{(n-1)(1-\beta^s)}{\lambda_s(\beta^s; z_s) + \xi_s(z_s)}\right]^{-1}. \tag{42}$$

From (23) and (26), and using the definition $z_s = \frac{\gamma}{(n-2)\pi_s}$, we have

$$\frac{n}{n-2} = z_s \left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1(\beta^s; z_s) + \xi_1)} + \frac{\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon} + \frac{(n-2)z_s^2}{\tau_u}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{z_s^2}{\tau_u}}}}{\lambda_s(\beta^s; z_s) + \xi_s(z_s)}\right]. \tag{43}$$

Observe that $\{\lambda_s(\beta^s; z_s)\}$ is uniformly bounded over $z_s \in (0, \infty)$, as (43) depends on z_s only through the terms ξ_1 and ξ_s, where ξ_1 is a constant and ξ_s satisfies the relation

$$\frac{\rho}{\tau_\theta + n\tau_\epsilon} \le \xi_s(z_s) \le \frac{\rho}{\tau_\theta + 2\tau_\epsilon}$$

for any $z_s > 0$ (see equation (2) and (15)). Consequently, $\{\lambda_1(\beta^s; z_s)\}$ is also uniformly bounded over $z_s \in (0, \infty)$. This uniform boundedness implies that the term on the right-hand side of (43) tends to infinity as $z_s \to \infty$ and to zero as $z_s \to 0$. Therefore, there exists a solution, denoted as $z_s(\beta^s)$, which is a function of the variable β^s, to the equation (43). From this solution, we immediately obtain the values of $\lambda_1(\beta^s; z_s(\beta^s))$ and $\lambda_s(\beta^s; z_s(\beta^s))$, and subsequently, the value of $\gamma(\beta^s, z_s(\beta^s))$ from (42). Additionally, we derive the value of $\pi_s(\beta^s)$ using the relation $\pi_s(\beta^s) = \frac{\gamma(\beta^s, z_s(\beta^s))}{(n-2)z_s(\beta^s)}$, where $\pi_s(\beta^s)$ and $z_s(\beta^s)$ are functions of β^s. Substituting $z_s(\beta^s)$ and $\pi_s(\beta^s)$ into the expression for β^s, we obtain an equation involving only β^s (along with other exogenous parameters):

$$\beta^s = \frac{\frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{(z_s(\beta^s))^2}{\tau_u}} \frac{1}{(n-2)\pi_s(\beta^s)}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{(z_s(\beta^s))^2}{\tau_u}}}. \tag{44}$$

To establish the existence of equilibrium, it suffices to show that equation (44) has a positive root within $(0, \beta^+)$. The proof relies on the intermediate value theorem by showing that the limit of the right-hand side of (44) exceeds (respectively, falls below) the left-hand side as $\beta^s \to 0$ (respectively, $\beta^s \to \beta^+$). First, consider the limit as $\beta^s \to 0$. In this case, (40) simplifies to

$$(2n-4)\lambda_s^2 + [(n-2)[(n-1)\xi_1 + \xi_s] + (n-1)(\xi_s - \xi_1) - 2\xi_s]\,\lambda_s - \left[(n-1)\xi_s\xi_1 + \xi_s^2\right] = 0,$$

which implies that λ_s, and consequently λ_1, γ, z_s, and π_s are bounded and bounded away from zero. This follows from (24), (21), (43), and the relation $\pi_s = \gamma/(z_s(n-2))$. Thus, the limit inferior of the right-hand side of (44) is strictly positive as $\beta^s \to 0$. Next, consider the limit of $\beta^s \to \beta^+ = \frac{2n-4}{2n-3}$. We first show that $\lambda_s \to \infty$ by contradiction. Otherwise, if $\{\lambda_s\}$ were bounded, then from (40), we would have

$$\lambda_s \to \frac{\frac{n-1}{2n-3}\xi_s\xi_1 + \xi_s^2}{\frac{n-2}{2n-3}\left(\frac{n-1}{2n-3}\xi_1 + \xi_s\right) + \frac{n-1}{2n-3}(\xi_s - \xi_1) - 2\xi_s} < 0,$$

which contradicts the fact that λ_s is a positive solution to (40). Therefore, $\lambda_s \to \infty$. We can further show that $\lambda_1 \to \infty$ by (24), $\gamma \to \infty$ by (21), and $z_s \to \infty$ by (43). Additionally, from

(23), (21) and (24), we derive

$$\pi_s = \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^s}{\lambda_s+\xi_s}}{\frac{1}{\lambda_1+\xi_1} + \frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}} = \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^s}{\lambda_1(n-1)(1-\beta^s)}}{\frac{1}{\lambda_1+\xi_1} + \frac{1}{\lambda_1}}$$
$$= \frac{\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^s}{(n-1)(1-\beta^s)}}{\frac{\lambda_1}{\lambda_1+\xi_1} + 1}, \tag{45}$$

which implies

$$\pi_s \to \frac{1}{2}\left(\frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon} + \frac{\tau_\epsilon}{(\tau_\theta + 2\tau_\epsilon)(n-1)(1-\beta^s)}\right),$$

where we use the limit $\alpha_o^s \to \frac{\tau_\epsilon}{\tau_\theta+2\tau_\epsilon}$. Consequently, the right-hand side of (44) tends to zero as $\beta^s \to \beta^+$. By the intermediate value theorem, Equation (44) must have a positive root $\beta^s \in (0, \beta^+)$. With this value of β^s, we can determine λ_s, λ_1, γ, z_s, and π_s. Finally, from (22), we obtain

$$\pi_1 = \frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{(n-1)\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}\frac{\pi_1}{\pi_s}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}}{\lambda_s+\xi_s}\right].$$

With the value of π_1, we get the value of π_1:

$$\pi_1 = \frac{\frac{\gamma}{n}\left[\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{(n-1)\tau_\epsilon}{(\lambda_s+\xi_s)\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\right)}\right]}{1+\frac{(n-1)(n-2)z_s}{n}\frac{\frac{\frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}}{\lambda_s+\xi_s}}.$$

The proof is completed. □

Proof of Proposition 2

Proof of (i). From (24) and (25), we have

$$\lambda_s = \left[\frac{n-2}{n-1}\frac{1}{\lambda_1} + \frac{1}{\lambda_1+\xi_1}\right]^{-1},$$

which implies that $\lambda_1 > \lambda_s$ if and only if

$$\frac{n-2}{n-1}\frac{1}{\lambda_1} + \frac{1}{\lambda_1+\xi_1} > \frac{1}{\lambda_1}.$$

This inequality simplifies to

$$(n-2)\lambda_1 > \xi_1.$$

Observe that $(n-2)\lambda_1 > \xi_1$ holds if and only if

$$\begin{aligned}
&(n-2)(\lambda_s+\xi_s) > (n-1)(1-\beta^s)\xi_1 \\
\Leftrightarrow &(n-2)(\lambda_s/\xi_s+1) > (n-1)(1-\beta^s)\xi_1/\xi_s \\
\Leftrightarrow &\frac{\lambda_s}{\xi_s} > \frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s} - 1.
\end{aligned}$$

This inequality is satisfied if

$$\frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s} - 1 \le 0,$$

or if $\frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s} - 1 > 0$ and (from (40))

$$\begin{aligned}
&[(2n-3)(1-\beta^s)-1]\left[\frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s} - 1\right]^2 \\
&+\left[(n-2)(1-\beta^s)\left((n-1)(1-\beta^s)\frac{\xi_1}{\xi_s}+1\right) + (n-1)(1-\beta^s)\left(1-\frac{\xi_1}{\xi_s}\right) - 2\right] \\
&\quad\times\left[\frac{(n-1)(1-\beta^s)}{n-2}\frac{\xi_1}{\xi_s} - 1\right] - \left[(n-1)(1-\beta^s)\frac{\xi_1}{\xi_s}+1\right] < 0.
\end{aligned}$$

After simplification, this inequality reduces to

$$(n-1)^3\xi_1^2(1-\beta^s) - \left[(n-1)\xi_1^2 + (n-2)(n-1)^2\xi_s\xi_1 + (n-2)(n-1)\xi_1^2\right] < 0,$$

or equivalently,

$$1-\beta^s < \frac{(n-1)\xi_1^2 + (n-1)(n-2)\xi_s\xi_1}{(n-1)^2\xi_1^2} = \frac{\xi_1+(n-2)\xi_s}{(n-1)\xi_1}.$$

In summary, we have shown that $\lambda_1 > \lambda_s$ if and only if

$$1 - \beta^s < \frac{\xi_1 + (n-2)\xi_s}{(n-1)\xi_1} = \frac{1 + (n-2)\xi_s/\xi_1}{n-1}.$$

Note that $\xi_1 \leq \xi_s$ (see equations (14) and (15)), which implies that $\lambda_1 > \lambda_s$.

Proof of (ii). From (24), we have

$$\lambda_1 + \xi_1 = \frac{\lambda_s + \xi_s}{(n-1)(1-\beta^s)} + \xi_1. \tag{46}$$

This implies that when $(n-1)(1-\beta^s) \leq 1$, it holds that $\lambda_1 + \xi_1 \geq \lambda_s + \xi_s$. Next, suppose $(n-1)(1-\beta^s) > 1$. In this case, $\lambda_1 + \xi_1 < \lambda_s + \xi_s$ if and only if

$$\lambda_s + \xi_s > \frac{\xi_1}{1 - \frac{1}{(n-1)(1-\beta^s)}}. \tag{47}$$

Note that (40) can be rewritten as

$$\begin{aligned}
&[(2n-3)(1-\beta^s) - 1](\lambda_s + \xi_s)^2 \\
&- \left[(n-1)(1-\beta^s)\xi_1 + (2n-3)(1-\beta^s)\xi_s - (n-1)(n-2)(1-\beta^s)^2\xi_1\right](\lambda_s + \xi_s) \\
&- (n-1)(n-2)(1-\beta^s)^2\xi_s\xi_1 = 0.
\end{aligned} \tag{48}$$

From (48), we deduce that (47) holds if and only if

$$\begin{aligned}
&[(2n-3)(1-\beta^s) - 1]\left(\frac{\xi_1}{1 - \frac{1}{(n-1)(1-\beta^s)}}\right)^2 \\
&- \left[(n-1)(1-\beta^s)\xi_1 + (2n-3)(1-\beta^s)\xi_s - (n-1)(n-2)(1-\beta^s)^2\xi_1\right]\frac{\xi_1}{1 - \frac{1}{(n-1)(1-\beta^s)}} \\
&- (n-1)(n-2)(1-\beta^s)^2\xi_s\xi_1 < 0.
\end{aligned}$$

After simplification, this inequality reduces to

$$(n-1)(n-2)(1-\beta^s)^2\xi_1 - \left[(n-1)(n-2)(1-\beta^s)^2 - \beta^s\right]\xi_s < 0.$$

The proof is completed. $\square$

Proof of Proposition 3

Proof of (i). To begin with, from (28), (29), (30) and (31), we observe that the institutional investor beats the sophisticated individual investors if and only if

$$\frac{\lambda_s + \xi_s}{\lambda_1 + \xi_1} > \frac{\Psi_s}{\Psi_1}. \tag{49}$$

From (43), we have

$$\begin{aligned} \frac{n}{n-2} &\le z_s \left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)\xi_1} + \frac{\tau_\epsilon}{(\tau_\theta + 2\tau_\epsilon)\xi_s(z_s)} \right] \\ &\le z_s \left[\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)\xi_1} + \frac{\tau_\epsilon(\tau_\theta + n\tau_\epsilon)}{(\tau_\theta + 2\tau_\epsilon)\rho} \right]. \end{aligned} \tag{50}$$

Combining this with the expression for α_o^s (see (26)), we deduce that

$$\alpha_o^s \to \frac{\tau_\epsilon}{\tau_\theta + 2\tau_\epsilon}$$

as $\tau_u \to 0$. Consequently, from (23) and (21), we obtain

$$\pi_s = \frac{\frac{\tau_\epsilon}{(\tau_\theta + n\tau_\epsilon)(\lambda_1 + \xi_1)} + \frac{\alpha_o^s}{\lambda_s + \xi_s}}{\frac{1}{\lambda_1 + \xi_1} + \frac{(n-1)(1-\beta^s)}{\lambda_s + \xi_s}} > \min\left\{ \frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon}, \frac{\tau_\epsilon}{2(\tau_\theta + 2\tau_\epsilon)(n-1)} \right\} > 0$$

for all sufficiently small τ_u. As a result, it follows from the expression of β^s that $\beta^s \to 0$ as $\tau_u \to 0$. Moreover, from (21), we have

$$\gamma \ge n \left(\frac{1}{\xi_1} + \frac{n-1}{\xi_s} \right)^{-1} \ge n \left[\frac{\tau_\theta + n\tau_\epsilon}{\rho} + \frac{(n-1)(\tau_\theta + n\tau_\epsilon)}{\rho} \right]^{-1}.$$

This implies that $\mathrm{Var}(\theta - p) \ge \gamma^2/\tau_u \to \infty$ as $\tau_u \to 0$. Thus, the term on the right-hand side of (49) tends to one as $\tau_u \to 0$.

Since we have shown that $\beta^s \to 0$, it follows from (48) and (24) that $\{\lambda_s + \xi_s\}$ and $\{\lambda_1 + \xi_1\}$ are bounded and bounded away from zero. Part (ii) of Proposition 2 indicates that the limit of $(\lambda_s+\xi_s)/(\lambda_1+\xi_1)$ is greater than one, as $\frac{1}{\tau_\theta+n\tau_\epsilon} < \frac{1}{\tau_\theta+2\tau_\epsilon}$, noting that $\xi_s \to \frac{\rho}{\tau_\theta+2\tau_\epsilon}$ as $\tau_u \to 0$ and $\xi_1 = \frac{\rho}{\tau_\theta+n\tau_\epsilon}$. Part (i) thus follows from (49).

Proof of (ii). We analyze the term on the left-hand side of (49). First, we show by contradiction that $(2n-3)(1-\beta^s) \to 1$, i.e., $\beta^s \to \beta^+ = \frac{2n-4}{2n-3}$ as $\tau_u \to \infty$. If this were not the case, it would follow from (40) that $\{\lambda_s\}$ is bounded, and consequently, $\{\lambda_1\}$ is also bounded and bounded away from zero by (24). This would imply that $\{z_s\}$ is bounded by (43). As a result, $\alpha_o^s \to 0$, and $\beta^s - \frac{\tau_\epsilon}{\pi_s(\tau_\theta+n\tau_\epsilon)} \to 0$ by (26) and (27), leading to

$$\beta^s - \frac{\frac{\lambda_1}{\lambda_1+\xi_1}+1}{\frac{\lambda_1}{\lambda_1+\xi_1}} \to 0$$

by (45). This creates a contradiction, as we have established that the equilibrium parameter β^s is less than one, i.e., $\beta^s \in (0, \beta^+)$. Therefore, $\beta^s \to \beta^+$ as $\tau_u \to \infty$.

As shown in Proposition 1, we can further show by contradiction that $\lambda_s \to \infty$, and $\lambda_1 \to \infty$. Considering these limits, along with (45) and (44), we conclude that $(n-2)^2 z_s^2/\tau_u$ converges to a finite positive number, denoted as $\hat{d}$, which satisfies the following equation:

$$\beta^+ = \frac{\frac{1}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}}{\tau_\theta+2\tau_\epsilon+\frac{n-2}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}} \frac{2}{\frac{\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}+\frac{1}{(n-1)(1-\beta^+)}\frac{\tau_\epsilon-\frac{1}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}}{\tau_\theta+2\tau_\epsilon+\frac{n-2}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{n-2}}}}.$$

From this and the relation $\beta^+ = \frac{2n-4}{2n-3}$, we get

$$\hat{d} = \frac{1}{\tau_\epsilon}\frac{(n-1)^2(\tau_\theta+n\tau_\epsilon)}{(3n-4)\tau_\theta+(2n^2-n-2)\tau_\epsilon}. \tag{51}$$

Next, from (46), we have

$$\begin{aligned}\frac{\lambda_s+\xi_s}{\lambda_1+\xi_1} = \frac{\lambda_s+\xi_s}{\frac{\lambda_s+\xi_s}{(n-1)(1-\beta^s)}+\xi_1} = \frac{1}{\frac{1}{(n-1)(1-\beta^s)}+\frac{\xi_1}{\lambda_s+\xi_s}} &\to (n-1)(1-\beta^+) \\ &= \frac{n-1}{2n-3},\end{aligned} \tag{52}$$

since we have shown that $\lambda_s \to \infty$ and $\beta^s \to \beta^+$ as $\tau_u \to \infty$. Given that the limit of the term on the left-hand side of (49) is a constant independent of any other parameters, the proof is completed by the following two steps.

Step one: Here, we demonstrate that the limit of the term on the right-hand side of (49) as $\tau_u \to \infty$ strictly decreases with τ_θ. To this end, we first derive the limits of π_s and π_1 as $\tau_u \to \infty$. We have already established that $\beta^s \to \beta^+$, $(n-2)^2 z_s^2/\tau_u \to \hat{d}$, and $\lambda_1 \to \infty$. From (45) and (51), it follows that

$$\begin{aligned}
\pi_s &= \frac{\frac{\tau_\epsilon \lambda_1}{(\tau_\theta + n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}}{(n-1)(1-\beta^s)}}{\frac{\lambda_1}{\lambda_1+\xi_1}+1} \\
&\to \frac{1}{2}\left[\frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon} + \frac{2n-3}{n-1}\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{\hat{d}}{(n-2)}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{\hat{d}}{(n-2)^2}}}\right] \\
&=: \hat{\pi}_o < \infty.
\end{aligned} \tag{53}$$

We also have

$$\begin{aligned}
\pi_1 &\overset{(22),(21)}{=} \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{(n-1)\alpha_1^s}{\lambda_s+\xi_s}}{\frac{1}{\lambda_1+\xi_1} + \frac{(n-1)(1-\beta^s)}{\lambda_s+\xi_s}} \\
&\overset{(24)}{=} \frac{\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_1^s}{1-\beta^s}}{\frac{\lambda_1}{\lambda_1+\xi_1}+1} \\
&\overset{(26)}{=} \frac{\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}}\frac{\pi_1}{\pi_s}}{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}}}{1-\beta^s}}{\frac{\lambda_1}{\lambda_1+\xi_1}+1},
\end{aligned}$$

which leads to

$$\pi_1 = \frac{\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)\left(\frac{\lambda_1}{\lambda_1+\xi_1}+1\right)} + \frac{\tau_\epsilon}{(1-\beta^s)\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\right)\left(\frac{\lambda_1}{\lambda_1+\xi_1}+1\right)}}{\frac{1}{\pi_s\left(\frac{1}{\tau_\epsilon}+\frac{(n-2)z_s^2}{\tau_u}\right)(1-\beta^s)\left(\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_s^2}{\tau_u}}\right)\left(\frac{\lambda_1}{\lambda_1+\xi_1}+1\right)}+1}$$

$$= \frac{\frac{\tau_\epsilon \lambda_1}{(\tau_\theta + n\tau_\epsilon)(\lambda_1+\xi_1)} \left(\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{z_s^2}{\tau_u}} \right) + \frac{\tau_\epsilon}{1-\beta^s}}{\frac{1}{\pi_s \left(\frac{1}{\tau_\epsilon} + \frac{(n-2)z_s^2}{\tau_u} \right)(1-\beta^s)} + \left(\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{z_s^2}{\tau_u}} \right) \left(\frac{\lambda_1}{\lambda_1+\xi_1} + 1 \right)}$$

$$\to \frac{\frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon} \left(\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{\hat{d}}{(n-2)^2}} \right) + \tau_\epsilon(2n-3)}{\frac{2(2n-3)}{\frac{1}{\tau_\epsilon} + \frac{\hat{d}}{n-2}} \left[\frac{\tau_\epsilon}{\tau_\theta + n\tau_\epsilon} + \frac{2n-3}{n-1} \frac{\tau_\epsilon - \frac{1}{\frac{1}{\tau_\epsilon} + \frac{\hat{d}}{n-2}}}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{\hat{d}}{(n-2)^2}}} \right]^{-1} + 2 \left(\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{\hat{d}}{(n-2)^2}} \right)}$$

$$=: \hat{\pi}_1 < \infty, \tag{54}$$

where the limit follows from $\beta^s \to \beta^+$, $(n-2)^2 z_s^2/\tau_u \to \hat{d}$, $\lambda_1 \to \infty$, and the limit of π_s given by (53).

Next, we show that the limit of the term on the right-hand side of (49) as $\tau_u \to \infty$ decreases with τ_θ. Using the limits of π_s and π_1 and $(n-2)^2 z_s^2/\tau_u$ from (53), (54), and (51), we obtain

$$\begin{aligned} \mathrm{Var}(\theta - p) &= \frac{(\pi_1 + (n-1)\pi_s - 1)^2}{\tau_\theta} + \frac{\pi_1^2 + (n-1)\pi_s^2}{\tau_\epsilon} + \frac{\gamma^2}{\tau_u} \\ &\to \frac{(\hat{\pi}_1 + (n-1)\hat{\pi}_s - 1)^2}{\tau_\theta} + \frac{\hat{\pi}_1^2 + (n-1)\hat{\pi}_s^2}{\tau_\epsilon} + \hat{\pi}_s^2 \hat{d} \\ &=: V_1(\tau_\theta), \end{aligned} \tag{55}$$

$$\begin{aligned} \mathrm{Var}[\theta | y_1, y_i, p] &= \frac{1}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{z_s^2}{\tau_u}}} \\ &\to \frac{1}{\tau_\theta + 2\tau_\epsilon + \frac{1}{\frac{1}{(n-2)\tau_\epsilon} + \frac{\hat{d}}{(n-2)^2}}} =: V_2(\tau_\theta), \end{aligned} \tag{56}$$

$$\mathrm{Var}[\theta | y_1, ..., y_n, p] = \mathrm{Var}[\theta | y_1, ..., y_n] = \frac{1}{\tau_\theta + n\tau_\epsilon} =: V_3(\tau_\theta) \tag{57}$$

as $\tau_u \to \infty$. Hence, the term on the right-hand side of (49)

$$\frac{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta | y_1, y_i, p]}{\mathrm{Var}(\theta - p) - \mathrm{Var}[\theta | y_1, y_2, ..., y_n]} \to \frac{V_1(\tau_\theta) - V_2(\tau_\theta)}{V_1(\tau_\theta) - V_3(\tau_\theta)}.$$

Using (55), (56), (57), and the definition of $\hat{d}$ (see (51)), with some calculations we can show that

$$\frac{\partial\left(\frac{V_1(\tau_\theta)-V_2(\tau_\theta)}{V_1(\tau_\theta)-V_3(\tau_\theta)}\right)}{\partial\tau_\theta}<0$$

is equivalent to

$$-\frac{4\tau_\epsilon(n-1)^3(n-2)^2}{(2n-3)(8\tau_\theta+6\tau_\epsilon-11n\tau_\theta-5n\tau_\epsilon+4n^2\tau_\theta-2n^2\tau_\epsilon+2n^3\tau_\epsilon)^2}<0,$$

which is indeed true. This completes the proof of this step.

Step two: Here, we demonstrate that the limit of the term on the right-hand side of (49) as $\tau_u\to\infty$ satisfies the relation (49) when $\tau_\theta\to 0$. First, we derive the following expressions

$$\begin{aligned}
&\mathrm{Var}(\theta-p)-\mathrm{Var}[\theta|y_1,y_i,p]=\mathbb{E}[(\theta-p)(\mathbb{E}[\theta|y_1,y_i,p]-p)]\\
&=\mathbb{E}\Bigg\{\left[(1-\pi_1-(n-1)\pi_s)\theta-\pi_1\epsilon_1-\pi_s\sum_{i=2}^{n}\epsilon_i-\gamma u\right]\\
&\qquad\times\Bigg[(\alpha_o^s+\alpha_1^s-(1-\beta^s)(\pi_1+(n-1)\pi_s))\theta+(\alpha_1^s-(1-\beta^s)\pi_1)\epsilon_1\\
&\qquad\quad+(\alpha_o^s-(1-\beta^s)\pi_s)\epsilon_i-\pi_s(1-\beta^s)\sum_{j\notin\{1,i\}}^{n}\epsilon_j-(1-\beta^s)\gamma u\Bigg]\Bigg\}\\
&=\left[(1-\pi_1-(n-1)\pi_s)(\alpha_o^s+\alpha_1^s-(1-\beta^s)(\pi_1+(n-1)\pi_s))\right]/\tau_\theta+(1-\beta^s)\gamma^2/\tau_u\\
&\qquad+\left[-\pi_1(\alpha_1^s-(1-\beta^s)\pi_1)-\pi_s(\alpha_o^s-(1-\beta^s)\pi_s)+\pi_s^2(1-\beta^s)(n-2)\right]/\tau_\epsilon,
\end{aligned}\tag{58}$$

and

$$\begin{aligned}
&\mathrm{Var}(\theta-p)-\mathrm{Var}[\theta|y_1,y_2,...,y_n]=\mathbb{E}[(\theta-p)(\mathbb{E}[\theta|y_1,...,y_n]-p)]\\
&=\mathbb{E}\Bigg\{\left[(1-\pi_1-(n-1)\pi_s)\theta-\pi_1\epsilon_1-\pi_s\sum_{i=2}^{n}\epsilon_i-\gamma u\right]\\
&\qquad\times\left[(nb\tau_\epsilon-(\pi_1+(n-1)\pi_s))\theta+(b\tau_\epsilon-\pi_1)\epsilon_1+(b\tau_\epsilon-\pi_s)\sum_{i=2}^{n}\epsilon_i-\gamma u\right]\Bigg\}\\
&=\left[(1-\pi_1-(n-1)\pi_s)(nb\tau_\epsilon-(\pi_1+(n-1)\pi_s))\right]/\tau_\theta\\
&\qquad-\left[\pi_1(b\tau_\epsilon-\pi_1)+\pi_s(b\tau_\epsilon-\pi_s)(n-1)\right]/\tau_\epsilon+\gamma^2/\tau_u,
\end{aligned}\tag{59}$$

where $b = \frac{1}{\tau_\theta + n\tau_\epsilon}$. Next, applying (52), (58), and (59), we find that

$$\frac{\lambda_s + \xi_s}{\lambda_1 + \xi_1}\mathbb{E}[(\theta - p)(\mathbb{E}[\theta|y_1, ..., y_n, p] - p)] - \mathbb{E}[(\theta - p)(\mathbb{E}[\theta|y_1, y_i, p] - p)]$$

tends to

$$\begin{aligned}
&(n-1)(1-\beta^+)\Big\{\Big[(1-\hat{\pi}_1-(n-1)\hat{\pi}_s)(nb\tau_\epsilon-(\hat{\pi}_1+(n-1)\hat{\pi}_s))\Big]/\tau_\theta \\
&\qquad\qquad - \Big[\hat{\pi}_1(b\tau_\epsilon-\hat{\pi}_1)+\hat{\pi}_s(b\tau_\epsilon-\hat{\pi}_s)(n-1)\Big]/\tau_\epsilon + \hat{d}\hat{\pi}_s^2\Big\} \\
&-\Big\{\Big[(1-\hat{\pi}_1-(n-1)\hat{\pi}_s)(\hat{\alpha}_o^s+\hat{\alpha}_1^s-(1-\beta^+)(\hat{\pi}_1+(n-1)\hat{\pi}_s))\Big]/\tau_\theta \\
&\quad + \Big[-\hat{\pi}_1(\hat{\alpha}_1^s-(1-\beta^+)\hat{\pi}_1)-\hat{\pi}_s(\hat{\alpha}_o^s-(1-\beta^+)\hat{\pi}_s)+\hat{\pi}_s^2(1-\beta^+)(n-2)\Big]/\tau_\epsilon + (1-\beta^+)\hat{d}\hat{\pi}_s^2\Big\} \\
&\propto (1-\hat{\pi}_1-(n-1)\hat{\pi}_s)[n(n-1)(1-\beta^+)b\tau_\epsilon-\hat{\alpha}_1^s-\hat{\alpha}_o^s-(\hat{\pi}_1+(n-1)\hat{\pi}_s)(1-\beta^+)(n-2)]\tau_\epsilon \\
&\quad - [\hat{\pi}_s^2(1-\beta^+)(n-2)+\hat{\pi}_1((n-1)(1-\beta^+)(b\tau_\epsilon-\hat{\pi}_1)-(\hat{\alpha}_1^s-(1-\beta^+)\hat{\pi}_1)) \\
&\qquad + \hat{\pi}_s((n-1)^2(1-\beta^+)(b\tau_\epsilon-\hat{\pi}_s)-(\hat{\alpha}_o^s-(1-\beta^+)\hat{\pi}_s))]\tau_\theta + (1-\beta^+)(n-2)\hat{d}\hat{\pi}_s^2\tau_\epsilon\tau_\theta. \qquad (60)
\end{aligned}$$

We define $\tilde{d}$, $\tilde{\pi}_s$, $\tilde{\pi}_1$, $\tilde{\alpha}_o^s$, and $\tilde{\alpha}_1^s$ as the limits of $\hat{d}$, $\hat{\pi}_s$, $\hat{\pi}_1$, $\hat{\alpha}_0^s$, and $\hat{\alpha}_1^s$ as $\tau_\theta \to 0$. Then, the term in (60) tends to

$$\begin{aligned}
&\big(n(n-1)(1-\beta^+)b\tau_\epsilon-\tilde{\alpha}_1^s-\tilde{\alpha}_o^s\big)+(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)^2(1-\beta^+)(n-2) \\
&\quad -(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)(n(n-1)(1-\beta^+)b\tau_\epsilon-\tilde{\alpha}_1^s-\tilde{\alpha}_o^s)-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)(1-\beta^+)(n-2) \\
&=\big(n(n-1)(1-\beta^+)b\tau_\epsilon-\tilde{\alpha}_1^s-\tilde{\alpha}_o^s\big)[1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)] \\
&\quad -(1-\beta^+)(n-2)(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)[1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s)]. \qquad (61)
\end{aligned}$$

From (22), (23), (21), and (24), we have

$$\begin{aligned}
\pi_1+(n-1)\pi_s = \frac{1}{\frac{\lambda_1}{\lambda_1+\xi_1}+1}\Bigg[&\frac{\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{(n-1)\alpha_1^s}{(n-1)(1-\beta^s)} \\
&+\frac{(n-1)\tau_\epsilon\lambda_1}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)}+\frac{(n-1)\alpha_o^s}{(n-1)(1-\beta^s)}\Bigg],
\end{aligned}$$

which implies

$$\tilde{\pi}_1 + (n-1)\tilde{\pi}_s = \frac{1}{2}\left(\frac{1}{n} + \frac{\tilde{\alpha}_1^s}{1-\beta^+} + \frac{n-1}{n} + \frac{\tilde{\alpha}_o^s}{1-\beta^+}\right) = \frac{1}{2}\left(1 + \frac{\tilde{\alpha}_o^s + \tilde{\alpha}_1^s}{1-\beta^+}\right).$$

Therefore, (61) can be rewritten as

$$\begin{aligned}
&(1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s))\left[(n-1)(1-\beta^+) - (\tilde{\alpha}_1^s+\tilde{\alpha}_o^s) - \frac{1}{2}(n-2)((1-\beta^+)+(\tilde{\alpha}_1^s+\tilde{\alpha}_o^s))\right] \\
&= (1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s))\left[\frac{n}{2}(1-\beta^+) - \frac{n}{2}(\tilde{\alpha}_1^s+\tilde{\alpha}_o^s)\right] \\
&= n(1-\beta^+)(1-(\tilde{\pi}_1+(n-1)\tilde{\pi}_s))^2 \geq 0,
\end{aligned}$$

which implies that the term in (61) is non-negative. The proof is completed. □

Proof of Proposition 4

The proof follows a similar approach to that of Proposition 1; therefore, we present only the proof outline and omit the detailed derivations.

To establish the existence of equilibrium, it suffices to demonstrate that the system of equilibrium equations admits a positive solution. Following the structure of (41), (42), and (43), and utilizing the relation $z_n = \frac{\gamma}{(n-2)\pi_n}$, we first express the key variables λ_1, γ, z_n, and π_n as functions of the variable β^n. Substituting these expressions into the equilibrium condition for β^n (analogous to (44)), we obtain a univariate equation in β^n. To establish the existence of a solution, we analyze the limiting cases as $\beta^n \to 0$ and $\beta^n \to 1$. By applying the intermediate value theorem for continuous functions, we show that the equation admits a positive solution. This, in turn, determines the equilibrium values of the other endogenous variables, confirming the existence of equilibrium. □

Proof of Proposition 5

Note that the institutional investor cannot beat the naive individual investors if and only if

$$\frac{\xi_n}{\lambda_1+\xi_1} \leq \frac{\Psi_n}{\Psi_1}. \tag{62}$$

Moreover, following a similar line of reasoning as in the proof of Proposition 3, we can show that $\alpha_o^n \to \frac{\tau_\epsilon}{\tau_\theta+2\tau_\epsilon}$ and $\beta^n \to 0$ (see (36) and (37)). Hence, from (34) and (33), we have

$$\pi_n = \frac{\frac{\tau_\epsilon}{(\tau_\theta+n\tau_\epsilon)(\lambda_1+\xi_1)} + \frac{\alpha_o^n}{\xi_n}}{\frac{1}{\lambda_1+\xi_1} + \frac{(n-1)(1-\beta^n)}{\xi_n}} > \min\left\{\frac{\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}, \frac{\tau_\epsilon}{2(\tau_\theta+2\tau_\epsilon)(n-1)}\right\} > 0$$

for all sufficiently small τ_u. As a result, we conclude that $\beta^n \to 0$ as $\tau_u \to 0$. Furthermore, from (33), we have

$$\gamma \geq n\left(\frac{1}{\xi_1} + \frac{n-1}{\xi_n}\right)^{-1} \geq n\left[\frac{\tau_\theta+n\tau_\epsilon}{\rho} + \frac{(n-1)(\tau_\theta+n\tau_\epsilon)}{\rho}\right]^{-1},$$

as $\tau_u \to 0$. Consequently, both Ψ_n and Ψ_1 tend to one as $\tau_u \to 0$.

For the left-hand side of (62), we obtain

$$\frac{\xi_n}{\lambda_1+\xi_1} = \frac{\xi_n}{\frac{\xi_n}{(n-1)(1-\beta^n)} + \xi_1} = \frac{1}{\frac{1}{(n-1)(1-\beta^n)} + \frac{\tau_\theta+2\tau_\epsilon+\frac{1}{\frac{1}{(n-2)\tau_\epsilon}+\frac{z_n^2}{\tau_u}}}{\tau_\theta+n\tau_\epsilon}} \to \frac{1}{\frac{1}{n-1} + \frac{\tau_\theta+2\tau_\epsilon}{\tau_\theta+n\tau_\epsilon}},$$

where the first equality follows from (35), and the limit follows from $\xi_n \to \frac{\rho}{\tau_\theta+2\tau_\epsilon}$. Note that similar to (50), $z_n \not\to 0$ as $\tau_u \to 0$, and $\xi_1 = \frac{\rho}{\tau_\theta+n\tau_\epsilon}$. Thus, $\frac{\xi_n}{\lambda_1+\xi_1}$ is smaller than one if

$$\frac{1}{n-1} + \frac{\tau_\theta+2\tau_\epsilon}{\tau_\theta+n\tau_\epsilon} > 1,$$

which is equivalent to

$$(n^2-4n+2)\tau_\epsilon/\tau_\theta < 1.$$

The conclusion follows immediately. This completes the proof. $\square$

Proof of Lemma 1

Since each investor has only prior information about the fundamental value and does not infer information from the price, the optimal demand functions for institutional investors ($l = 1, ..., k$)

and naive individual investors ($j = k+1, ..., n$) are respectively given by

$$x_l^* = \frac{\mathbb{E}(\theta) - p}{\lambda_1 + \xi_1} = -\frac{p}{\lambda_1 + \xi_1}, \tag{63}$$

$$x_j^* = \frac{\mathbb{E}(\theta) - p}{\xi_n} = -\frac{p}{\xi_n}, \tag{64}$$

where $\xi_1 = \xi_n = \rho \operatorname{Var}(\theta) = \rho/\tau_\theta$. From (63) and (64), the market-clearing condition $\sum_{l=1}^{k} x_l^* + \sum_{j=k+1}^{n} x_j^* + nu = 0$ simplifies to

$$-\frac{kp}{\lambda_1 + \xi_1} - \frac{(n-k)p}{\xi_n} + nu = 0,$$

which leads to the equilibrium price

$$p = n\left(\frac{k}{\lambda_1 + \xi_1} + \frac{n-k}{\xi_n}\right)^{-1} u =: \gamma u.$$

Furthermore, the price impact parameter satisfies

$$\lambda_1 = \left(\frac{k-1}{\lambda_1 + \xi_1} + \frac{n-k}{\xi_n}\right)^{-1}.$$

Then the expected trading profits for an institutional investor ($l = 1, ..., k$) are given by

$$\mathbb{E}[(\theta - p)x_l^*] \overset{(63)}{=} \frac{\mathbb{E}[(\theta - p)(\mathbb{E}(\theta) - p)]}{\lambda_1 + \xi_1} = \frac{\mathbb{E}[-(\theta - \gamma u)\gamma u]}{\lambda_1 + \xi_1} = \frac{\gamma^2}{(\lambda_1 + \xi_1)\tau_u}.$$

Similarly, the expected trading profits for a naive individual investor ($j = k+1, ..., n$) are given by

$$\mathbb{E}[(\theta - p)x_j^*] \overset{(64)}{=} \frac{\mathbb{E}[(\theta - p)(\mathbb{E}(\theta) - p)]}{\xi_n} = \frac{\mathbb{E}[-(\theta - \gamma u)\gamma u]}{\xi_n} = \frac{\gamma^2}{\xi_n \tau_u}.$$

This completes the proof. □

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